Saturn & his rings. Moon 5 days old.

Doggett Hinman & Co. Sc

Jupiter and his Moons

TELESCOPIC VIEW OF THE FULL MOON

F. E. Brasch
1919.

AN

INTRODUCTION

TO

ASTRONOMY;

DESIGNED AS A

TEXT BOOK

FOR THE

STUDENTS OF YALE COLLEGE.

BY DENISON OLMSTED, A. M.
PROFESSOR OF NATURAL PHILOSOPHY AND ASTRONOMY.

NEW YORK:
COLLINS, KEESE, & Co.
1839.

PREFACE.

Nearly all who have written Treatises on Astronomy, designed for young learners, appear to have erred in one of two ways; they have either disregarded demonstrative evidence, and relied on mere popular illustration, or they have exhibited the elements of the science in naked mathematical formulæ. The former are usually diffuse and superficial; the latter, technical and abstruse.

In the following Treatise, we have endeavored to unite the advantages of both methods. We have sought, first, to establish the great principles of astronomy on a mathematical basis; and, secondly, to render the study interesting and intelligible to the learner, by easy and familiar illustrations. We would not encourage any one to believe that he can enjoy a full view of the grand edifice of astronomy, while its noble foundations are hidden from his sight; nor would we assure him that he can contemplate the structure in its true magnificence, while its basement alone is within his field of vision. We would, therefore, that the student of astronomy should confine his attention neither to the exterior of the building, nor to the mere analytic investigation of its structure. We would desire that he should not only study it in models and diagrams, and mathematical formulæ, but should at the same time acquire a love of nature herself, and cultivate the habit of raising his views to the grand originals. Nor is the effort to form a clear conception of the motions and dimensions of the heavenly bodies, less favorable to the improvement of the intellectual powers, than the study of pure geometry.

But it is evidently possible to follow out all the intricacies of an analytical process, and to arrive at a full conviction of the great truths of astronomy, and yet know very little of nature. According to our experience, however, but few students in the course of a liberal education will feel satisfied with this. They do not need so much to be convinced that the assertions of astronomers are true, as they desire to know what the truths are, and how they were ascertained; and they will derive from the study of astronomy little of that moral and intellectual elevation which they had anticipated, unless they learn to look upon the heavens with new views, and a clear comprehension of their wonderful mechanism.

Much of the difficulty that usually attends the early progress of the astronomical student, arises from his being too soon introduced to the most perplexing part of the whole subject,—the planetary motions. In this work, the consideration of these is for the most part postponed until the learner has become familiar with the artificial circles of the sphere, and conversant with the celestial bodies. We then first take the most simple view possible of the planetary motions by contemplating them as they really are in nature, and afterwards proceed to the more difficult inquiry, why they appear as they do.

ANALYSIS.

DESIGNED AS A BASIS FOR REVIEW AND EXAMINATION.

Chapter V.—The Moon—Lunar Geography—Phases of the Moon—Her Revolutions.

Chapter VI.—Lunar Irregularities.

ERRATA.—Art. 84, line 6, for *subtracted* read *added*.
153, " 25, for *eastward* read *westward*.
218, " 18, for *or* read *on*.

On page 247, the place of *Vindemiatrix* is incorrectly stated. It should be, 15° east of Denebola, and 20° north of Spica, in the arm of *Virgo*.

☞ *Diagrams for public recitations.*

As many of the figures of this work are too complicated to be drawn on the black board at each recitation, we have found it very convenient to provide a set of permanent cards of pasteboard, on which the diagrams are inscribed on so large a scale, as to be distinctly visible in all parts of the lecture room. The letters may be either made with a pen, or better procured of the printer, and pasted on.

The cards are made by the bookbinder, and consist of a thick paper board about 18 by 14 inches, on each side of which a white sheet is pasted, with a neat finish around the edges. A loop attached to the top is convenient for hanging the card on a [redacted]

INTRODUCTION TO ASTRONOMY.

PRELIMINARY OBSERVATIONS.

1. ASTRONOMY *is that science which treats of the heavenly bodies.* More particularly, its object is to teach what is known respecting the Sun, Moon, Planets, Comets, and Fixed Stars; and also to explain the methods by which this knowledge is acquired. Astronomy is sometimes divided into Descriptive, Physical, and Practical. Descriptive Astronomy respects *facts;* Physical Astronomy, *causes;* Practical Astronomy, the *means of investigating the facts*, whether by instruments, or by calculation. It is the province of Descriptive Astronomy to observe, classify, and record, all the phenomena of the heavenly bodies, whether pertaining to those bodies individually, or resulting from their motions and mutual relations. It is the part of Physical Astronomy to explain the causes of these phenomena, by investigating and applying the general laws on which they depend; especially by tracing out all the consequences of the law of universal gravitation. Practical Astronomy lends its aid to both the other departments.

2. Astronomy is the most ancient of all the sciences. At a period of very high antiquity, it was cultivated in Egypt, in Chaldea, in China, and in India. Such knowledge of the heavenly bodies as could be acquired by close and long continued observation, without the aid of instruments, was diligently amassed; and tables of the celestial motions were constructed, which could be used in predicting eclipses, and other astronomical phenomena.

About 500 years before the Christian era, *Pythagoras*, of Greece, taught astronomy at the celebrated school at Crotona, and exhibited more correct views of the nature of the celestial motions, than were entertained by any other astronomer of the ancient world. His views, however, were not generally adopted,

but lay neglected for nearly 2000 years, when they were revived and established by Copernicus and Galileo. The most celebrated astronomical school of antiquity, was at Alexandria in Egypt, which was established and sustained by the Ptolemies, (Egyptian princes,) about 300 years before the Christian era. The employment of instruments for measuring angles, and bringing in trigonometrical calculations to aid the naked powers of observation, gave to the Alexandrian astronomers great advantages over all their predecessors. The most able astronomer of the Alexandrian school was ***Hipparchus***, who was distinguished above all the ancients for the accuracy of his astronomical measurements and determinations. The knowledge of astronomy possessed by the Alexandrian school, and recorded in the ***Almagest***, or great work of ***Ptolemy***, constituted the chief of what was known of our science during the middle ages, until the fifteenth and sixteenth centuries, when the labors of ***Copernicus*** of Prussia, ***Tycho Brahe*** of Denmark, ***Kepler*** of Germany, and ***Galileo*** of Italy, laid the solid foundations of modern astronomy. Copernicus expounded the true theory of the celestial motions; Tycho Brahe carried the use of instruments and the art of astronomical observation to a far higher degree of accuracy than had ever been done before; Kepler discovered the great laws of physical astronomy; and Galileo, having first enjoyed the aid of the telescope, made innumerable discoveries in the solar system. Near the beginning of the eighteenth century, ***Sir Isaac Newton*** discovered, in the law [redacted]

4. Astronomers of every age, have been distinguished for their persevering industry, and their great love of accuracy. They have uniformly aspired to an exactness in their inquiries, far beyond what is aimed at in most geographical investigations, satisfied with nothing short of numerical accuracy wherever this is attainable; and years of toilsome observation, or laborious calculation, have been spent with the hope of attaining a few seconds nearer to the truth. Moreover, a severe but delightful labor is imposed on all, who would arrive at a clear and satisfactory knowledge of the subject of astronomy. Diagrams, artificial globes, orreries, and familiar comparisons and illustrations, proposed by the author or the instructor, may afford essential aid to the learner, but nothing can convey to him a perfect comprehension of the celestial motions, without much diligent study and reflection.

5. In expounding the doctrines of astronomy, we do not, as in Geometry, claim that every thing shall be proved as soon as asserted. We may first put the learner in possession of the leading facts of the science, and afterwards explain to him the methods by which those facts were discovered, and by which they may be verified; we may assume the principles of the true system of the world, and employ those principles in the explanation of many subordinate phenomena, while we reserve the discussion of the merits of the system itself, until the learner is extensively acquainted with astronomical facts, and therefore better able to appreciate the evidence by which the system is established.

6. The *Copernican system* is that which is held to be the true system of the world. It maintains (1,) That the *apparent* diurnal revolution of the heavenly bodies, from east to west, is owing to the *real* revolution of the earth on its own axis from west to east, in the same time; and (2,) That the sun is the center around which the earth and planets all revolve from west to east, contrary to the opinion that the earth is the center of motion of the sun and planets.

7. We shall treat, first, of the Earth in its astronomical relations; secondly, of the Solar System; thirdly, of the Fixed Stars; and fourthly, of Astronomical Observations and Calculations.

PART I.—OF THE EARTH.

CHAPTER I.

OF THE FIGURE AND DIMENSIONS OF THE EARTH, AND THE DOCTRINE OF THE SPHERE.

8. *The figure of the earth is nearly globular.* This fact is known, first, by the circular form of its shadow cast upon the moon in a lunar eclipse; secondly, from analogy, each of the other planets being seen to be spherical; thirdly, by our seeing the tops of distant objects while the other parts are invisible, as the topmast of a ship, while either leaving or approaching the shore, or the lantern of a light-house, which when first descried at a distance at sea, appears to glimmer upon the very surface of the water; fourthly, by the depression or *dip of the horizon* when the spectator is on an eminence; and, finally, by actual observations and measurements, made for the express purpose of ascertaining the figure of the earth, by means of which astronomers are enabled to compute the distances from the center of the earth of various places on its surface, which distances are found to be nearly equal.

center of the earth, DAE the portion of the earth's surface seen from O; OD, OE, lines drawn from the place of the spectator to the most distant parts of the horizon, and CD a radius of the earth. The dip of the horizon is the angle HOD or ROE. Now the angle made between the direction of the plumb line and that of the extreme line of the horizon or the surface of the sea, namely, the angle ZOD, can be easily measured; and subtracting the right angle ZOH from ZOD, the remainder is the dip of the horizon, from which the length of the line OD may be calculated, the height of the spectator, that is, the line OA, being known. This length, to whatever point of the horizon the line is drawn, is always found to be the same; and hence it is inferred, that the boundary which limits the view on all sides, is a circle. Moreover, at whatever elevation the dip of the horizon is taken, in any part of the earth, the space seen by the spectator is always circular. Hence the surface of the earth is spherical.

10. The earth being a sphere, the dip of the horizon HOD= OCD. Therefore, to find the dip of the horizon corresponding to any given height AO, (the diameter of the earth being known,) we have in the triangle OCD, the right angle at D, and the two sides CD, CO, to find the angle OCD. Therefore,

CO : rad. : : CD : cos. OCD. Learning the dip corresponding to different altitudes, by giving to the line AO different values, we may arrange the results in a table.

Table showing the Dip of the Horizon at different elevations, from 1 foot to 100 feet.

Feet.	′ ″	Feet.	′ ″	Feet.	′ ″
1	0.59	13	3.33	26	5.01
2	1.24	14	3.41	28	5.13
3	1.42	15	3.49	30	5.23
4	1.58	16	3.56	35	5.49
5	2.12	17	4.03	40	6.14
6	2.25	18	4.11	45	6.36
7	2.36	19	4.17	50	6.58
8	2.47	20	4.24	60	7.37
9	2.57	21	4.31	70	8.14
10	3.07	22	4.37	80	8.48
11	3.16	23	4.43	90	9.20
12	3.25	24	4.49	100	9.51

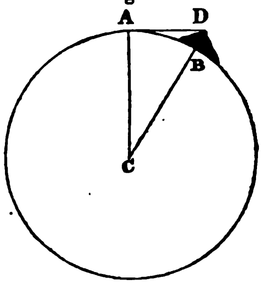
Such a table is of use in estimating the altitude of a body above the horizon, when the instrument (as usually happens) is more or less elevated above the general level of the earth. For if it be a star whose altitude above the horizon is required, the instrument being situated at O, (Fig. 1,) the inquiry is how far the star is elevated above the level HOR, but the angle taken is that above the visible horizon OD. The dip, therefore, or the angle HOD, corresponding to the height of the point O, must be subtracted, to obtain the true altitude. On the Peak of Teneriffe, Humboldt observed the surface of the sea to be depressed on all sides nearly 2 degrees. The sun arose to him 12 minutes sooner than to an inhabitant of the plain; and from the plain, the top of the mountain appeared enlightened 12 minutes before the rising or after the setting of the sun.

11. The foregoing considerations show that the form of the earth is spherical; but more exact determinations prove, that the earth, though nearly globular, is not exactly so: its diameter from the north to the south pole is about 26 miles less than through the equator, giving to the earth the form of an oblate spheroid,* or a flattened sphere resembling an orange. We shall reserve the explanations of the methods by which this fact is established, until the learner is better prepared than at present to understand them.

[redacted]

volume of the globe, as hardly to occasion any sensible deviation from a surface uniformly curvilinear. The irregularities of the earth's surface, in this view, are no greater than the rough points on the rind of an orange, which do not perceptibly interrupt its continuity; for the highest mountain on the globe is only about five miles above the general level; and the deepest mine hitherto opened is only about half a mile.* Now $\frac{5}{7912}=\frac{1}{1582}$, or about one sixteen hundredth part of the whole diameter, an inequality which, in an artificial globe of eighteen inches diameter, amounts to only the eighty eighth part of an inch.

Fig. 2.



13. The diameter of the earth, *considered as a perfect sphere*, may be determined by means of observations on a mountain of known elevation, seen in the horizon from the sea. Let BD (Fig. 2,) be a mountain of known height a, whose top is seen in the horizon by a spectator at A, b miles from it. Let x denote the radius of the earth. Then $x^2+b^2=(x+a)^2=x^2+2ax+a^2$. Hence, $2ax=b^2-a^2$, and $x=\frac{b^2-a^2}{2a}$. For example, suppose the height of the mountain is just one mile; then it will be found, by observation, to be visible on the horizon at the distance of 89 miles$=b$. Hence, $\frac{b^2-a^2}{2a}=\frac{(89)^2-1}{2}=\frac{7921-1}{2}=3960=$ radius of the earth, and $7920=$the earth's diameter.

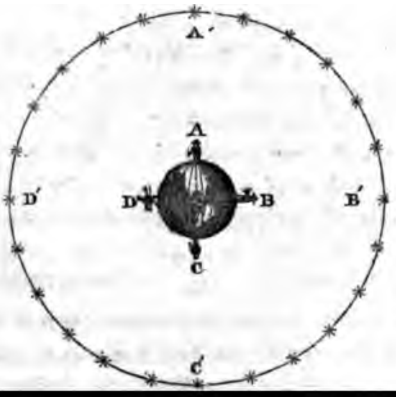
14. Another method, and the most ancient, is to ascertain the distance on the surface of the earth, corresponding to a degree of latitude. Let us select two convenient places, one lying directly north of the other, whose difference of latitude is known. Suppose this difference to be $1^\circ\ 30'$, and the distance between the two places, as measured by a chain, to be 104 miles. Then, since there are 360 degrees of latitude in the entire circumference, $1^\circ\ 30' : 104 :: 360^\circ : 24960$. And $\frac{24960}{3.1416}=7944$.

* Sir John Herschel.

The foregoing *approximations* are sufficient to show that the earth is about 8,000 miles in diameter.

15. The greatest difficulty in the way of acquiring correct views in astronomy, arises from the erroneous notions that preoccupy the mind. To divest himself of these, the learner should conceive of the earth as a huge globe occupying a small portion of space, and encircled on all sides with the starry sphere. He should free his mind from its habitual proneness to consider one part of space as naturally *up* and another *down*, and view himself as subject to a force which binds him to the earth as truly as though he were fastened to it by some invisible cords or wires, as the needle attaches itself to all sides of a spherical loadstone. He

Fig. 3.



17. A section of a sphere by a plane cutting it in any manner, is a circle. *Great circles* are those which pass through the center of the sphere, and divide it into two equal hemispheres: *Small circles*, are such as do not pass through the center, but divide the sphere into two unequal parts. Every circle, whether great or small, is divided into 360 equal parts called *degrees*. A degree, therefore, is not any fixed or definite quantity, but only a certain aliquot part of any circle.

18. The *axis* of a circle, is a straight line passing through its center at right angles to its plane.

19. The *pole* of a great circle, is the point on the sphere where its axis cuts through the sphere. Every great circle has two poles, each of which is every where 90° from the great circle. For, the measure of an angle at the center of a sphere, is the arc of a great circle intercepted between the two lines that contain the angle; and, since the angle made by the axis and any radius of the circle is a right angle, consequently its measure on the sphere, namely, the distance from the pole to the circumference of the circle, must be 90°. If two great circles cut each other at right angles, the poles of each circle lie in the circumference of the other circle. For each circle passes through the axis of the other.

20. All great circles of the sphere cut each other in two points diametrically opposite, and consequently, their points of section are 180° apart. For the line of common section, is a diameter in both circles, and therefore bisects both.

21. A great circle which passes through the pole of another great circle, cuts the latter at right angles. For, since it passes through the pole and the center of the circle, it must pass through the axis; which being at right angles to the plane of the circle, every plane which passes through it is at right angles to the same plane.

The great circle which passes through the pole of another great circle and is at right angles to it, is called a *secondary* to that circle.

22. The angle made by two great circles on the surface of the sphere, is measured by the arc of another great circle, of which the angular point is the pole, being the arc of that great circle intercepted between those two circles. For this arc is the measure of the angle formed at the center of the sphere by two radii, drawn at right angles to the line of common section of the two circles, one in one plane and the other in the other, which angle is therefore that of the inclination of those planes.

23. In order to fix the position of any plane, either on the surface of the earth or in the heavens, both the earth and the heavens are conceived to be divided into separate portions by circles, which are imagined to cut through them in various ways. The earth thus intersected is called the *terrestrial*, and the heavens the *celestial* sphere. The learner will remark, that these circles have no existence in nature, but are mere landmarks, artificially contrived for convenience of reference. On account of the immense distance of the heavenly bodies, they appear to us, wherever we are placed, to be fixed in the same concave surface, or celestial vault. The great circles of the globe, extended every way to meet the concave surface of the heavens, become circles of the celestial sphere.

24. The *Horizon* is the great circle which divides the earth into upper and lower hemispheres, and separates the visible heav-

[redacted]

rectly under our feet. The plumb line is in the axis of the horizon, and consequently directed towards its poles.

Every place on the surface of the earth has its own horizon; and the traveller has a new horizon at every step, always extending 90 degrees from him in all directions.

26. *Vertical circles* are those which pass through the poles of the horizon, perpendicular to it.

The *Meridian* is that vertical circle which passes through the north and south points.

The *Prime Vertical*, is that vertical circle which passes through the east and west points.

27. As in geometry, we determine the position of any point by means of rectangular coordinates, or perpendiculars drawn from the point to planes at right angles to each other, so in astronomy we ascertain the place of a body, as a fixed star, by taking its angular distance from two great circles, one of which is perpendicular to the other. The horizon and the meridian, or the horizon and the prime vertical, are coordinate circles used for such measurements.

The *Altitude* of a body, is its elevation above the horizon, measured on a vertical circle.

The *Azimuth* of a body, is its distance measured on the horizon from the meridian to a vertical circle passing through the body.

The *Amplitude* of a body, is its distance on the horizon, from the prime vertical, to a vertical circle passing through the body.

Azimuth is reckoned 90° from either the north or south point; and amplitude 90° from either the east or west point. Azimuth and amplitude are mutually complements of each other. When a point is *on* the horizon, it is only necessary to count the number of degrees of the horizon between that point and the meridian, in order to find its azimuth; but if the point is *above* the horizon, then its azimuth is estimated by passing a vertical circle through it, and reckoning the azimuth from the point where this circle cuts the horizon.

The *Zenith Distance* of a body is measured on a vertical circle, passing through that body. It is the complement of the altitude.

28. The *Axis of the Earth* is the diameter, on which the earth is conceived to turn in its diurnal revolution. The same line continued until it meets the starry concave, constitutes the *axis of the celestial sphere.*

The *Poles of the Earth* are the extremities of the earth's axis: *the Poles of the Heavens*, the extremities of the celestial axis.

29. The *Equator* is a great circle cutting the axis of the earth at right angles. Hence the axis of the earth is the axis of the equator, and its poles are the poles of the equator. The intersection of the plane of the equator with the surface of the earth, constitutes the *terrestrial*, and with the concave sphere of the heavens, the *celestial* equator. The latter, by way of distinction, is sometimes denominated the *equinoctial.*

30. The secondaries to the equator, that is, the great circles passing through the poles of the equator, are called *Meridians*, because that secondary which passes through the zenith of any place is the meridian of that place, and is at right angles both to the equator and the horizon, passing as it does through the poles of both. These secondaries are also called *Hour Circles*, because the arcs of the equator intercepted between them are used as measures of time.

31. The *Latitude* of a place on the earth, is its distance from the equator north or south. The *Polar Distance*, or angular

[redacted]

33. The *Ecliptic* is a great circle in which the earth performs its annual revolution around the sun. It passes through the center of the earth and the center of the sun. It is found by observation that the earth does not lie with its axis at right angles to the plane of the ecliptic, but that it is turned about 23½ degrees out of a perpendicular direction, making an angle with the plane itself of 66½°. The equator, therefore, must be turned the same distance out of a coincidence with the ecliptic, the two circles making an angle with each other of 23½°. It is particularly important for the learner to form correct ideas of the ecliptic, and of its relations to the equator, since to these two circles a great number of astronomical measurements and phenomena are referred.

34. The *Equinoctial Points*, or *Equinoxes*,* are the intersections of the ecliptic and equator. The time when the sun crosses the equator in returning northward is called the *vernal*, and in going southward, the *autumnal* equinox. The vernal equinox occurs about the 21st of March, and the autumnal the 22d of September.

35. The *Solstitial Points* are the two points of the ecliptic most distant from the equator. The times when the sun comes to them are called *solstices*. The summer solstice occurs about the 22d of June, and the winter solstice about the 22d of December.

The ecliptic is divided into twelve equal parts of 30° each, called *signs*, which, beginning at the vernal equinox, succeed each other in the following order:

Northern.		*Southern.*	
1. Aries	♈	7. Libra	♎
2. Taurus	♉	8. Scorpio	♏
3. Gemini	♊	9. Sagittarius	♐
4. Cancer	♋	10. Capricornus	♑
5. Leo	♌	11. Aquarius	♒
6. Virgo	♍	12. Pisces	♓

* The term Equinoxes strictly denotes the *times* when the sun arrives at the equinoctial points, but it is also frequently used to denote those points themselves.

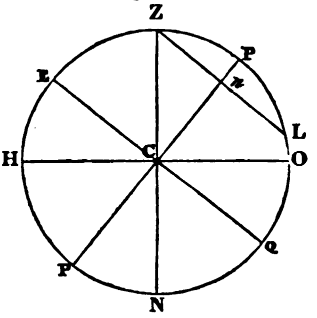
The mode of reckoning on the ecliptic, is by signs, degrees, minutes, and seconds. The sign is denoted either by its name or its number. Thus 100° may be expressed either as the 10th degree of Cancer, or as 3^s 10°.

36. Of the various meridians, two are distinguished by the name of *Colures*. The *Equinoctial Colure*, is the meridian which passes through the equinoctial points. From this meridian, right ascension and celestial longitude are reckoned, as longitude on the earth is reckoned from the meridian of Greenwich. The *Solstitial Colure*, is the meridian which passes through the solstitial points. As the solstitial points are 90° from the equinoctial points, so the solstitial colure is 90° from the equinoctial colure. It is also at right angles, or a secondary, to both the ecliptic and equator. For, like every other meridian, it is of course perpendicular to the equator, passing through its poles. Moreover, the equinox, being a point both in the equator and in the ecliptic, is 90° from the solstice, from the pole of the equator, and from the pole of the ecliptic. Hence the solstitial colure, which passes through the solstice and the pole of the equator, passes also through the pole of the ecliptic, being the great circle of which the equinox itself is the pole. Consequently, the solstitial colure is a secondary to both the equator and the ecliptic. (See Arts. 19, 20, 21.)

[redacted]

reckoned from the meridian of Greenwich. On the other hand, celestial longitude and latitude are referred, not to the equator, but to the ecliptic. *Celestial Longitude*, is the distance of a body from the vernal equinox reckoned on the ecliptic. *Celestial Latitude*, is distance from the ecliptic measured on a secondary to the latter. Or, more briefly, Longitude is distance *on* the ecliptic; Latitude, distance *from* the ecliptic. The *North Polar Distance* of a star, is the complement of its declination.

Fig. 4.



38. *Parallels of Latitude* are small circles parallel to the equator. They constantly diminish in size as we go from the equator to the pole, the radius being always equal to the cosine of the latitude. In fig. 4, let HO be the horizon, EQ the equator, PP the axis of the earth, ZN the prime vertical, and ZL a parallel of latitude of any place Z. Then ZE is the latitude, (Art. 31.) and ZP the complement of the latitude; but Z*n* the radius of the parallel of latitude ZL, is the sine of ZP, and therefore the cosine of the latitude.

39. The *Tropics* are the parallels of latitude that pass through the solstices. The northern tropic is called the tropic of Cancer; the southern, the tropic of Capricorn.

40. The *Polar Circles* are the parallels of latitude that pass through the poles of the ecliptic, at the distance of 23½ degrees from the pole of the earth. (Art. 33.)

41. The earth is divided into five zones. That portion of the earth which lies between the tropics, is called the *Torrid Zone;* that between the tropics and polar circles, the *Temperate Zones;* and that between the polar circles and the poles, the *Frigid Zones.*

42. The *Zodiac* is the part of the celestial sphere which lies about 8 degrees on each side of the ecliptic. This portion of the heavens is thus marked off by itself, because all the planets move within it.

43. *The elevation of the pole is equal to the latitude of the place.* The arc PE (Fig. 4.) $= ZO \therefore PO = ZE$ which equals the latitude.

44. *The elevation of the equator is equal to the complement of the latitude.*
$ZH = 90°$. But $ZE = \text{Lat.} \therefore EH = 90 - \text{Lat.}$

45. *The distance of any place from the pole (or the polar distance) equals the complement of the latitude.*
$EP = 90°$. But $EZ = \text{Lat.} \therefore ZP = 90 - \text{Lat.}$

CHAPTER II.

DIURNAL REVOLUTION—ARTIFICIAL GLOBES—ASTRONOMICAL PROBLEMS.

46. THE apparent diurnal revolution of the heavenly bodies from east to west, is owing to the actual revolution of the earth on its own axis from west to east. If we conceive of a radius of the earth's equator extended until it meets the concave sphere of the heavens, then as the earth revolves, the extremity of this line [redacted]

day, we shall find the intervals exactly equal to one another; that is, *the sidereal days are all equal.** Whatever star we select for the observation, the same result will be obtained. The stars, therefore, always keep the same relative position, and have a common movement round the earth,—a consequence that naturally flows from the hypothesis, that their *apparent* motion is all produced by a single *real* motion, namely, that of the earth. The sun, moon, and planets, as well the fixed stars, revolve in like manner, but their returns to the meridian are not, like those of the fixed stars, at exactly equal intervals.

48. The *appearances* of the diurnal motions of the heavenly bodies are different in different parts of the earth, since every place has its own horizon, (Art. 15,) and different horizons are variously inclined to each other. Let us suppose the spectator viewing the diurnal revolutions from several different positions on the earth.

49. On the *equator*, his horizon would pass through both poles; for the horizon cuts the celestial vault at 90 degrees in every direction from the zenith of the spectator; but the pole is likewise 90 degrees from his zenith, and consequently, the pole must be in the horizon. The celestial equator would coincide with the Prime Vertical, being a great circle passing through the east and west points. Since all the diurnal circles are parallel to the equator, consequently, they would all, like the equator, be perpendicular to the horizon. Such a view of the heavenly bodies, is called a right sphere; or,

A RIGHT SPHERE *is one in which all the daily revolutions of the stars, are in circles perpendicular to the horizon.*

A right sphere is seen only at the equator. Any star situated in the celestial equator, would appear to rise directly in the east, at noon to be in the zenith of the spectator, and to set directly in the west; in proportion as stars are at a greater distance from the equator towards the pole, they describe smaller and smaller circles, until, near the pole, their motion is hardly perceptible. Every star remains an equal time above and below the horizon; and since the

* Allowance is here supposed to be made for the effects of precession, &c.

times of their revolutions are equal, the velocities are as the lengths of the circles they describe. Consequently, as the stars are more remote from the equator towards the pole, their motions become slower, until, at the pole, the north star appears stationary.

50. If the spectator advances one degree towards the north pole, his horizon reaches one degree beyond the pole of the earth, and cuts the starry sphere one degree below the pole of the heavens, or below the north star, if that be taken as the place of the pole. As he moves onward towards the pole, his horizon continually reaches farther and farther beyond it, until when he comes to the pole of the earth, and under the pole of the heavens, his horizon reaches on all sides to the equator and coincides with it. Moreover, since all the circles of daily motion are parallel to the equator, they become, to the spectator at the pole, parallel to the horizon. This is what constitutes a parallel sphere. Or,

A Parallel Sphere *is that in which all the circles of daily motion are parallel to the horizon.*

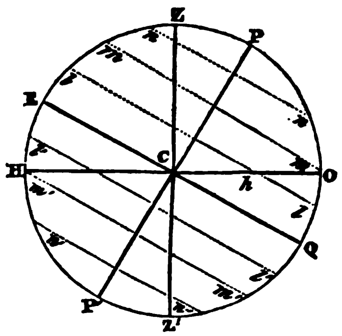
51. To render this view of the heavens familiar, the learner should follow round in his mind a number of separate stars, one near the horizon, one a few degrees above it, and a third near the zenith. To one who stood upon the north pole, the stars of the northern hemisphere would all be perpetually in view when not obscured by clouds or lost in the sun's light, and none of those of the southern hemisphere would ever be seen. The sun would

between the two, the diurnal motions are oblique to the horizon. This aspect of the heavens constitutes an oblique sphere, which is thus defined:

An OBLIQUE SPHERE *is that in which the circles of daily motion are oblique to the horizon.*

Suppose for example the spectator is at the latitude of fifty degrees. His horizon reaches 50° beyond the pole of the earth, and gives the same apparent elevation to the pole of the heavens. It cuts the equator, and all the circles of daily motion, at an angle of 40°, being always equal to the co-altitude of the pole. Thus, let HO (Fig. 5,) represent the horizon, EQ the equator, and PP′ the axis of the earth. Also, *ll*, *mm*, &c. parallels of latitude. Then the horizon of a spectator at Z, in latitude 50° reaches to 50° beyond the pole (Art. 50); and the angle ECH, is 40°. As we advance still farther north the elevation of the diurnal circles grows less and less, and consequently the motions of the heavenly bodies more and more oblique, until finally, at the pole, where the latitude is 90°, the angle of elevation of the equator vanishes, and the horizon and equator coincide with each other, as before stated.

Fig. 5.



54. The CIRCLE OF PERPETUAL APPARITION, *is the boundary of that space around the elevated pole, where the stars never set.* Its distance from the pole is equal to the latitude of the place. For, since the altitude of the pole is equal to the latitude, a star whose polar distance is just equal to the latitude, will when at its lowest point only just reach the horizon; and all the stars nearer the pole than this will evidently not descend so far as the horizon.

Thus, *mm* (Fig. 5,) is the circle of perpetual apparation, between which and the north pole, the stars never set, and its distance from the pole OP is evidently equal to the elevation of the pole, and of course to the latitude.

55. In the opposite hemisphere, a similar part of the sphere adjacent to the depressed pole never rises. Hence,

The CIRCLE OF PERPETUAL OCCULTATION, ***is the boundary of that space around the depressed pole, within which the stars never rise.*** Thus, m' m' (Fig. 5,) is the circle of perpetual occultation, between which and the south pole, the stars never rise.

56. In an oblique sphere, the horizon cuts the circles of daily motion unequally. Towards the elevated pole, more than half the circle is above the horizon, and a greater and greater portion as the distance from the equator is increased, until finally, within the circle of perpetual apparition, the whole circle is above the horizon. Just the opposite takes place in the hemisphere next the depressed pole. Accordingly, when the sun is in the equator, as the equator and horizon, like all other great circles of the sphere, bisect each other, the days and nights are equal all over the globe. But when the sun is north of the equator, the days become longer than the nights, but shorter when the sun is south of the equator. Moreover, the higher the latitude, the greater is the inequality in the lengths of the days and nights. All these points will be readily understood by inspecting figure 5.

57. Most of the phenomena of the diurnal revolution can be explained, either on the supposition that the celestial sphere actually all turns around the earth once in 24 hours, or that this motion of the heavens is merely apparent, arising from the revolu-

58. While we retain the same place on the earth, the diurnal revolution occasions no change in our horizon, but our horizon goes round as well as ourselves. Let us first take our station on the equator at sunrise; our horizon now passes through both the poles, and through the sun, which we are to conceive of as at a great distance from the earth, and therefore as cut, not by the terrestrial but by the celestial horizon. As the earth turns, the horizon dips more and more below the sun, at the rate of 15 degrees for every hour, and, as in the case of the polar star, the sun appears to rise at the same rate. In six hours, therefore, it is depressed 90 degrees below the sun, which brings us directly under the sun, which, for our present purpose, we may consider as having all the while maintained the same fixed position in space. The earth continues to turn, and in six hours more, it completely reverses the position of our horizon, so that the western part of the horizon which at sunrise was diametrically opposite to the sun now cuts the sun, and soon afterwards it rises above the level of the sun, and the sun sets. During the next twelve hours, the sun continues on the invisible side of the sphere, until the horizon returns to the position from which it started, and a new day begins.

59. Let us next contemplate the similar phenomena at the *poles*. Here the horizon, coinciding as it does with the equator, would cut the sun through its center, and the sun would appear to revolve along the surface of the sea, one half above and the other half below the horizon. This supposes the sun in its annual revolution to be at one of the equinoxes. When the sun is north of the equator, it revolves continually round in a circle which, during a single revolution, appears parallel to the equator, and it is constantly day; and when the sun is south of the equator, it is, for the same reason, continual night.

60. We have endeavored to conceive of the manner in which the apparent diurnal movements of the sun are *really* produced at two stations, namely, in the right sphere, and in the parallel sphere. These two cases being clearly understood, there will be little difficulty in applying a similar explanation to an oblique sphere.

ARTIFICIAL GLOBES.

61. Artificial globes are of two kinds, terrestrial and celestial. The first exhibits a miniature representation of the earth; the second, of the visible heavens; and both show the various circles by which the two spheres are respectively traversed. Since all globes are similar solid figures, a small globe, imagined to be situated at the center of the earth or of the celestial vault, may represent all the visible objects and artificial divisions of either sphere, and with great accuracy and just proportions, though on a scale greatly reduced. The study of artificial globes, therefore, cannot be too strongly recommended to the student of astronomy.*

62. An artificial globe is encompassed from north to south by a strong brass ring to represent the meridian of the place. This ring is made fast to the two poles and thus supports the globe, while it is itself supported in a vertical position by means of a frame, the ring being usually let into a socket in which it may be easily slid, so as to give any required elevation to the pole. The brass meridian is graduated each way from the equator to the pole 90°, to measure degrees of latitude or declination, according as the distance from the equator refers to a point on the earth or in the heavens. The horizon is represented by a broad zone, made broad for the convenience of carrying on it a circle of azimuth, another of amplitude, and a wide space on which are delineated the signs of the ecliptic, and the sun's place for every day in the year;

being a secondary to the equinoctial, becomes an hour circle of any star which, by turning the globe, is brought under it.

64. The *Hour Index* is a small circle described around the pole of the equator, on which are marked the hours of the day. As this circle turns along with the globe, it makes a complete revolution in the same time with the equator; or, for any less period, the same number of degrees of this circle and of the equator pass under the meridian. Hence the hour index measures arcs of right ascension.

65. The *Quadrant of Altitude* is a flexible strip of brass, graduated into ninety equal parts, corresponding in length to degrees on the globe, so that when applied to the globe and bent so as closely to fit its surface, it measures the angular distance between any two points. When the zero, or the point where the graduation begins, is laid on the pole of any great circle, the 90th degree will reach to the circumference of that circle, and being therefore a great circle passing through the pole of another great circle, it becomes a secondary to the latter. (Art. 21.) Thus the quadrant of altitude may be used as a secondary to any great circle on the sphere; but it is used chiefly as a secondary to the horizon, the point marked 90° being screwed fast to the pole of the horizon, that is, the zenith, and the other end, marked 0, being slid along between the surface of the sphere and the wooden horizon. It thus becomes a vertical circle, *on* which to measure the altitude of any star through which it passes, or *from* which to measure the azimuth of the star, which is the arc of the horizon intercepted between the meridian and the quadrant of altitude passing through the star, (Art. 27.)

66. To *rectify the globe for any place*, the north pole must be elevated to the latitude of the place (Art. 43); then the equator and all the diurnal circles will have their due inclination in respect to the horizon; and, on turning the globe, every point on either globe will revolve as the same point does in nature; and the relative situations of all places will be the same as on the native spheres.

PROBLEMS ON THE TERRESTRIAL GLOBE.

67. *To find the Latitude and Longitude of a place:* Turn the globe so as to bring the place to the brass meridian; then the degree and minute on the meridian directly over the place will indicate its latitude, and the point of the equator under the meridian, will show its longitude.

Ex. What is the Latitude and Longitude of the city of New York?

68. *To find a place having its latitude and longitude given:* Bring to the brass meridian the point of the equator corresponding to the longitude, and then at the degree of the meridian denoting the latitude, the place will be found.

Ex. What place on the globe is in Latitude 39 N. and Longitude 77 W.?

69. *To find the bearing and distance of two places:* Rectify the globe for one of the places (Art. 66); screw the quadrant of altitude to the zenith,* and let it pass through the other place. Then the azimuth will give the bearing of the second place from the first, and the number of degrees on the quadrant of altitude, multiplied by 69, (the number of miles in a degree,) will give the distance between the two places.

Ex. What is the bearing of New Orleans from New York, and what is the distance between these places?

globe, until the other place comes under the meridian, and the index will point to the required hour.

Ex. What time is it at Canton, in China, when it is 9 o'clock A. M. at New York ?

72. *To find the antœci,* the periœci,† and the antipodes‡ of any place:* Bring the given place to the meridian; then in the opposite hemisphere, in the same degree of latitude, will be found the antœci. The same place remaining under the meridian, set the index to XII, and turn the globe until the other XII is under the index; then the periœci will be on the meridian under the same degree of latitude with the given plane, and the antipodes will be under the meridian, in the same latitude, in the opposite hemisphere.

Ex. Find the antœci, the periœci, and the antipodes of the citizens of New York.

The antœci have the same hour of the day, but different seasons of the year; the periœci have the same seasons, but opposite hours; and the antipodes have both opposite hours and opposite seasons.

73. *To rectify the globe for the sun's place:* On the wooden horizon, find the day of the month, and against it is given the the sun's place in the ecliptic, expressed by signs and degrees.§ Look for the same sign and degree on the ecliptic, bring that point to the meridian and set the hour index to XII. To all places under the meridian it will then be noon.

Ex. Rectify the globe for the sun's place on the 1st of September.

74. ***The latitude of the place being given, to find the time of the sun's rising and setting on any given day at that place:*** Having rectified the globe for the latitude, (Art. 66,) bring the sun's place in the ecliptic to the graduated edge of the meridian, and set the hour index to XII; then turn the globe so as to bring the sun to the eastern and then to the western horizon, .and the hour index will show the times of rising and setting respectively.

* αντι οικος. † περι οικος. ‡ αντι πους.

§ The larger globes have the day of the month marked against the corresponding sign on the ecliptic itself.

Ex. At what time will the sun rise and set at New Haven, Lat. 41° 18′ on the 10th of July?

PROBLEMS ON THE CELESTIAL GLOBE.

75. *To find the Declination and Right Ascension of a heavenly body:* Bring the place of the body (whether the sun or a star) to the meridian. Then the degree and minute standing over it will show its declination, and the point of the equinoctial under the meridian will give its right ascension. It will be remarked, that the declination and right ascension are found in the same manner as latitude and longitude on the terrestrial globe. Right ascension is expressed either in degrees or in hours; both being reckoned from the vernal equinox, (Art. 37.)

Ex. What is the declination and right ascension of the bright star Lyra?—also of the sun on the 5th of June?

76. *To represent the appearance of the heavens at any time:* Rectify the globe for the latitude, bring the sun's place in the ecliptic to the meridian, and set the hour index to XII; then turn the globe westward until the index points to the given hour, and the constellations would then have the same appearance to an eye situated at the center of the globe, as they have at that moment in the sky.

Ex. Required the aspect of the stars at New Haven, Lat. 41° 18′, at 10 o'clock, on the evening of December 5th.

[redacted]

Ex. What is the distance between the two largest stars of the Great Bear.*

79. *To find the sun's meridian altitude, the latitude and day of the month being given:* Having rectified the globe for the latitude, (Art. 66,) bring the sun's place in the ecliptic to the meridian, and count the number of degrees and minutes between that point of the meridian and the zenith. The complement of this arc will be the sun's meridian altitude.

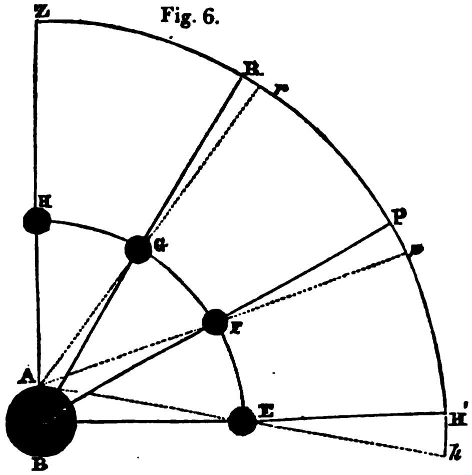
Ex. What is the sun's meridian altitude at noon on the 1st of August, in Lat. 41° 18′?

CHAPTER III.

OF PARALLAX, REFRACTION, AND TWILIGHT.

80. Parallax *is the apparent change of place which bodies undergo by being viewed from different points.* Thus in figure 6, let A represent the earth, CH′ the horizon. H′Z a quadrant of

Fig. 6.



* These two stars are sometimes called "the Pointers," from the line which passes through them being always nearly in the direction of the north star. The angular distance between them is about 5°, and may be learned as a standard for reference in estimating by the eye, the distance between any two points on the celestial vault.

a great circle of the heavens, extending from the horizon to the zenith ; and let E, F, G, H, be successive positions of the moon at different elevations, from the horizon to the meridian. Now a spectator on the surface of the earth at A, would refer the place of E to *h*, whereas, if seen from the center of the earth, it would appear at H′. The arc H′*h* is called the parallactic arc, and the angle H′E*h*, or its equal AEC, is the angle of parallax. The same is true of the angles at F, G, and H, respectively.

81. Since then a heavenly body is liable to be referred to different points on the celestial vault, when seen from different parts of the earth, and thus some confusion occasioned in the determination of points on the celestial sphere, astronomers have agreed to consider the true place of a celestial object to be that where it would appear if seen from the center of the earth. The doctrine of parallax teaches how to reduce observations made at any place on the surface of the earth, to such as they would be if made from the center.

82. The angle AEC is called the horizontal parallax, which may be thus defined. *Horizontal Parallax*, is the change of position which a celestial body, appearing in the horizon as seen from the surface of the earth, would assume if viewed from the earth's center. It is the angle subtended by the semi-diameter of the earth, as viewed from the body itself. If we consider any [redacted]

with the sines, it increases much slower than in the simple ratio of the distance from the zenith,) and diminishes, as the distance from the spectator increases. Again, since the parallax AGC is as the sine of the zenith distance, let P represent the horizontal parallax, and P′ the parallax at any altitude; then,

$$P' : P :: \text{sin. zenith dist.} : \text{sin. } 90^\circ \therefore P = \frac{P'}{\text{sin. zen. dist.}}.$$

Hence, the horizontal parallax of a body equals its parallax at any altitude, divided by the sine of its distance from the zenith.

83. From observations, therefore, on the parallax of a body at any elevation, we are enabled to find the angle subtended by the semi-diameter of the earth as seen from the body. Or if the horizontal parallax is given, the parallax at any altitude may be found, for

$$P' = P \times \text{sin. zenith distance.}$$

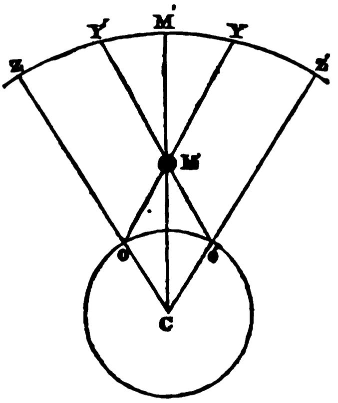
Hence, in the zenith the parallax is nothing, and is at its maximum in the horizon.

84. It is evident from the figure, that the effect of parallax upon the place of a celestial body is *to depress it.* Thus, in consequence of parallax, E is depressed by the arc $H'h$; F by the arc Pp; G by the arc Rr; while H sustains no change. Hence, in all calculations respecting the altitude of the sun, moon, or planets, the amount of parallax is to be subtracted: the stars, as we shall see hereafter, have no sensible parallax. As the depression which arises from parallax is in the direction of a vertical circle, when the body is on the meridian, the body has only a parallax in declination; but in other situations, there is at the same time a parallax in declination and right ascension; for its direction being *oblique* to the equinoctial, it can be resolved into two parts, one of which (the declination) is perpendicular, and the other (the right ascension) is parallel to the equinoctial.

85. *The mode of determining the horizontal parallax,* is as follows:

Let O, O′, (Fig. 7,) be two places on the earth, situated under the same meridian, at a great distance from each other; one place, for example, at the Cape of Good Hope, and the other in the north

Fig. 7.



of Europe. The latitude of each place being known, the arc of the meridian OO' is known, and the angle OCO' also is known. Let the celestial body M, (the moon for example,) be observed simultaneously at O and O', and its zenith distance at each place accurately taken, namely, ZY and Z'Y'; then the angles ZOM and Z'O'M, and of course their supplements COM, CO'M are found. Then in the quadrilateral figure COMO', we have all the angles and the two radii, CO, CO', whence the side CM may be easily found. But, CM : CO::sin. ZOM : sin. CMO=sine of the angle of parallax; or (since the arc is very small) equals the parallax P'. But when M as seen from O is in the horizon, ZOM becomes a right angle, and its sine equal to radius. Then,

$$CM : CO :: 1 : P = \text{horizontal parallax} = \frac{CO}{CM}.$$

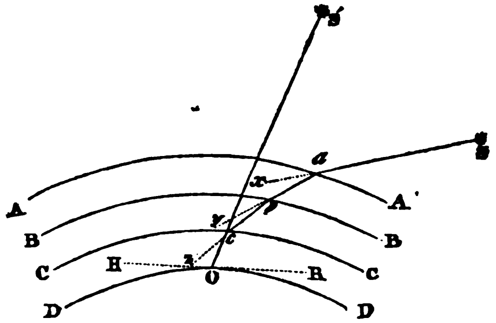
On this principle, the horizontal parallax of the moon was determined by La Caille and La Lande, two French astronomers, one stationed at the Cape of Good Hope, the other at Berlin; and and in a similar way the parallax of Mars was ascertained, by observations made simultaneously at the Cape of Good Hope and

thenuse CE, which is the distance of the moon from the center of the earth.

REFRACTION.

88. While parallax depresses the celestial bodies subject to it, *refraction elevates them;* and it affects alike the most distant as well as nearer bodies, being occasioned by the change of direction which light undergoes in passing through the atmosphere. Let us conceive of the atmosphere as made up of a great number of concentric strata, as AA, BB, CC, and DD, (Fig. 8,)

Fig. 8.



increasing rapidly in density (as is known to be the fact) in approaching near to the surface of the earth. Let S be a star, from which a ray of light S*a* enters the atmosphere at *a*, where, being turned towards the radius of the convex surface, it would change its direction into the line *ab*, and again into *bc*, and *c*O, reaching the eye at O. Now, since an object always appears in the direction in which the light finally strikes the eye, the star would be seen in the direction of the last ray *c*O, and consequently, the star would apparently change its place, in consequence of refraction, from S to S′, being elevated out of its true position. Moreover, since on account of the continual increase of density in descending through the atmosphere, the light would be continually turned out of its course more and more, it would therefore move, not in the polygon represented in the figure, but in a corresponding curve, whose curvature is rapidly increased near the surface of the earth.

89. When a body is in the zenith, since a ray of light from it enters the atmosphere at right angles to the refracting medium, it suffers no refraction. Consequently, the position of the heavenly bodies, when in the zenith, is not changed by refraction, while near the horizon, when a ray of light strikes the medium very obliquely, and traverses the atmosphere through its densest part, the refraction is greatest. The following numbers, taken at different altitudes, will show how rapidly refraction diminishes from the horizon upwards. The amount of refraction at the horizon is 34′ 00″. At different elevations it is as follows.

Elevation.	Refraction.	Elevation.	Refraction.
0° 10′	32′ 00″	30°	1′ 40″
0° 30′	30′ 00″	40°	1′ 09″
1° 00′	24′ 25″	45°	0′ 58″
5° 00′	10′ 00″	60°	0′ 33″
10° 00′	5′ 20″	80°	0′ 10″
20° 00′	2′ 39″	90°	0′ 00″

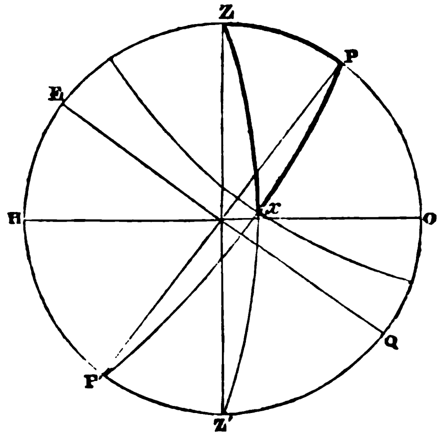
From this table it appears, that while refraction at the horizon is 34 minutes, at so small an elevation as only 10 minutes above the horizon it loses 2 minutes, more than the entire change from the elevation of 30° to the zenith. From the horizon to 1° above, the refraction is diminished nearly 10 minutes. The amount at the horizon, at 45°, and at 90°, respectively, is 34′, 58″, and 0. In finding the altitude of a heavenly body, the effect of parallax must be added, but that of refraction subtracted.

between the two will show the amount of refraction at the lower culmination, the latitude of the place being known.

Example. At Paris, latitude 48° 50′, a star was observed to pass the meridian 6′ north of the zenith, and consequently, 41° 4′, from the pole.* It should have passed the meridian at the same distance below the pole, but the distance was found to be only 40° 57′ 35″. Hence, 41° 4′ − 40° 57′ 35″ = 6′ 25″ is the refraction due to that altitude, that is, at the altitude of 7° 46′. By taking similar observations in various places situated in high latitudes, the amount of refraction might be ascertained for a number of different altitudes, and thus the law of increase in refraction as we proceed from the zenith towards the horizon, might be ascertained.

91. Another method of finding the refraction at different altitudes, is as follows. Take the altitude of the sun or a star, whose right ascension and declination are known, and note the time by the clock. Observe also when it crosses the meridian, and the difference of time between the two observations will give the hour angle Z P x, (Fig. 9.) In this triangle Z P x we also know PZ the

Fig. 9.



co-latitude and Px the co-declination. Hence we can find the co-altitude Zx, and of course the true altitude. Compare the alti-

* For the polar distance of the place = 90 − 48° 50′ = 41° 10′; and 41° 10′ − 6′ = 41° 4′.

tude thus found with that before determined by observation, and the difference will be the refraction due to the apparent altitude.

Ex. On May 1, 1738, at 5h. 20m. in the morning, Cassini observed the altitude of the sun's center at Paris to be 5° 0′ 14″. The latitude of Paris being 48° 50′ 10″, and the sun's declination at that time being 15° 0′ 25″: *Required the refraction.*

By spherical trigonometry, Zx will be found equal to 85° 10′ 8″; consequently, the true altitude was 4° 49′ 52″. Now to 5° 0′ 14″, the apparent altitude, 9″ must be added for parallax, and we have 5° 0′ 23″ the apparent altitude corrected for parallax. Hence, 5° 0′ 23″ − 4° 49′ 52″ = 10′ 31″, the refraction at the apparent altitude 5° 0′ 14″.*

92. By these and similar methods, we could easily determine the refraction due to any elevation above the horizon, provided the refracting medium (the atmosphere) were always uniform. But this is not the fact: the refracting power of the atmosphere is altered by changes in density and temperature.† Hence in delicate observations, it is necessary to take into the account the state of the barometer and of the thermometer, the influence of the variations of each having been very carefully investigated, and rules having been given accordingly. With every precaution to insure accuracy, on account of the variable character of the refracting medium, the tables are not considered as entirely accurate to a greater distance from the zenith than 74°; but almost all astronomical observations are made at a greater altitude than this.

when light clouds enable us to view the solar disk. Were all parts of the sun equally raised by refraction, there would be no change of figure ; but since the lower side is more refracted than the upper, the effect is to shorten the vertical diameter and thus to give the disk an oval form. This effect is particularly remarkable when the sun, at his rising or setting, is observed from the top of a mountain, or at an elevation near the sea shore ; for in such situations the rays of light make a greater angle than ordinary with a perpendicular to the refracting medium, and the amount of refraction is proportionally greater. In some cases of this kind, the shortening of the vertical diameter of the sun has been observed to amount to 6′, or about one fifth of the whole.

95. The apparent *enlargement of the sun and moon in the horizon*, arises from an optical illusion. These bodies in fact are not seen under so great an angle when in the horizon, as when on the meridian, for they are nearer to us in the latter case than in the former. The distance of the sun is indeed so great that it makes very little difference in his apparent diameter, whether he is viewed in the horizon or on the meridian ; but with the moon the case is otherwise ; its angular diameter, when measured with instruments, is perceptibly larger at the time of its culmination. Why then do the sun and moon appear so much larger when near the horizon ? It is owing to that general law, explained in optics, by which we judge of the magnitudes of distant objects, not merely by the angle they subtend at the eye, but also by our impressions respecting their distance, allowing, under a given angle, a greater magnitude as we imagine the distance of a body to be greater. Now, on account of the numerous objects usually in sight between us and the sun, when on the horizon, he appears much farther removed from us than when on the meridian, and we assign to him a proportionally greater magnitude. If we view the sun, in the two positions, through smoked glass, no such difference of size is observed, for here no objects are seen but the sun himself.

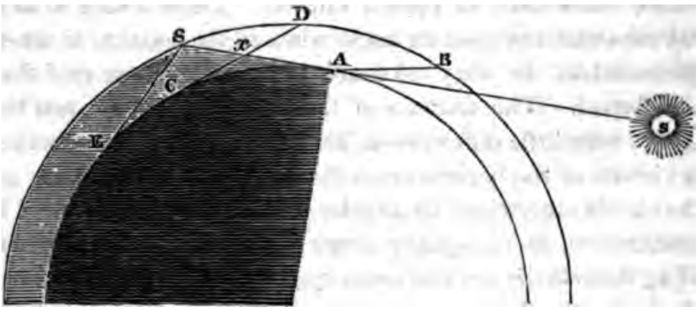
96. The extraordinary enlargement of the sun or moon, particularly the latter, when seen at its rising through a grove of trees,

depends on a different principle. Through the various openings between the trees, we see different images of the sun, a great number of which overlapping each other, swell the dimensions of the moon, under the most favorable circumstances, to a very unusual size.

TWILIGHT.

97. *Twilight* also is another phenomenon depending upon the agency of the earth's atmosphere. It is due partly to refraction and partly to reflexion, but mostly to the latter. While the sun is within 18° of the horizon, before it rises or after it sets, some portion of its light is conveyed to us by means of numerous reflections from the atmosphere. Let AB (Fig. 10,) be the horizon

Fig. 10.



of the spectator at A, and let SS be a ray of light from the sun when it is two or three degrees below the horizon. Then to

gradual. To the inhabitants of an oblique sphere, the twilight is longer in proportion as the place is nearer the elevated pole.

99. Were it not for the power the atmosphere has of dispersing the solar light, and scattering it in various directions, no objects would be visible to us out of direct sunshine; every shadow of a passing cloud would be pitchy darkness; the stars would be visible all day, and every apartment into which the sun had not direct admission, would be involved in the obscurity of night. This scattering action of the atmosphere on the solar light, is greatly increased by the irregularity of temperature caused by the sun, which throws the atmosphere into a constant state of undulation, and by thus bringing together masses of air of different temperatures, produces partial reflections and refractions at their common boundaries, by which means much light is turned aside from the direct course, and diverted to the purposes of general illumination. In the upper regions of the atmosphere, as on the tops of very high mountains, where the air is too much rarefied to reflect much light, the sky assumes a black appearance, and stars become visible in the day time.

CHAPTER IV.

OF TIME.

100. Time *is a measured portion of indefinite duration.*

The great standard of time is the period of the revolution of the earth on its axis, which, by the most exact observations, is found to be always the same. The time of the earth's revolution on its axis is called a *sidereal day*, and is determined by the revolution of a star from the instant it crosses the meridian, until it comes round to the meridian again. This interval being called a sidereal day, it is divided into 24 *sidereal hours.* Observations taken upon numerous stars, in different ages of the world, show that they all perform their diurnal revolutions in the same time,

and that their motion during any part of the revolution is perfectly uniform.

101. *Solar time* is reckoned by the apparent revolution of the sun, from the meridian round to the same meridian again. Were the sun stationary in the heavens, like a fixed star, the time of its apparent revolution would be equal to the revolution of the earth on its axis, and the solar and the sidereal days would be equal. But since the sun passes from west to east, through 360° in 365¼ days, it moves eastward nearly 1° a day, (59′ 8″.3). While, therefore, the earth is turning round on its axis, the sun is moving in the same direction, so that when we have come round under the same celestial meridian from which we started, we do not find the sun there, but he has moved eastward nearly a degree, and the earth must perform so much more than one complete revolution, in order to come under the sun again. Now since a place on the earth gains 359° in 24 hours, how long will it take to gain 1°?

$$359 : 24 :: 1 : \frac{24}{359} = 4^{m} \text{ nearly.}$$

Hence the solar day is about 4 minutes longer than the sidereal; and if we were to reckon the sidereal day 24 hours, we should reckon the solar day 24h. 4m. To suit the purposes of society at large, however, it is found most convenient to reckon the solar day 24 hours, and to throw the fraction into the sidereal day. Then,

24h. 4m. : 24::24 : 23h. 56m. nearly (23h. 56^{m} 4^{s}.09)=the [redacted]

the lower meridian, namely, at 12 o'clock at night, and counted by 12 hours from the lower to the upper culmination, and from the upper to the lower. The *astronomical* day is the apparent solar day counted through the whole 24 hours, instead of by periods of 12 hours each, and begins at noon. Thus 10 days and 14 hours of astronomical time, would be 11 days and 2 hours of apparent time.

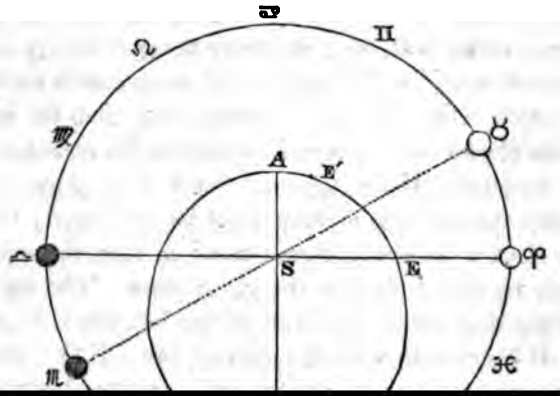
104. Clocks are usually regulated so as to indicate mean solar time; yet as this is an artificial period, not marked off, like the sidereal day, by any natural event, it is necessary to know how much is to be added to or subtracted from the apparent solar time, in order to give the corresponding mean time. The interval by which apparent time differs from mean time, is called the *equation of time.* If a clock were constructed (as it may be) so as to keep exactly with the sun, going faster or slower according as the increments of right ascension were greater or smaller, and another clock were regulated to mean time, then the difference of the two clocks, at any period, would be the equation of time for that moment. If the apparent clock were *faster* than the mean, then the equation of time must be subtracted; but if the apparent clock were *slower* than the mean, then the equation of time must be added, to give the mean time. The two clocks would differ most about the 3d of November, when the apparent time is $16\frac{1}{4}^{m}$ greater than the mean ($16^{m}\ 16^{s}.7$). But, since apparent time is sometimes greater and sometimes less than mean time, the two must obviously be sometimes equal to each other. This is in fact the case four times a year, namely, April 15th, June 15th, September 1st, and December 22d. These epochs, however, do not remain constant; for, on account of the change in the position of the perihelion, or the point where the earth is nearest the sun, (which shifts its place from west to east about 12″ a year,) the period when the sun's motions are most rapid, as well as that when they are slowest, will occur at different parts of the year. The change is indeed exceedingly small in a single year; but in the progress of ages, the time of year when the sun's motion in its orbit is most accelerated, will not be, as at present, on the first of January, but may fall on the first of March, June, or

any other day of the year, and the amount of the equation of time is obviously affected by the sun's distance from its perihelion, since the sun moves most rapidly when nearest the perihelion, and slowest when farthest from that point.

105. *The inequality of the solar days depends on two causes, the unequal motion of the earth in its orbit, and the inclination of the equator to the ecliptic.*

First, on account of the eccentricity* of the earth's orbit, the earth actually moves faster from the autumnal to the vernal equinox, than from the vernal to the autumnal, the difference of the two periods being about eight days (7d. 17h. 17m.) Thus, let AEB (Fig. 11,) represent the earth's orbit, S being the place of

Fig. 11.



[redacted]

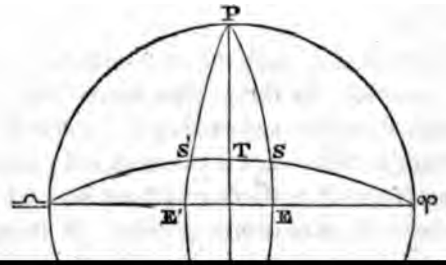
would be referred if seen from the sun; and the place of the sun is the part of the heavens to which it is referred as seen from the earth. Thus, when the earth is at E, it is said to be in Aries; and as it moves from E through E′ to A, its path in the heavens is through Aries, Taurus, Gemini, &c. Meanwhile the sun takes its place successively in Libra, Scorpio, Sagittarius, &c. Now, as will be shown more fully hereafter, the earth moves faster when proceeding from Aries through its perihelion to Libra, than from Libra through its aphelion to Aries, and, consequently, describes the half of its apparent orbit in the heavens, ♈, ♋, ♎, sooner than the half ♎, ♑, ♈. The line of the apsides, that is, the major axis of the ellipse, is so situated at present, that the perihelion is in the sign Leo, nearly 100° (99° 30′ 5″) from the vernal equinox. The earth passes through it about the first of January, and then its velocity is the greatest in the whole year, being always greater as the distance is less, the angular velocity being inversely as the square of the distance, as will be shown by and by.

106. But differences of time are not reckoned on the ecliptic, but on the equinoctial; for the ecliptic being oblique to the meridian in the diurnal motion, and cutting it at different angles at different times, equal portions will not pass under the meridian in equal times, and therefore such portions could not be employed, as they are in the equinoctial, as measures of time. If therefore the sun moved uniformly in his orbit, so as to make the daily increments of longitude equal, still the corresponding arcs of right ascension, which determine the lengths of the solar day, would be unequal. Let us start from the equinox, from which both longitude and right ascension are reckoned, the former on the ecliptic, the latter on the equinoctial. Suppose the sun has described 70° of longitude; then to ascertain the corresponding arc of right ascension, we let a meridian pass through the sun: the point where it cuts the equator gives the sun's right ascension. Now since the ecliptic makes an acute angle with the meridian, while the equinoctial makes a right angle with it, consequently the arc of longitude is greater than the arc of right ascension. The difference, however, grows constantly less and less as we approach the tropic, as the

angle made between the ecliptic and the meridian constantly increases, until, when we reach the tropic, the meridian is at right angles to both circles, and the longitude and right ascension each equals 90°, and they are of course equal to each other. Beyond this, from the tropic to the other equinox, the arc of the ecliptic intercepted between the meridian and the autumnal equinox being greater than the corresponding arc of the equinoctial, of course its supplement, which measures the longitude, is less than the supplement of the corresponding arc of the equator which measures the right ascension. At the autumnal equinox again, the right ascension and longitude become equal. In a similar manner we might show that the *daily increments* of longitude and right ascension are unequal.

In order to illustrate the foregoing points, let ♈ ♎ (Fig. 12,) represent the equator, ♈ T ♎ the ecliptic, and PSE, PS′E′, two meridians meeting the sun in S and S′. Then in the trian-

Fig. 12.



[redacted]

arc of longitude, so the *daily increments* of right ascension differ from those of longitude. If we suppose in the quadrant ♈T, points taken to mark the progress of the sun from day to day, and let meridians like PSE pass through these points, the arc of the ecliptic embraced between the meridians will be the daily increments of longitude, while the corresponding parts of the equinoctial will be the daily increments of right ascension. Near ♈, the oblique direction in which the ecliptic cuts the meridian, will make the daily increments of longitude exceed those of right ascension; but this advantage is diminished as we approach the tropic, where the ecliptic becomes less oblique, and finally parallel to the equinoctial; while the convergence of the meridians contributes still farther to lessen the ratios of the daily increments of longitude to those of right ascension. Hence, at first, the diurnal arcs of right ascension are less than those of longitude, but afterwards greater; and they continue greater for a similar distance beyond the tropic.

108. From the foregoing considerations it appears, that the diurnal arcs of right ascension, by which the difference between the sidereal and the solar days is measured, are unequal, on account both of the unequal motion of the sun in his orbit, and of the inclination of his orbit to the equinoctial.

109. As astronomical time commences when the *sun* is on the meridian, so sidereal time commences when the vernal equinox is on the meridian, and is also counted from 0 to 24 hours. By 3 o'clock, for instance, of sidereal time, we mean that it is three hours since the vernal equinox crossed the meridian; as we say it is 3 o'clock of astronomical or of civil time, when it is three hours since the sun crossed the meridian.

THE CALENDAR.

110. The *astronomical year* is the time in which the sun makes one revolution in the ecliptic, and consists of 365d. 5h. 48m. $51^s.60$. The *civil year* consists of 365 days. The difference is nearly 6 hours, making one day in four years.

111. The most ancient nations determined the number of days in the year by means of the *stylus*, a perpendicular rod which

cast its shadow on a smooth plane, bearing a meridian line. The time when the shadow was shortest, would indicate the day of the summer solstice ; and the number of days which elapsed until the shadow returned to the same length again, would show the number of days in the year. This was found to be 365 whole days, and accordingly this period was adopted for the civil year. Such a difference, however, between the civil and astronomical years, at length threw all dates into confusion. For, if at first the summer solstice happened on the 21st of June, at the end of four years, the sun would not have reached the solstice until the 22d of June, that is, it would have been behind its time. At the end of the next four years the solstice would fall on the 23d; and in process of time it would fall successively on every day of the year. The same would be true of any other fixed date. Julius Cæsar made the first correction of the calendar, by introducing an intercalary day every fourth year, making February to consist of 29 instead of 28 days, and of course the whole year to consist of 366 days. This fourth year was denominated *Bissextile.** It is also called Leap Year.

112. But the true correction was not 6 hours, but 5h. 49m.; hence the intercalation was too great by 11 minutes. This small fraction would amount in 100 years to $\frac{3}{4}$ of a day, and in 1000 years to more than 7 days. From the year 325 to 1582, it had in fact amounted to about 10 days; for it was known that in

[redacted]

Thus the year 1838, not being divisible by 4, contains 365 days, while 1836 and 1840 are leap years. Yet to make every fourth year consist of 366 days would increase it too much by about $\frac{3}{4}$ of a day in 100 years; therefore every hundredth year has only 365 days. Thus 1800, although divisible by 4 was not a leap year, but a common year. But we have allowed a *whole* day in a hundred years, whereas we ought to have allowed only *three fourths* of a day. Hence, in 400 years we should allow a day too much, and therefore we let the 400th year remain a leap year. This rule involves an error of less than a day in 4237 years.* If the rule were extended by making every year divisible by 4,000 (which would now consist of 366 days) to consist of 365 days, the error would not be more than one day in 100,000 years.†

113. This reformation of the calendar was not adopted in England until 1752, by which time the error in the Julian calendar amounted to about 11 days. The year was brought forward, by reckoning the 3d of September the 14th. Previous to that time the year began the 25th of March; but it was now made to begin on the 1st of January, thus shortening the preceding year, 1751, one quarter.‡

114. As in the year 1582, the error in the Julian calendar amounted to 10 days, and increased by $\frac{3}{4}$ of a day in a century, at present the correction is 12 days; and the number of the year will differ by one with respect to dates between the 1st of January and the 25th of March.

Examples. General Washington was born Feb. 11, 1731, old style; to what date does this correspond in new style?

As the date is the earlier part of the 18th century, the correction is 11 days, which makes the birth day fall on the 22d of February; and since the year 1731 closed the 25th of March, while according to new style 1732 would have commenced on

* Woodhouse, p. 874. † Herschel's Ast. p. 384.

‡ Russia, and the Greek Church generally, adhere to the old style. In order to make the Russian dates correspond to ours, we must add to them 12 days. France and other Catholic countries, adopted the Gregorian calendar soon after it was promulgated.

the preceding 1st of January; therefore, the time required is Feb. 22, 1732. It is usual, in such cases, to write both years, thus: Feb. 11, 1731–2, O. S.

2. A great eclipse of the sun happened May 15th, 1836; to what date would this time correspond in old style?

Ans. May 3d.

115. *The common year begins and ends on the same day of the week; but leap year ends one day later in the week than it began.*

For $52 \times 7 = 364$ days; if therefore the year begins on Tuesday, for example, 364 days would complete 52 weeks, and one day would be left to begin another week, and the following year would begin on Wednesday. Hence, any day of the month is one day later in the week than the corresponding day of the preceding year. Thus, if the 16th of November, 1838, falls on Friday, the 16th of November, 1837, fell on Thursday, and will fall in 1839 on Saturday. But if leap year begins on Sunday, it ends on Monday, and the following year begins on Tuesday; while any given day of the month is two days later in the week than the corresponding date of the preceding year.

116. Fortunately for astronomy, the confusion of dates involved in different calendars affects recorded observations but little. Remarkable eclipses, for example, can be calculated back for several [redacted]

CHAPTER V.

OF ASTRONOMICAL INSTRUMENTS AND PROBLEMS—FIGURE AND DENSITY OF THE EARTH.

117. The most ancient astronomers employed no instruments for measuring angles, but acquired their knowledge of the heavenly bodies by long continued and most attentive inspection with the naked eye. In the Alexandrian school, about 300 years before the Christian era, instruments began to be freely used, and thenceforward trigonometry lent a powerful aid to the science of astronomy. Tycho Brahe, in the 16th century, formed a new era in practical astronomy, and carried the measurement of angles to 10″,—a degree of accuracy truly wonderful, considering that he had not the advantage of the telescope. By the application of the telescope to astronomical instruments, a far better defined view of objects was acquired, and a far greater degree of refinement was attainable. The astronomers royal of Great Britain perfected the art of observation, bringing the measurement of angles to 1″, and the estimation of differences of time to $\frac{1}{5}$ of a second. Beyond this degree of refinement it is supposed that we cannot advance, since unavoidable errors arising from the uncertainties of refraction, and the necessary imperfection of instruments, forbid us to hope for a more accurate determination than this. But a little reflection will show us, that 1″ on the limb of an astronomical instrument, must be a space exceedingly small. Suppose the circle, on which the angle is measured, be one foot in diameter. Then $\frac{12\times 3.14159}{360}=\frac{1}{10}$ inch = space occupied by 1°. Hence $\frac{1}{10\times 60}=\frac{1}{600}$ = space of 1′, and $\frac{1}{36000}$ = space of 1″. Such minute angles can be measured only by large circles. If, for example, a circle is 20 feet in diameter, a degree on its periphery would occupy a space 20 times as large as a degree on a circle of 1 foot. A degree therefore of the limb of such an instrument would occupy a space of 2 inches: one minute, $\frac{1}{30}$ of an inch; and one second, $\frac{1}{1800}$ of an inch.

118. But the actual divisions on the limb of an astronomical instrument never extend to seconds: in the smaller instruments they reach only to 10′, and on the largest rarely lower than 1′. The subdivisions of these spaces is carried on by means of the Vernier, which may be thus defined:

A VERNIER *is a contrivance attached to the graduated limb of an instrument, for the purpose of measuring aliquot parts of the smallest spaces, into which the instrument is divided.*

The vernier is usually a narrow zone of metal, which is made to slide on the graduated limb. Its divisions correspond to those on the limb, except that they are a little larger,* one tenth, for example, so that ten divisions on the vernier would equal eleven on the limb. Suppose now that our instrument is graduated to degrees only, but the altitude of a certain star is found to be 40° and *a fraction*, or $40°+x$. In order to estimate the amount of this fraction, we bring the zero point of the vernier to coincide with the point which indicates the exact altitude, or $40°+x$. We then look along the vernier until we find where one of its divisions coincides with one of the divisions of the limb. Let this be at the fourth division of the vernier. In four divisions, therefore, the vernier has gained upon the divisions of the limb, a space equal to x; and since, in the case supposed, it gains $\frac{1}{10}$ of a degree, or 6′ at each division, the entire gain is 24′, and the arc in question is 40° 24′.

119. As the vernier is used in the barometer, where its application is more [redacted]

Fig. 13.

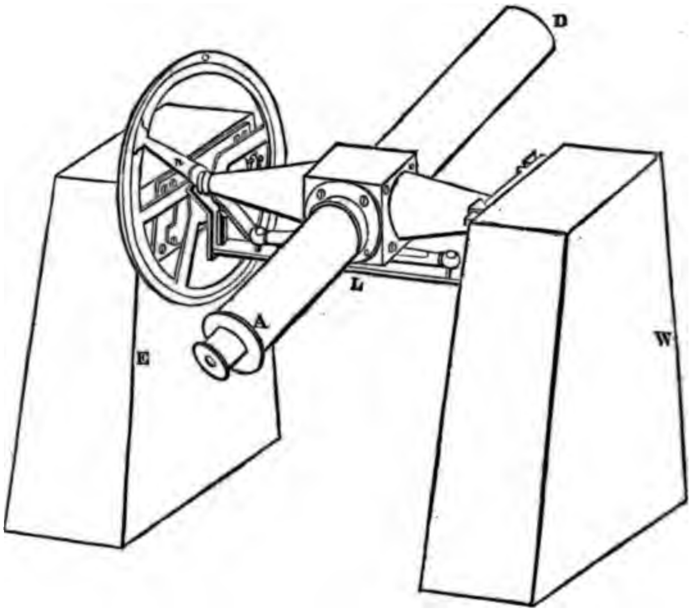
scale, until we find that the coincidence is at the 8th division of the vernier. Now as the vernier gains $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$ of an inch at each division upward, it of course gains $\frac{8}{100}$ in eight divisions. The fractional quantity, therefore, is .08 of an inch, and the height of the mercury is 30.38. If the divisions of the vernier were such, that each gained $\frac{1}{60}$ (when 60 on the vernier would equal 61 on the limb) on a limb divided into degrees, we could at once take off minutes; and were the limb graduated to minutes, we could in a similar way read off seconds.

120. The instruments most used for astronomical observations, are the Transit Instrument, the Astronomical Clock, the Mural Circle, and the Sextant. A large portion of all the observations made in an astronomical observatory, are taken *on the meridian.* When a heavenly body is on the meridian, being at its highest point above the horizon, it is then least affected by refraction and parallax; its zenith distance (from which its altitude and declination are easily derived) is readily estimated; and its right ascension may be very conveniently and accurately determined by means of the astronomical clock. Having the right ascension and declination of a heavenly body, various other particulars respecting its position may be found, as we shall see hereafter, by the aid of spherical trigonometry. Let us then first turn our attention to the instruments employed for determining the right ascension and declination. They are the Transit Instrument, the Astronomical Clock, and the Mural Circle.

121. The *Transit Instrument* is a telescope, which is fixed permanently in the meridian, and moves only in that plane. It rests on a horizontal axis, which consists of two hollow cones applied base to base, a form uniting lightness and strength. The two ends of the axis rest on two firm supports, as pillars of stone, for example, so connected with the building as to be as free as possible from all agitation. In figure 14, AD represents the telescope, E, W, massive stone pillars supporting the horizontal axis, beneath which is seen a spirit level, (which is used to bring the axis to a horizontal position,) and n a vertical circle graduated into degrees and minutes. This circle serves the purpose of pla-

cing the instrument at any required altitude or distance from the zenith, and of course for determining altitudes and zenith distances.

Fig. 14.



122. Various methods are described in works on practical astronomy, for placing the Transit Instrument accurately in the meridian. The following method by observations on the pole

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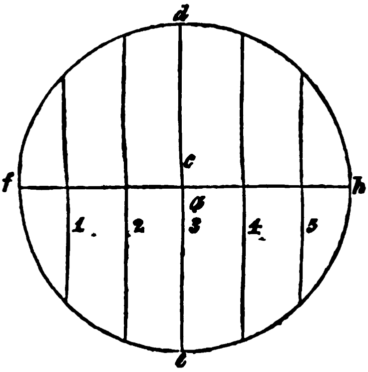
123. The *line of collimation* of a telescope, is a line joining the center of the object glass with the center of the eye glass. When the transit instrument is properly adjusted, this line, as the instrument is turned on its axis, moves in the plane of the meridian. Having, by means of the vertical circle *n*, set the instrument at the known altitude or zenith distance of any star, upon which we wish to make observations, we wait until the star enters the field of the telescope, and note the exact instant when it crosses the vertical wire in the center of the field, which wire marks the true plane of the meridian. Usually, however, there are placed in the focus of the eye glass five parallel wires or threads, two on each side of the central wire, and all at equal distances from each other, as is represented in the following diagram. The time of arriving at each of the wires being noted, and all the times added together and divided by the number of observations, the result gives the instant of crossing the central wire.

Fig. 15.



124. The *Astronomical Clock* is the constant companion of the Transit Instrument. This clock is so regulated as to keep exact pace with the stars, and of course with the revolution of the earth on its axis; that is, it is regulated to sidereal time. It measures the progress of a star, indicating an hour for every 15°, and 24 hours for the whole period of the revolution of the star. Sidereal time, it will be recollected, commences when the vernal equinox is on the meridian, just as solar time commences when the sun is on the meridian. Hence, the hour by the sidereal clock has no correspondence with the hour of the day, but simply indicates how long it is since the equinoctial point crossed the meridian. For example, the clock of an observatory points to 3h. 20m.; this may be in the morning, at noon, or any other time of the day, since it merely shows that it is 3h. 20m. since the equinox was on the meridian. Hence, when a star is on the meridian, the clock itself shows its right ascension; and the

interval of time between the arrival of any two stars upon the meridian, is the measure of their difference of right ascension.

125. Astronomical clocks are made of the best workmanship, with a compensation pendulum, and every other advantage which can promote their regularity. The Transit Instrument itself, when once accurately placed in the meridian, affords the means of testing the correctness of the clock, since one revolution of a star from the meridian to the meridian again, ought to correspond to exactly 24 hours by the clock, and to continue the same from day to day; and the right ascension of various stars as they cross the meridian, ought to be such by the clock as they are given in the tables, where they are stated according to the most accurate determinations of astronomers. Or by taking the difference of right ascension of any two stars on successive days, it will be seen whether the going of the clock is uniform for that part of the day; and by taking the right ascension of different pairs of stars, we may learn the rate of the clock at various parts of the day. We thus learn, not only whether the clock accurately measures the length of the sidereal day, but also whether it goes uniformly from hour to hour.

Although astronomical clocks have been brought to a great degree of perfection, so as to vary hardly a second for many months, yet none are absolutely perfect, and most are so far from it as to require to be corrected by means of the Transit Instru-

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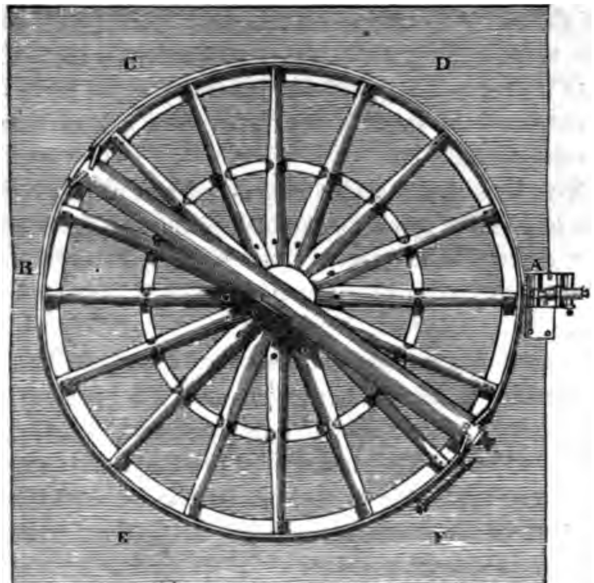
The Mural Circle is a graduated circle, usually of very large size, fixed permanently in the plane of the meridian, and attached firmly to a perpendicular *wall*. It is made of large size, sometimes 20 feet in diameter, in order that very small angles may be measured on its limb; and it is attached to a massive wall of solid masonry in order to insure perfect steadiness, a point the more difficult to attain in proportion as the instrument is heavier. The annexed diagram represents a Mural Circle fixed to its wall and ready for observations. It will be seen that every expedient is employed to give the instrument firmness of parts

Fig. 16.



and steadiness of position. Its radii are composed of hollow cones, uniting lightness and strength, and its telescope revolves on a large horizontal axis, fixed as firmly as possible in a solid wall. The graduations are made on the outer rim of the instrument, and are read off by six microscopes attached to the wall, one of which is represented at A, and the places of the five others

are marked by the letters B, C, D, E, F. Six are used in order that by taking the mean of such a number of readings, a higher degree of accuracy may be insured, than could be attained by a single reading. In a circle of six feet diameter, like that represented in the figure, the divisions may be easily carried to five minutes each. The microscope (which is of the variety called *compound microscope*) forms an enlarged image of each of these divisions in the focus of the eye glass. In the focus is also placed a delicate wire, which may be moved by means of a screw in a direction parallel to the divisions of the limb, and which is so adjusted to the screw as to move over the whole magnified space of five minutes by five revolutions of the screw. Of course one revolution of the screw measures one minute. Moreover, if the screw itself is made to carry an index attached to its axis and revolving with it over a disk graduated into sixty equal parts, then the space measured by moving the index over one of these parts, will be one second.

We have been thus minute in the description of this instrument, in order to give the learner some idea of the vast labor and great patience demanded of practical astronomers, in order to obtain measurements of such extreme accuracy as those to which they aspire.

On account of the great dimensions of this circle, and the expense attending it, as well as the difficulty of supporting it firmly, sometimes only one fourth of it is employed, constituting the

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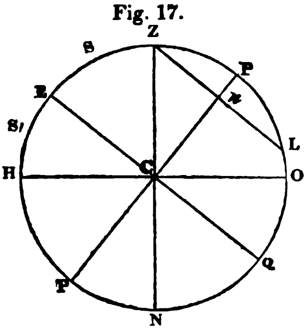
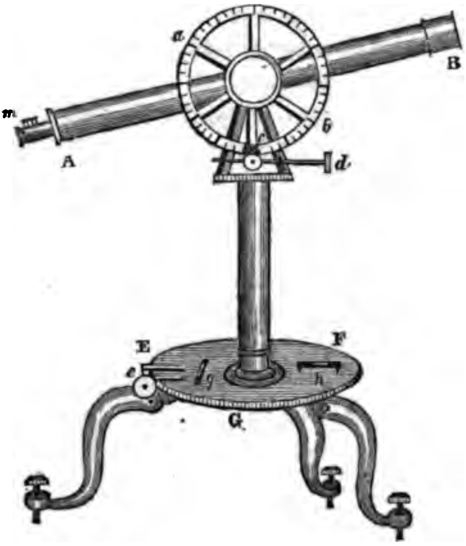
Fig. 17.



will best be found by taking its zenith distance ZS, of which it is the complement. From SH, subtract EH, the elevation of the equator, which equals the colatitude of the place of observation, (Art. 44,) and the remainder SE is the declination. Or if the star is nearer the horizon than the equator is, as at S′, subtract its meridian altitude from the colatitude, for the declination. Secondly, the declination may be found from the *north polar distance*, of which it is the complement. Thus from P to E is 90°. Therefore, PE−PS=90°−PS=SE=the declination. The height of the pole P is always known when the latitude of the place is known, being equal to the latitude.

128. The astronomical instruments already described are adapted to taking observations on the meridian only; but we sometimes require to know the *altitude* of a celestial body when it is not on the meridian, and its *azimuth*, or distance from the meridian measured on the horizon; and also the *angular distance* between two points on any part of the celestial sphere. An instrument especially designed to measure altitudes and azimuths, is called an *Altitude and Azimuth Instrument*, whatever may be its particular form. When a point is on the horizon its distance from the meridian, or its azimuth, may be taken by the common surveyor's compass, the direction of the meridian being determined by the needle; but when the object, as a star, is not on the horizon, its azimuth, it must be remembered, is the arc of the horizon from the meridian to a vertical circle passing through the star (Art. 27); at whatever different altitudes, therefore, two stars may be, and however the plane which passes through them may be inclined to the horizon, still it is their angular distance measured *on the horizon* which determines their difference of azimuth. Figure 18 represents an Altitude and Azimuth Instrument, several of the usual appendages and subordinate contrivances being omitted for the sake of distinctness and simplicity. Here *abo* is the vertical or altitude circle, and EFG the horizontal

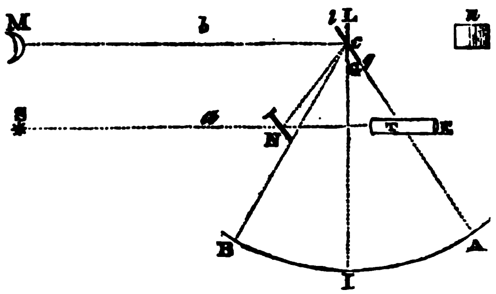
Fig. 18.



or azimuth circle; AB is a telescope mounted on a horizontal axis and capable of two motions, one in altitude parallel to the circle *abc*, and the other in azimuth parallel to EFG. Hence it can be easily brought to bear upon any object. At *m*, under the eye glass of the telescope, is a small mirror placed at an angle of 45° with the axis of the telescope, by means of which the image of the object is reflected upwards, so as to be conveniently pre-

[redacted]

Fig. 19.



and the index moves on a graduated arc BA. LG is called the *Index Glass*, and N the *Horizon Glass*. The under part only of the horizon glass is coated with quicksilver, the upper part being left transparent, as in *n*; so that while one object is seen through the upper part of *n* by direct vision, another may be seen through the lower part by reflexion from the two mirrors. The instrument is so contrived, that when the index is moved up to A, where the zero point is placed, or the graduation begins, the two reflectors LG and N are exactly parallel to each other, the index glass being then in the position *lg*. In this position of the mirrors, if the eye at E look through the telescope, T, so pointed as to see the star S through the transparent part of the horizon glass, it will see the same star, in the same place reflected from the silvered part; for the star (or any similar object) is at such a distance that the rays of light which strike upon the index glass LG are parallel to those which enter the eye directly. Therefore the angle of incidence *bc*N being equal to the angle of reflexion at *c*NE, the ray *b* will be made, by reflexion, to coincide with the ray *a*, and exhibit the object at the same place. Now, suppose it were the object to measure the angular distance between two bodies, as the moon and a star, and let the star be at S and the moon at M. The telescope being still directed to S, turn the index arm LI from A towards B until the image of the moon is brought down to S, its lower limb just touching S. By a principle in optics, the angular distance which the image of the moon passes over, is twice that of the mirror LG. But the mirror has passed over the graduated arc AI; therefore double that arc is the angular distance between the star and the moon's *lower* limb. If we then bring

the *upper* limb into contact with the star, the sum of both observations, divided by 2, will give the angular distance between the star and the moon's *center*. As each degree on the limb AB measures two degrees of angular distance, hence the divisions for single degrees are in fact only half a degree asunder; and a sextant, or the sixth part of the circle, measures an angular distance of 120°. The upper and lower points in the disk of the sun or of the moon may be considered as two separate objects, whose distance from each other may be taken in a similar manner, and thus their apparent diameters at any time be determined. We may select our points of observation either in a vertical, or in a horizontal direction.

130. If we make a star, or the limb of the sun or moon, one of the objects, and the point in the horizon directly beneath, the other, we thus obtain the *altitude* of the object. In this observation, the horizon is viewed through the transparent part of the horizon glass. At sea, where the horizon is usually well defined, the horizon itself may be used for taking altitudes; but on land, inequalities of the earth's surface, oblige us to have recourse to an *artificial horizon*. This in its simple state, is a basin of either water or quicksilver. By this means we see the image of the sun (or other body) just as far below the horizon as it is in reality above it. Hence, if we turn the index glass until the limb of the sun, as reflected from it, is brought into contact with [redacted]

observer; hence it is the chief instrument used for angular measurements at sea.

131. *Examples illustrating the use of the Sextant.*

Ex. 1.	Alt. ⊙'s lower limb, . .	49° 10′ 00″
	⊙'s semi-diameter, . .	0 15 51
		49° 25′ 51″
	Subtract Refraction, . .	00 00 49
		49° 25′ 02″
	Add Parallax, . . .	00 00 06
	True altitude ⊙'s center, .	49° 25′ 08″

Ex. 2. *With the Artificial Horizon.*

Altitude of ⊙'s upper limb above the image in the artificial horizon, 100° 2′ 47″.

True altitude, ...	50° 01′ 23.″5
⊙'s semi-diameter, . . .	00 15 50.
	49° 45′ 33.″5
Refraction, ...	00 00 48.
	49° 44′ 45.″5
Parallax, ...	00 00 05.
True altitude of ⊙'s center, . .	49° 44′ 50.″5

ASTRONOMICAL PROBLEMS.*

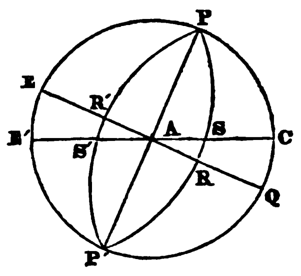
132. *Given the sun's Right Ascension and Declination, to find his Longitude and the Obliquity of the Ecliptic.*

Let PCP′ (Fig. 20,) represent the celestial meridian that passes through the first of Cancer and Capricorn, (the solstitial colure,) PP′ the axis of the sphere, EQ the equator, E′C the ecliptic, and PSP′ the declination circle (Art. 37,) passing through the sun S; then ARS is a right angle, and in the right angled spherical

* Young's Spherical Trigonometry, p. 136. Vince's Complete System, Vol. I.

triangle ARS, are given the right ascension AR (Art. 37,) and the declination RS, to find the longitude AS (Art. 37,) and the obliquity SAR.

Fig. 20.



As longitude and right ascension are measured from A, the first point of Aries, in the direction AS of the signs, quite round the globe, when, of the four quantities mentioned in the problem, the obliquity and the declination are given to find the others, we must know whether the sun is north, or whether it is south of the equator, the longitude being in the one case AS, and in the other, instead of AS', it is 360 – AS', that is, the supplement of AS'. We must also know on which side of the tropic the sun is, for the sun in passing from one of the tropics to the equinox, passes through the same degrees of declination as it had gone through in ascending from the other equinox to the tropic, although its longitude and right ascension go on continually increasing. From the 21st of March to the 21st of June, while describing the first quadrant from the vernal equinox, the declination is north and increasing; north but decreasing, in the second quadrant, until the 23d of September; south and increasing in the third quadrant, until the 21st of December; and finally, in the fourth quadrant, south but decreasing until the 21st of March.

Ex. 1. On the 17th of May, the sun's Right Ascension was [redacted]

Hence the computation for AS and A is as follows:

For the Longitude AS.		*For the Obliquity* A.	
cos. AR 53° 38′ 00″	9.7730185	sin. AR	9.9059247
cos. RS 19 15 57	9.9749710	tan. RS, ar. com.	0.4565209
cos. AS 55 57 43	9.7479895	cot. A 23° 27′ 50½″	10.3624456

Ex. 2. On the 31st of March, 1816, the sun's Declination was observed at Greenwich to be 4° 13′ 31½″: required his Right Ascension, the obliquity of the ecliptic being 23° 27′ 51″.

Ans. 9° 47′ 59″.

Ex. 3. What was the sun's Longitude on the 28th of November, 1810, when his Declination was 21° 16′ 4″, and his Right Ascension, in time, 16h. 14m. 58.4s.?

Ans. 245° 39′ 10″.

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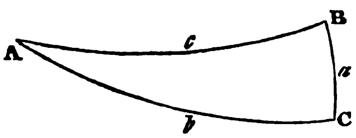
part, the two which lie next to it on each side will be *adjacent parts*. Thus, (in Fig. 21,) taking A for a middle part, b and c will be the adjacent parts; if we take c for the middle part, A and B will be the adjacent parts; if we take B for the middle part, c and a will be the adjacent parts; but if we take a for the middle part, then as the angle C is not considered as one of the circular parts, B and b are the adjacent parts; and, lastly, if b is the middle part, then the adjacent parts are A and a. The two parts immediately beyond the adjacent parts on each side, still disregarding the right angle, are called the *opposite parts*; thus if A is the middle part, the opposite parts are a and B. Napier's rule is as follows:

Fig. 21.



Radius into the sine of the middle part, equals the product of the tangents of the adjacent extremes, or of the cosines of the opposite extremes.

(The corresponding vowels are marked to aid the memory.) This rule is modified by using the *complements* of the two angles and the hypothenuse instead of the parts themselves. Thus instead of rad.$\times$sin. A, we say rad.$\times$cos. A, when A is the middle part; and rad.$\times$cos. AB, when the hypothenuse is the middle part.

Examples. 1. In the right angled triangle ABC, are given the two perpendicular sides, viz. a=48° 24′ 10″, b=59° 38′ 27″, to find the hypothenuse c. The hypothenuse being made the middle part, the other sides become the opposite parts, being separated from the middle part by the angles A and B. Hence, rad. cos. c= cos. a cos. b $\therefore$ cos. $c=\frac{\cos. a \cos. b}{\text{rad.}}$=70° 26′ 29″.

2. In the spherical triangle, right angled at C, are given two perpendicular sides, viz. a=116° 30′ 43″, b=29° 41′ 32″, to find the angle A.

Here, the required angle is *adjacent* to one of the given parts, viz. b, which make the middle part. Then,

$$\text{Rad.}\times\sin. b=\cot. A \tan. a \therefore \cot. A=\frac{\text{rad.}\times\sin. b}{\tan. a}=76°\ 7'\ 14''.$$

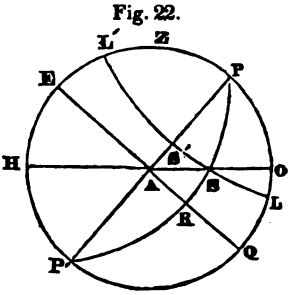
Ex. 4. The sun's Longitude being 8s. 7° 40′ 56″, and the Obliquity 23° 27′ 42¼″, what was the Right Ascension in time?

Ans. 16h. 23m. 34s.

133. ***Given the sun's Declination to find the time of his Rising and Setting at any place whose latitude is known.***

Let PEP′ (Fig. 22,) represent the meridian of the place, Z being the zenith, and HO the horizon; and let LL′ be the apparent path of the sun on the proposed day, cutting the horizon in S. Then the arc EZ will be the latitude of the place, and consequently EH, or its equal QO, will be the co-latitude, and this measures the angle OAQ; also RS will be the sun's declination, and AR expressed in time will be the time of rising before 6 o'clock. For it is evident that it will be sunrise when the sun arrives at the horizon at S; but PP′ being an hour circle whose plane is perpendicular to the meridian, (and of course projected into a straight line on the plane of projection,) the time the sun is passing from S to S′ taken from the time of describing S′L, which is six hours, must be the time from midnight to sunrise. But the time of describing SS′ is measured on the corresponding arc of the equinoctial AR.

Fig. 22.



In the right angled triangle ARS, we have the declination RS,

[redacted]

Ex. 2. Required the time of sunrise at latitude 57° 2′ 54″ N. when the sun's declination is 23° 28′.

Ans. 3h. 11m. 49s.

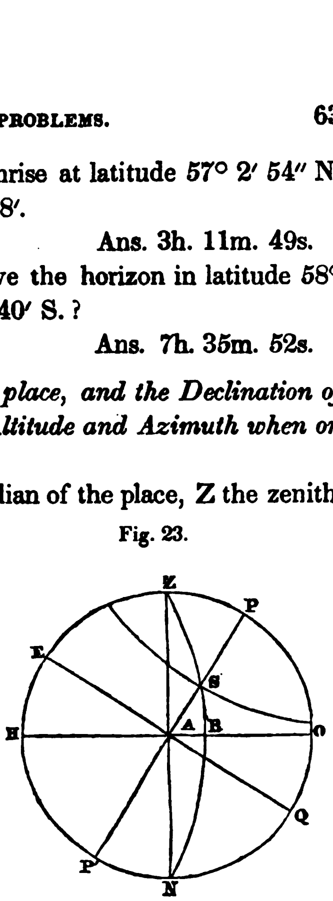
Ex. 3. How long is the sun above the horizon in latitude 58° 12′ N. when his declination is 18° 40′ S. ?

Ans. 7h. 35m. 52s.

134. *Given the Latitude of the place, and the Declination of a heavenly body, to determine its Altitude and Azimuth when on the six o'clock hour circle.*

Let HZO (Fig. 23,) be the meridian of the place, Z the zenith, HO the horizon, S the place of the object on the 6 o'clock hour circle PSP′, which of course cuts the equator in the east and west points, and ZSB the vertical circle passing through the sun. Then in the right angled triangle SBA, the given quantities are AS, which is the declination, and the arc OP or angle SAB, the latitude of the place, to find the altitude BS, and the azimuth BO, or the amplitude AB, which is its complement.

Fig. 23.



Ex. 1. What was the altitude and azimuth of Arcturus, when upon the six o'clock hour circle of Greenwich, lat. 51° 28′ 40″ N. on the first of April, 1822; its declination being 20° 6′ 50″ N. ?

For the Altitude.		*For the Azimuth.*	
Rad. sin. BS = sin. AS sin. A		Rad. cos. A = cot. BO cot. AS	
Rad. . .	10.	Cot. 20° 06′ 50″	10.4362545
Sin. 20° 06′ 50″	9.5364162	Cos. 51 28 40	9.7943612
Sin. 51 28 40	9.8934103	Rad. . .	10.
Sin. 15 36 27	9.4298265	Cot. 77° 09′ 04″	9.3581067

Ex. 2. At latitude 62° 12′ N. the altitude of the sun at 6 o'clock in the morning was found to be 18° 20′ 23″ : required his declination and azimuth.

Ans. Dec. 20° 50′ 12″ N. Az. 79° 56′ 11″.

135. *The Latitudes and Longitudes of two celestial objects being given, to find their Distance apart.*

Let P (Fig. 24,) represent the pole of the ecliptic, and PS, PS′, two arcs of celestial latitude (Art. 37,) drawn to the two objects SS′; then will these arcs represent the co-latitudes, the angle P will be the difference of longitude, and the arc SS′ will be the distance sought. Here we have the two sides and included angle given to find the third side. By Napier's Rules for the solution of oblique angled spherical triangles, (see Spherical Trigonometry,) the sum and difference of the two angles opposite the given sides may be found, and thence the angles themselves. The required side may then be found by the theorem, that the sines of the sides are as the sines of their opposite angles.* The computation is omitted here on account of its great length. If P be the pole of the *equator* instead of the ecliptic, then PS and PS′ will represent arcs of co-declination, and the angle P will denote difference of right ascension. From these data, also, we may therefore derive the distance between any two stars. Or, finally, if P be the pole of the *horizon*, the angle at P will denote difference of azimuth, and the sides PS, PS′, zenith distances, from which the side SS′ may likewise be determined.

Fig. 24.

FIGURE AND DENSITY OF THE EARTH.

136. We have already shown, (Art. 8,) that the figure of the earth is *nearly* globular; but since the semi-diameter of the earth [redacted]

the earth on its axis—by measuring *arcs of the meridian*—by experiments with the *pendulum*—and by the unequal action of the earth on the *moon*, arising from the redundance of matter about the equatorial regions. We will briefly consider each of these methods.

137. First, *the known effects of the centrifugal force, would give to the earth a spheroidal figure, elevated in the equatorial, and flattened in the polar regions.*

Had the earth been originally constituted (as geologists suppose) of yielding materials, either fluid or semi-fluid, so that its particles could obey their mutual attraction, while the body remained at rest it would spontaneously assume the figure of a perfect sphere; as soon, however, as it began to revolve on its axis, the greater velocity of the equatorial regions would give to them a greater centrifugal force, and cause the body to swell out into the form of an oblate spheroid. Even had the solid part of the earth consisted of unyielding materials and been created a perfect sphere, still the waters that covered it would have receded from the polar and have been accumulated in the equatorial regions, leaving bare extensive regions on the one side, and ascending to a mountainous elevation on the other.

On estimating, from the known dimensions of the earth and the velocity of its rotation, the amount of the centrifugal force in different latitudes, and the figure of equilibrium which would result, Newton inferred that the earth must have the form of an oblate spheroid before the fact had been established by observation; and he assigned nearly the true ratio of the polar and equatorial diameters.

138. Secondly, *the spheroidal figure of the earth is proved, by actually measuring the length of a degree on the meridian in different latitudes.*

Were the earth a perfect sphere, the section of it made by a plane passing through its center in any direction would be a perfect circle, whose curvature would be equal in all parts; but if we find by actual observation, that the curvature of the section is not uniform, we infer a corresponding departure in the earth from

the figure of a perfect sphere. This task of measuring portions of the meridian, has been executed in different countries by means of a system of triangles with astonishing accuracy.* The result is, that the length of a degree increases as we proceed from the equator towards the pole, as may be seen from the following table:

Places of observation.	Latitude.	Length of a degree in miles.
Peru,	00° 00′ 00″	68.732
Pennsylvania,	39 12 00	68,896
Italy,	43 01 00	68.998
France,	46 12 00	69.054
England,	51 29 54½	69.146
Sweden,	66 20 10	69.292

Combining the results of various estimates, the dimensions of the terrestrial spheroid are found to be as follows:

Equatorial diameter, ...	7925.648
Polar diameter, ...	7899.170
Mean diameter, ...	7912.409

The difference between the greatest and least, is $26.478 = \frac{1}{300}$ of the greatest. This fraction ($\frac{1}{300}$) is denominated the *ellipticity* of the earth, being the excess of the transverse over the conjugate axis, on the supposition that the section of the earth coinciding with the meridian, is an ellipse; and that such is the case, is proved by the fact that calculations on this hypothesis, of the lengths of arcs of the meridian in different latitudes, agree with the lengths obtained by actual measurement.

[redacted]

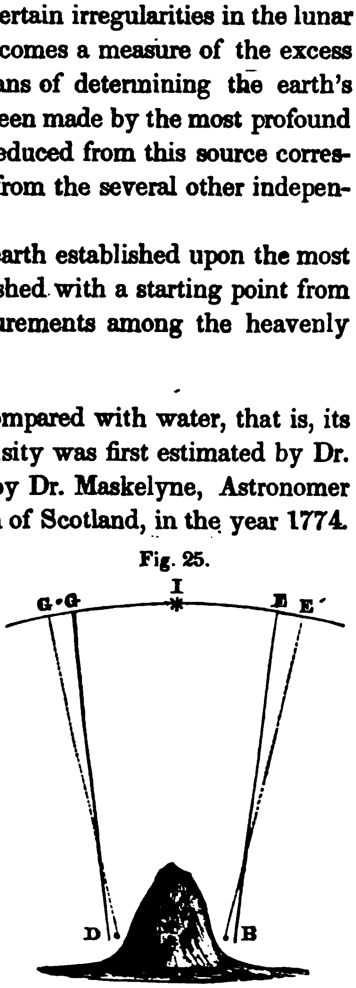
140. Fourthly, *that the earth is of a spheroidal figure, is inferred from the motions of the moon.*

These are found to be affected by the excess of matter about the equatorial regions, producing certain irregularities in the lunar motions, the amount of which becomes a measure of the excess itself, and hence affords the means of determining the earth's ellipticity. This calculation has been made by the most profound mathematicians, and the figure deduced from this source corresponds very nearly to that derived from the several other independent methods.

We thus have the shape of the earth established upon the most satisfactory evidence, and are furnished with a starting point from which to determine various measurements among the heavenly bodies.

141. The *density* of the earth compared with water, that is, its specific gravity, is $5\frac{1}{2}$.* The density was first estimated by Dr. Hutton, from observations made by Dr. Maskelyne, Astronomer Royal, on Schehallien, a mountain of Scotland, in the year 1774. Thus, let M (Fig. 25,) represent the mountain, D, B, two stations on opposite sides of the mountain, and I a star; and let IE and IG be the zenith distances as determined by the differences of latitudes of the two stations. But the apparent zenith distances as determined by the plumb line are IE′ and IG′. The deviation towards the mountain on each side exceeded 7″.† The attraction of the mountain being observed on both sides of it, and its mass being computed from a number of sections taken in all directions, these data, when compared with the known attraction and magnitude of the earth, led to a knowledge of its mean density. According to Dr. Hutton, this is to that of water as 9 to 2;

Fig. 25.



* Bailly, Ast. Tables, p. 21. † Robison's Phys. Ast.

but later and more accurate estimates have made the specific gravity of the earth as stated above. But this density is nearly double the average density of the materials that compose the exterior crust of the earth, showing a great increase of density towards the center.

The density of the earth is an important element, as we shall find that it helps us to a knowledge of the density of each of the other members of the solar system.

PART II.—OF THE SOLAR SYSTEM.

142. Having considered the Earth, in its astronomical relations, and the Doctrine of the Sphere, we proceed now to a survey of the Solar System, and shall treat successively of the Sun, Moon, Planets, and Comets.

CHAPTER I.

OF THE SUN—SOLAR SPOTS—ZODIACAL LIGHT.

143. The *figure* which the sun presents to us is that of a perfect circle, whereas most of the planets exhibit a disk more or less elliptical, indicating that the true shape of the body is an oblate spheroid. So great, however, is the distance of the sun, that a line 400 miles long would subtend an angle of only 1″ at the eye, and would therefore be the least space that could be measured. Hence, were the difference between two conjugate diameters of the sun any quantity less than this, we could not determine by actual measurement that it existed at all. Still we learn from theoretical considerations, founded upon the known effects of centrifugal force, arising from the sun's revolution on his axis, that his figure is not a perfect sphere, but is slightly spheroidal.*

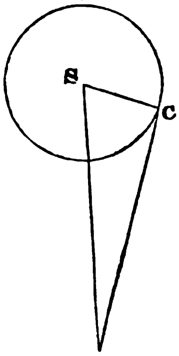
144. The *distance of the sun from the earth,* is nearly 95,000,000 miles. For, its horizontal parallax being 8.″6, (Art. 86,) and the semi-diameter of the earth 3956 miles,

Sin. 8.″6 : 3956 : : Rad. : 95,000,000 nearly. In order to form some faint conception at least of this vast distance, let us reflect that a railway car, moving at the rate of 20 miles per hour, would require more than 500 years to reach the sun.

* See Mecanique Celeste, III, 165. Delambre, t. I, p. 483.

145. The apparent *diameter* of the sun may be found either by the Sextant, (Art. 129,) by an instrument called the *Heliometer*, specially designed for measuring its angular breadth, or by the time it occupies in crossing the meridian. If, for example, it occupied 4^m, its angular diameter would be $1°$. It in fact occupies a little more than 2^m, and hence its apparent diameter is a little more than half a degree, $(32' 3'')$. Having the distance and angular diameter, we can easily find its *linear* diameter. Let E (Fig. 26,) be the earth, S the sun, ES a line drawn to the center of the disk, and EC a line drawn touching the disk at C. Join SC; then

Fig. 26.



Rad. : ES (95,000,000) : : sin. $16' 1.''5$: 442840 = semi-diameter, and 885680 = diameter. And $\frac{885680}{7912} = 112$ nearly; that is, it would require one hundred and twelve bodies like the earth, if laid side by side, to reach across the diameter of the sun; and a ship sailing at the rate of ten knots an hour, would require more than ten years to sail across the solar disk. Since spheres are to each other as the cubes of their diameters,

$1^3 : 112^3 :: 1 : 1,400,000$ nearly; that is, the sun is about 1,400,000 times as large as the earth. The distance of the moon from the earth being 237,000 miles, were the center of the sun made to coincide with the center of the earth, the sun would

[redacted]

distance of the center of force were the same in both cases; but since the attraction of a sphere is the same as though all the matter were collected in the center, consequently, the weight of a body, so far as it depends on its distance from the center of force, would be the square of 112 times less at the sun than at the earth. Or, putting W for the weight at the earth, and W′ for the weight at the sun, then

$$W : W' :: \frac{1}{1^2} : \frac{350000}{(112)^2} = 27.9 \text{ lbs.}$$

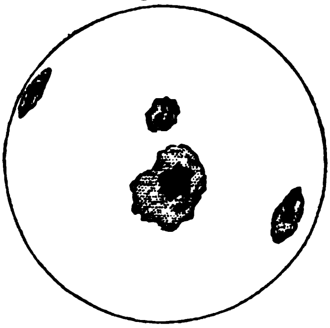
Hence a body would weigh nearly 28 times as much at the sun as at the earth. A man weighing 200 lbs. would, if transported to the surface of the sun, weigh 5,580 lbs., or nearly $2\frac{1}{2}$ tons. To lift one's limbs, would, in such a case, be beyond the ordinary power of the muscles. At the surface of the earth, a body falls through $16\frac{1}{12}$ feet in a second; and since the spaces are as the velocities, the times being equal, and the velocities as the forces, therefore a body would fall at the sun in one second, through $16\frac{1}{12} \times 27\frac{9}{10} = 448.7$ feet.

SOLAR SPOTS.

147. The surface of the sun, when viewed with a telescope, usually exhibits dark spots, which vary much, at different times, in number, figure, and extent. One hundred or more, assembled in several distinct groups, are sometimes visible at once on the solar disk. The greatest part of the solar spots are commonly very small, but occasionally a spot of enormous size is seen occupying an extent of 50,000 miles in diameter. They are sometimes even visible to the naked eye, when the sun is viewed through colored glass, or, when near the horizon, it is seen through light clouds or vapors. When it is recollected that 1″ of the solar disk implies an extent of 400 miles, (Art. 143,) it is evident that a space large enough to be seen by the naked eye, must cover a very large extent.

A solar spot usually consists of two parts, the *nucleus* and the *umbra*, (Fig. 27.) The nucleus is black, of a very irregular shape, and is subject to great and sudden changes, both in form and size. Spots have sometimes seemed to burst asunder, and to project fragments in different directions. The umbra is a wide margin of lighter

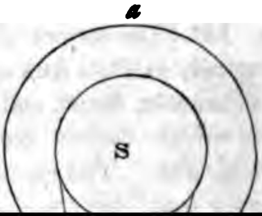
Fig. 27.



shade, and is often of greater extent than the nucleus. The spots are usually confined to a zone extending across the central regions of the sun, not exceeding 60° in breadth. When the spots are observed from day to day, they are seen *to move across the disk of the sun*, occupying about two weeks in passing from one limb to the other. After an absence of about the same period, the spot returns, having taken 27d. 7h. 37m. in the entire revolution.

148. The spots must be nearly or quite *in contact with the body of the sun.* Were they at any considerable distance from it, the time during which they would be seen on the solar disk, would be less than that occupied in the remainder of the revolution. Thus, let S (Fig. 28,) be the sun, E the earth, and *abc* the path of the body, revolving about the sun.

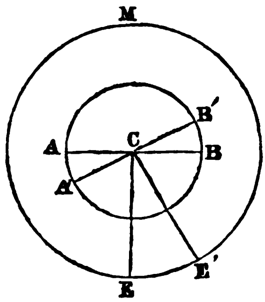
Fig. 28.



Unless the spot were nearly or quite in contact with the body of the sun, being projected upon his disk only while passing from *b* to *c*, and being invisible while describing the arc *cab*, it would of course be out of sight lon-

exceeds it by nearly two days. For, let AA′B (Fig. 29,) represent the sun, and EE′M the orbit of the earth. Thus, when the earth is at E, the visible disk of the sun will be AA′B; and if the earth remained stationary at E, the time occupied by a spot after leaving A until it returned to A, would be just equal to the time of the sun's revolution on his axis. But during the 27½ days in which the spot has been performing its apparent revolution, the earth has been advancing in his orbit from E to E′, where the visible disk of the sun is A′B′. Consequently, before the spot can appear again on the limb from which it set out, it must describe so much more than an entire revolution as equals the arc AA′, which equals the arc EE′. Hence,

Fig. 29.



365d. 5h. 48m.+27d. 7h. 37m. : 365d. 5h. 48m. : : 27d. 7h. 37m. : 25d. 9h. 56m. = the time of the sun's revolution on his axis.

149. If the path which the spots appear to describe by the revolution of the sun on his axis left each a visible trace on his surface, they would form, like the circles of diurnal revolution on the earth, so many parallel rings, of which that which passed through the center would constitute the solar equator, while those on each side of this great circle would be small circles, corresponding to parallels of latitude on the earth. Let us conceive of an artificial sphere to represent the sun, having such rings plainly marked on its surface. Let this sphere be placed at some distance from the eye, with its axis perpendicular to the axis of vision, in which case the equator would coincide with the line of vision, and its edge be presented to the eye. It would therefore be projected into a straight line. The same would be the case with all the smaller rings, the distance being supposed such that the rays of light come from them all to the eye nearly parallel. Now let the axis, instead of being perpendicular to the line of vision, be inclined to that line, then all the rings being seen obliquely would be projected into ellipses. If, however, while the sphere remained in a fixed position, the eye were carried around it,

10

(being always in the same plane,) twice during the circuit it would be in the plane of the equator, and project this and all the smaller circles into straight lines; and twice, at points 90° distant from the foregoing positions, the eye would be at a distance from the planes of the rings equal to the inclination of the equator of the sphere to the line of vision. Here it would project the rings into wider ellipses than at other points; and the ellipses would become more and more acute as the eye departed from either of these points, until they vanished again into straight lines.

150. It is in a similar manner that the eye views the paths described by the spots on the sun. If the sun revolved on an axis perpendicular to the plane of the earth's orbit, the eye being situated in the plane of revolution, and at such a distance from the sun that the light comes to the eye from all parts of the solar disk nearly parallel, the paths described by the spots would be projected into straight lines, and each would describe a straight line across the solar disk, parallel to the axis of vision. But the axis of the sun is inclined to the ecliptic about 7½° from a perpendicular, so that usually all the circles described by the spots are projected into ellipses. The breadth of these, however, will vary as the eye, in the annual revolution, is carried around the sun, and when the eye comes into the plane of the rings, as it does twice a year, they are projected into straight lines, and for a short time a spot seems moving in a straight line inclined to the [redacted]

151. With regard to the *cause* of the solar spots, various hypotheses have been proposed, none of which is entirely satisfactory. That which ascribes their origin to *volcanic action*, appears to us the most reasonable.*

Besides the dark spots on the sun, there are also seen, in different parts, places that are brighter than the neighboring portions of the disk. These are called *faculæ*. Other inequalities are observable in powerful telescopes, all indicating that the surface of the sun is in a state of constant and powerful agitation.

ZODIACAL LIGHT.

152. The Zodiacal Light is a faint light resembling the tail of a comet, and is seen at certain seasons of the year following the course of the sun after evening twilight, or preceding his approach in the morning sky. Figure 30 represents its appearance as seen in the evening in March, 1836. The following are the leading facts respecting it.

Fig. 30.

1. *Its form is that of a luminous pyramid*, having its base towards the sun. It reaches to an immense distance from the sun, sometimes even beyond the orbit of the earth. It is brighter in the parts nearer the sun than in those that are more remote, and terminates in an obtuse apex, its light fading away by insensible gradations, until it becomes too feeble for distinct vision. Hence its limits are at the same time, fixed at different distances from the sun by different observers, according to their respective powers of vision.

2. *Its aspects vary very much with the different seasons of the year*. About the first of October, in our climate (Lat. 41° 18′) it becomes visible before the dawn of day, rising along north of

* In the system of instruction in Yale College, subjects of this kind are discussed in a course of astronomical lectures, addressed to the class after they have finished the perusal of the text-book.

the ecliptic, and terminating above the nebula of Cancer. About the middle of November, its vertex is in the constellation Leo. At this time no traces of it are seen in the west after sunset, but about the first of December it becomes faintly visible in the west, crossing the Milky Way near the horizon, and reaching from the sun to the head of Capricornus, forming, as its brightness increases, a counterpart to the Milky Way, between which on the right, and the Zodiacal Light on the left, lies a triangular space embracing the Dolphin. Through the month of December, the Zodiacal Light is seen on both sides of the sun, namely, before the morning and after the evening twilight, sometimes extending 50° westward, and 70° eastward of the sun at the same time. After it begins to appear in the western sky, it increases rapidly from night to night, both in length and brightness, and withdraws itself from the morning sky, where it is scarcely seen after the month of December, until the next October.

3. *The Zodiacal Light moves through the heavens in the order of the signs.* It moves with unequal velocity, being sometimes stationary and sometimes retrograde, while at other times it advances much faster than the sun. In February and March, it is very conspicuous in the west, reaching to the Pleiades and beyond; but in April it becomes more faint, and nearly or quite disappears during the month of May. It is scarcely seen in this latitude during the summer months.

4. *It is remarkably conspicuous at certain periods of a few* [redacted]

CHAPTER II.

OF THE APPARENT ANNUAL MOTION OF THE SUN—SEASONS—FIGURE OF THE EARTH'S ORBIT.

153. The revolution of the earth around the sun once a year, produces an apparent motion of the sun around the earth in the same period. When bodies are at such a distance from each other as the earth and the sun, a spectator on either would project the other body upon the concave sphere of the heavens, always seeing it on the opposite side of a great circle, 180° from himself. Thus when the earth arrives at Libra (Fig. 11,) we see the sun in the opposite sign Aries. When the earth moves from Libra to Scorpio, as we are unconscious of our own motion, the sun it is that appears to move from Aries to Taurus, being always seen in the heavens, where a line drawn from the eye of the spectator through the body meets the concave sphere of the heavens. Hence the line of projection carries the sun forward on one side of the ecliptic, at the same rate as the earth moves on the opposite side; and therefore, although we are unconscious of our own motion, we can read it from day to day in the motions of the sun. If we could see the stars at the same time with the sun, we could actually observe from day to day the sun's progress through them, as we observe the progress of the moon at night; only the sun's rate of motion would be nearly fourteen times slower than that of the moon. Although we do not see the stars when the sun is present, yet after the sun is set, we can observe that it makes daily progress eastward, as is apparent from the constellations of the Zodiac occupying, successively, the western sky after sunset, proving that either all the stars have a common motion eastward independent of their diurnal motion, or that the sun has a motion past them, from west to east. We shall see hereafter abundant evidence to prove, that this change in the relative position of the sun and stars, is owing to a change in the apparent place of the sun, and not to any change in the stars.

154. Although the apparent revolution of the sun is in a direction opposite to the real motion of the earth, as regards absolute space, yet both are nevertheless from west to east, since these terms do not refer to any directions in absolute space, but to the order in which certain constellations (the constellations of the Zodiac) succeed one another. The earth itself, on opposite sides of its orbit, does in fact move towards directly opposite points of space; but it is all the while pursuing its course in the order of the signs. In the same manner, although the earth turns on its axis from west to east, yet any place on the surface of the earth is moving in a direction in space exactly opposite to its direction twelve hours before. If the sun left a visible trace on the face of the sky, the ecliptic would of course be distinctly marked on the celestial sphere as it is on an artificial globe; and were the equator delineated in a similar manner, (by any method like that supposed in Art. 46,) we should then see at a glance the relative position of these two circles, the points where they intersect one another constituting the equinoxes, the points where they are at the greatest distance asunder, or the solstices, and various other particulars, which, for want of such visible traces, we are now obliged to search for by indirect and circuitous methods. It will even aid the learner to have constantly before his mental vision, an imaginary delineation of these two important circles on the face of the sky.

[redacted]

clination is readily found, by subtracting that distance from the latitude. By thus taking the sun's declination for every day of the year at noon, and comparing the results, we learn its motion *to and from the equator.*

156. To obtain the motion in right ascension, we observe, with a transit instrument, the instant when the center of the sun is on the meridian. Our sidereal clock gives us the right ascension in time (Art. 124,) which we may easily, if we choose, convert into degrees and minutes, although it is more common to express right ascension by hours, minutes, and seconds. The differences of right ascension from day to day throughout the year, give us the sun's annual motion *parallel to the equator.* From the daily records of these two motions, at right angles to each other, arranged in a table,* it is easy to trace out the path of the sun on the artificial globe; or to calculate it with the greatest precision by means of spherical triangles, since the declination and right ascension constitute two sides of a right angled spherical triangle, the corresponding arc of the ecliptic, that is, the longitude, being the third side, (Art. 132.) By inspecting a table of observations, we shall find that the declination attains its greatest value on the 22d of December, when it is 23° 27′ 54″ south; that from this period it diminishes daily and becomes nothing on the 21st of March; that it then increases towards the north, and reaches a similar maximum at the northern tropic about the 22d of June; and, finally, that it returns again to the southern tropic by gradations similar to those which marked its northward progress. Our table of observations also shows us, that the daily differences of declination are very unequal; that, for several days, when the sun is near either tropic, its declination scarcely varies at all; while near the equator, the variations from day to day are very rapid,—a fact which is easily understood, when we reflect, that at the solstices the equator and the ecliptic are parallel to each other,† both being at right angles to the meridian; while at the

* Such a table may be found in Biot's Astronomy, in Delambre, and in most collections of Astronomical Tables.

† Or, more properly, the *tangents* of the two circles (which denote the directions of the curves at those points) are parallel.

equinoxes, the ecliptic departs most rapidly from the direction of the equator.

On examining, in like manner, a table of observations of the right ascension, we find that the daily differences of right ascension are likewise unequal; that the mean of them all is 3′ 56″, or 236″, but that they have varied between 215″ and 266″. On examining, moreover, the right ascension at each of the equinoxes, we find that the two records differ by 180°; which proves that the path of the sun is a great circle, since no other would bisect the equinoctial as this does.

157. *The obliquity of the ecliptic is equal to the sun's greatest declination.* For, by article 22, the inclination of any two great circles is equal to their greatest distance asunder, as measured on the sphere. The obliquity of the ecliptic may be determined from the sun's meridian altitude, or zenith distance, on the day of the solstice. The exact instant of the solstice, however, will not of course occur when the sun is on the meridian, but may happen at some other meridian; still, the changes of declination near the solstice are so exceedingly small, that no material error will result from this source. The obliquity may also be found, without knowing the latitude, by observing the greatest and least meridian altitudes of the sun, and taking half the difference. This is the method practiced in ancient times by Hipparchus, (Art. 2.) On comparing observations made at different periods for more than two thousand years, it is found, that the obliquity

[redacted]

Since the distance of the earth from the sun is 95,000,000 miles, and the length of the entire orbit nearly 600,000,000 miles, it will be found, on calculation, that the earth moves 1,640,000 miles per day, 68,000 miles per hour, 1,100 miles per minute, and nearly 19 miles every second, a velocity nearly sixty times as great as the maximum velocity of a cannon ball. A place on the earth's equator turns, in the diurnal revolution, at the rate of about 1,000 miles an hour and $\frac{5}{17}$ of a mile per second. The motion around the sun, therefore, is nearly 70 times as swift as the greatest motion around the axis.

THE SEASONS.

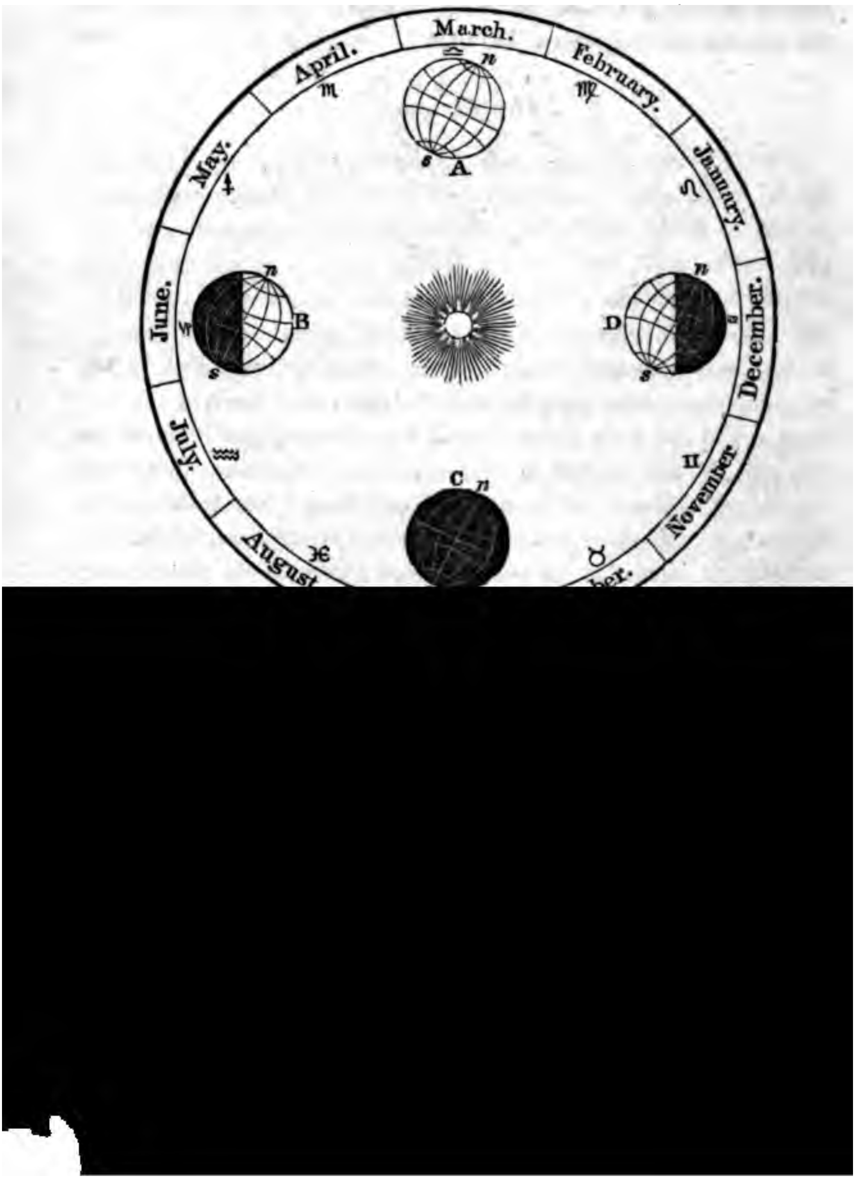
159. *The change of seasons depends on two causes,* (1) *the obliquity of the ecliptic, and* (2) *the earth's axis always remaining parallel to itself.* Had the earth's axis been perpendicular to the plane of its orbit, the equator would have coincided with the ecliptic, and the sun would have constantly appeared in the equator. To the inhabitants of the equatorial regions, the sun would always have appeared to move in the prime vertical; and to the inhabitants of either pole, he would always have been in the horizon. But the axis being turned out of a perpendicular direction 23° 28′, the equator is turned the same distance out of the ecliptic; and since the equator and ecliptic are two great circles which cut each other in two opposite points, the sun, while performing his circuit in the ecliptic, must evidently be once a year in each of those points, and must depart from the equator of the heavens to a distance on either side equal to the inclination of the two circles, that is, 23° 28′. (Art. 22.)

160. The earth being a globe, the sun constantly enlightens the half next to him,* while the other half is in darkness. The boundary between the enlightened and the unenlightened part, is called *the circle of illumination.* When the earth is at one of the equinoxes, the sun is at the other, and the circle of illumina-

* In fact, the sun enlightens a little more than half the earth, since on account of his vast magnitude the tangents drawn from opposite sides of the sun to opposite sides of the earth, converge to a point behind the earth, as will be seen by and by in the representation of eclipses.

tion passes through both the poles. When the earth reaches one of the tropics, the sun being at the other, the circle of illumination cuts the earth, so as to pass 23° 28′ beyond the nearer, and the same distance short of the remoter pole. These results would not be uniform, were not the earth's axis always to remain parallel to itself. The following figure will illustrate the foregoing statements.

Fig. 31.



son of every place, of course the day and night will be equal in all parts of the globe.* Again, at B when the earth is at the southern tropic, the sun shines 23½° beyond the north pole *n*, and falls the same distance short of the south pole *s*. The case is exactly reversed when the earth is at the northern tropic and the sun at the southern. While the earth is at one of the tropics, at B for example, let us conceive of it as turning on its axis, and we shall readily see that all that part of the earth which lies within the north polar circle will enjoy continual day, while that within the south polar circle will have continual night, and that all other places will have their days longer as they are nearer to the enlightened pole, and shorter as they are nearer to the unenlightened pole. This figure likewise shows the successive positions of the earth at different periods of the year, with respect to the signs, and what months correspond to particular signs. Thus the earth enters Libra and the sun Aries on the 21st of March, and on the 21st of June the earth is just entering Capricorn and the sun Cancer.

161. Had the axis of the earth been perpendicular to the plane of the ecliptic, then the sun would always have appeared to move in the equator, the days would every where have been equal to the nights, and there could have been no change of seasons. On the other hand, had the inclination of the ecliptic to the equator been much greater than it is, the vicissitudes of the seasons would have been proportionally greater than at present. Suppose, for instance, the equator had been at right angles to the ecliptic, in which case the poles of the earth would have been situated in the ecliptic itself; then in different parts of the earth the appearances would have been as follows. To a spectator on the equator, the sun as he left the vernal equinox would every day perform his diurnal revolution in a smaller and smaller circle, until he reached the north pole, when he would halt for a moment and then wheel about and return to the equator in the reverse order. The progress of the sun through the southern signs, to the south pole, would be similar to that already described. Such would be the

* At the pole, the solar disk, at the time of the equinox, appears bisected by the horizon.

appearances to an inhabitant of the equatorial regions. To a spectator living in an oblique sphere, in our own latitude for example, the sun while north of the equator would advance continually northward, making his diurnal circuits in parallels farther and farther distant from the equator, until he reached the circle of perpetual apparition, after which he would climb by a spiral course to the north star, and then as rapidly return to the equator. By a similar progress southward, the sun would at length pass the circle of perpetual occultation, and for some time (which would be longer or shorter according to the latitude of the place of observation) there would be continual night.

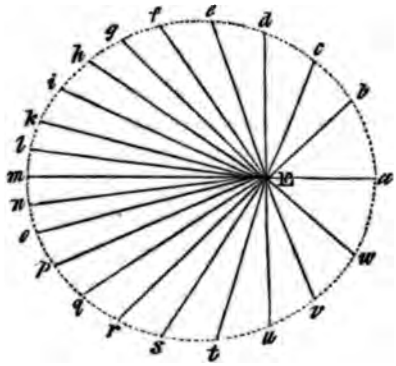
The great vicissitudes of heat and cold which would attend such a motion of the sun, would be wholly incompatible with the existence of either the animal or the vegetable kingdoms, and all terrestrial nature would be doomed to perpetual sterility and desolation. The happy provision which the Creator has made against such extreme vicissitudes, by confining the changes of the seasons within such narrow bounds, conspires with many other express arrangements in the economy of nature to secure the safety and comfort of the human race.

FIGURE OF THE EARTH'S ORBIT.

162. Thus far we have taken the earth's orbit as a great circle, such being the projection of it on the celestial sphere; but we

on further trial, we find that it has the properties of an ellipse. Thus, let E (Fig. 32,) be the place of the earth, and *a*, *b*, *c*, &c. successive positions of the sun; the *relative* lengths of the lines E*a*, E*b*, &c. being known: on connecting the points, *a*, *b*, *c*, &c. the resulting figure indicates the true shape of the earth's orbit.

Fig. 32.



163. These relative distances are found in two different ways; first, *by changes in the sun's apparent diameter*, and, secondly, *by variations in his angular velocity*. Were the variations in the sun's horizontal parallax considerable, as is the case with the moon's, this might be made the measure of the relative distances, for the parallax varies inversely as the distance, (Art. 82); but the whole horizontal parallax of the sun is only 9″, and its variations are too slight and delicate, and too difficult to be found, to serve as a criterion of the changes in the sun's distance from the earth. But the changes in the *sun's apparent diameter*, are much more sensible, and furnish a better method of measuring the relative distances of the earth from the sun. By a principle in optics, the apparent diameter of an object, at different distances from the spectator, is inversely as the distance.* Hence, the apparent diameters of the sun, taken at different periods of the year, become measures of the different lengths of the radius vector.

* More exactly, the *tangent* of the apparent diameter is inversely as the distance; but in small angles like those concerned in the present inquiry, the angle itself may be taken for the tangent.

164. The point where the earth, or any planet, in its revolution, is nearest the sun, is called its *perihelion:* the point where it is farthest from the sun, its *aphelion*. The place of the earth's perihelion is known, since there the apparent magnitude of the sun is greatest; and when the sun's magnitude is least, the earth is known to be at its aphelion. The sun's apparent diameter when greatest is 32′ 35.″6; and when least, 31′ 31″; hence the radius vector at the aphelion : rad. vector at the perihelion::32.5933 : 31.5167::1.034 : 1. Half the difference of the two is equal to the distance of the focus of the ellipse from the center, a quantity which is always taken as the measure of the *eccentricity* of a planetary orbit.

165. The differences of *angular velocity* in the sun in the different parts of his apparent revolution, are still more remarkable. At the perihelion, the sun moves in twenty four hours over an arc of 61′, while at the aphelion he describes in the same time an arc of only 57′, these being the daily increments of longitude in those two points respectively. If the apparent motions of the sun depended alone on our different distances from him, the angular velocity would vary inversely as the distance, and the ratio expressed by these two numbers would be the same as that of the two numbers which denote the differences of apparent diameter in these two points. That is, $\frac{61}{57}$ ($= 1.07$) would equal 32.5933

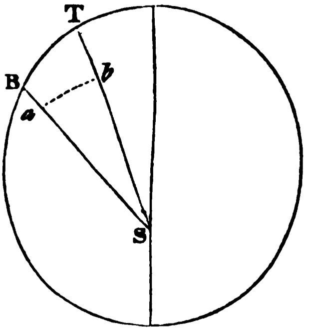
[redacted]

this principle, the relative distances of the earth from the sun, in every point of its revolution, may be easily calculated. Thus, we have seen that the arcs described by the sun in one day at the perihelion and aphelion are as 61 to 57. Hence the distances of the earth from the sun at those two points are as $\sqrt{57}$ to $\sqrt{61}$, or as 1 to 1.034. From twenty four observations made with the greatest care by Dr. Maskelyne at the Royal Observatory of Greenwich, the following distances of the earth from the sun are determined for each month in the year.

Time of Observation.		*Distances.*	*Time of Observation.*		*Distances.*
January	12–13,	0.98448	July	18–19,	1.01658
February	17–18,	0.98950	August	26–27,	1.01042
March	14–15,	0.99622	September	22–23,	1.00283
April	28–29,	1.00800	October	24–25,	0.99303
May	15–16,	1.01234	November	18–20,	0.98746
June	17–18,	1.01654	December	17–18,	0.98415

167. The angular velocity being inversely as the square of the distance in all parts of the solar orbit, it follows that *the product of the angle described in any given time, by the square of the distance, is always the same constant quantity.* For if of two factors, A×B, A is increased as B is diminished, the product of A and B is always the same. If, therefore, from the sun S (Fig. 33,) two radii be drawn to T, B, the extremities of the arc described in one day, then $ST^2 \times TB$ gives the same product in all parts of the orbit.*

Fig. 33.



168. *The radius vector of the solar orbit describes equal spaces in equal times, and in unequal times, spaces proportional to the times.*

Let TB (Fig. 33,) be the arc described by the sun in one day; then, Sector TSB=½SB×TB.

* TB, as seen from the earth, would be projected into a circular arc, equal to the measure of the angle at S.

Taking Sb as any radius, describe the circular arc ab, which is the measure of the angle at S. Now,

$\mathrm{S}b : ab :: \mathrm{SB} : \mathrm{BT} = \mathrm{SB} \times \frac{ab}{\mathrm{S}b}$; and substituting this value of BT in the above equation, we have $\mathrm{TSB} = \frac{1}{2}\mathrm{SB} \times \mathrm{SB} \times \frac{ab}{\mathrm{S}b} = \frac{1}{2}\mathrm{SB}^2 \times \frac{ab}{\mathrm{S}b}$. But S$b$ is constant, and the product of $\mathrm{SB}^2 \times ab$ is likewise constant; therefore the sector is always equal to a constant quantity, and therefore the radius vector passes over equal spaces in equal times.*

The sun's orbit may be accurately represented by taking some point as the perihelion, drawing the radius vector to that point, and, considering this line as unity, drawing other radii making angles with each other such that the included areas shall be proportional to the times, and of a length required by the distance of each point as given in the table (Art. 166.) On connecting these radii, we shall thus see at once how little the earth's orbit departs from a perfect circle. Small as the difference appears between the greatest and least distances, yet it amounts to nearly $\frac{1}{30}$ of the perihelion distance, a quantity no less than 3,000,000 of miles.

169. The foregoing method of determining the figure of the earth's orbit is founded on *observation;* but this figure is subject to numerous irregularities, the nature of which cannot be clearly understood without a knowledge of the leading principles of [redacted]

CHAPTER III.

OF UNIVERSAL GRAVITATION.

170. UNIVERSAL GRAVITATION, *is that influence by which every body in the universe, whether great or small, tends towards every other, with a force which is directly as the quantity of matter, and inversely as the square of the distance.*

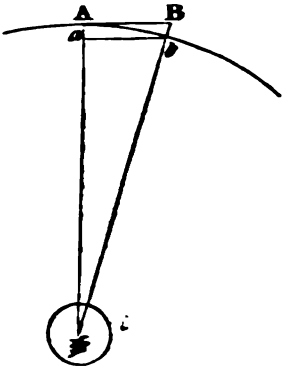
As this force acts as though bodies were drawn towards each other by a mutual attraction, the force is denominated *attraction;* but it must be borne in mind, that this term is figurative, and implies nothing respecting the nature of the force.

The *existence* of such a force in nature was distinctly asserted by several astronomers previous to the time of Sir Isaac Newton, but its *laws* were first promulgated by this wonderful man in his Principia, in the year 1687. It is related, that while sitting in a garden, and musing on the cause of the falling of an apple, he reasoned thus:* that, since bodies far removed from the earth fall towards it, as from the tops of towers, and the highest mountains, why may not the same influence extend even to the moon; and if so, may not this be the reason why the moon is made to revolve around the earth, as would be the case with a cannon ball were it projected horizontally near the earth with a certain velocity. According to the first law of motion, the moon, if not continually drawn or impelled towards the earth by some force, would not revolve around it, but would proceed on in a straight line. But going around the earth as she does, in an orbit that is nearly circular, she must be urged towards the earth by some force, which, in a given time, may be represented by the versed sine of the arc described in that time. For let the earth (Fig. 34,) be at E, and let the arc described by the moon in one second of time be A*b*. Were the moon influenced by no extraneous force, to turn her aside, she would have described, not the arc A*b*, but the straight line AB, and would have been found at

* Pemberton's View of Newton's Philosophy.

the end of the given time at B instead of *b*. She therefore departs from the line in which she tends naturally to move, by the line B*b*, which in small angles may be taken as equal to the versed sine A*a*. This deviation from the tangent must be owing to *some* extraneous force. Does this force correspond to what the force of gravity exerted by the earth, would be at the distance of the moon? Now we know the distance of the moon from the earth, and of course the circumference of her orbit. We also know the time of her revolution around the earth. Hence we may estimate the length of the arc A*b* described in one second; and knowing the arc we can calculate its versed sine. For the moon being 60 times as far from the center of the earth, as the surface of the earth is from the center, consequently, since the force of gravity decreases as the square of the distance increases, the space through which the moon would fall by the force of the earth's attraction alone, would be $\frac{16\frac{1}{12}}{60^2}=.05$ inches. On calculating the value of the versed sine of the arc described in one second, it proves to be the same. Hence gravity, and no other force than gravity, causes the moon to circulate around the sun.

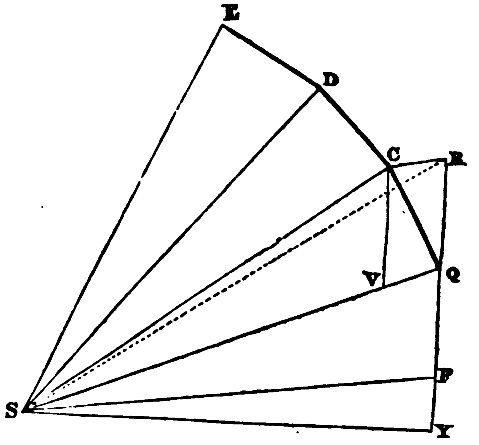
Fig. 34.



172. *If a body revolves about an immovable center of force, and is constantly attracted to it, it will always move in the same plane, and describe areas about the center proportional to the times.**

Let S (Fig. 35,) be the center of force, and suppose a body to be projected at P in the direction PQR, and take PQ = QR; then by the first law of motion, the body would move uniformly in the direction PQR, and describe PQ, QR, in the same time, if no other force acted upon it. But when the body comes to Q,

Fig. 35.



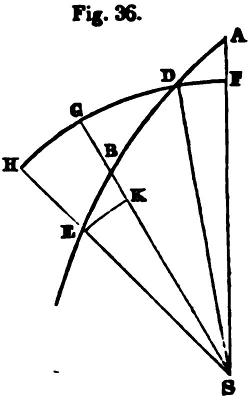
let a single impulse act at S, sufficient to draw the body through QV, in the time it would have described QR; and complete the parallelogram VQRC, and the body in the same time will describe QC; therefore, PQ, QC, are described in the same time. But the triangle SCQ = SRQ = SPQ; that is, *equal areas are described in equal times.* For the same reason, if a single impulse act at C, D, E, &c. at equal intervals of time, the several areas SPQ, SQC, SCD, SDE, &c. will all be equal to each other. Now this demonstration is independent of any particular dimensions in the several triangles, and consequently, holds good when they are taken indefinitely small, in which case we may consider the force as acting, not by separate impulses, but *constantly*, causing the body to describe a curve around S. And as no force acts

* The learner will remark that what has been before proved (Art. 168,) respecting the radius vector of the earth, is here shown to hold good with respect to *every* body which revolves around a center of force; and the same is true of several other propositions demonstrated in this chapter.

out of the plane SPQ, the whole curve must lie in that plane; that is, *the body moves always in the same plane.*

173. *If a body describes a curve around a center towards which it tends by any force, the angular velocity of the body around that center is reciprocally as the square of the distance from it.**

Fig. 36.



Let ABE (Fig. 36,) be any curve described about the center S; draw SA, SB, to any two points of the curve A and B; and let AD, BE, be described in indefinitely small equal times. Join SD and SE, and with the center S and distance SD, describe a circle meeting SA, SB, SE, in F, G, H; and with the center S and distance SE describe a circle meeting SB in K.

Because AD and BE are described in equal times, the triangles ASD, BSE, are equal. Hence, (Euc. 15. 6.)

$$DF : EK :: KS : FS :: BS : AS\dagger :: BS^2 : BS \times AS \quad (1)$$

$$GH : EK :: SH : SE :: SF : SE :: SA : SB :: SA^2 : BS \times AS \quad (2)$$

Hence, (1) $DF : BS^2 :: EK : BS \times AS$

(2) $GH : AS^2 :: EK : BS \times AS$

$\therefore\ DF : GH :: BS^2 : AS^2.$

But DF and GH measure the respective angular velocities at A and B, while AS and BS represent the distance at the same [redacted]

in the curve described from P, with a constant force, SY becomes a perpendicular to the tangent to the curve. But by article 172, the area described in a given time is constant. Therefore SPQ is constant, and $V \propto \frac{1}{SY}$; that is, the velocity varies inversely as the perpendicular upon the tangent. Hence, the velocity of a revolving body increases as it approaches the center of force.

175. *If equal areas be described about a center in equal times, the force must tend towards that center.*

Let SPQ (Fig. 35,) = SQC; now SPQ = SQR $\therefore$ SQC = SQR $\therefore$ CR is parallel to QS. Complete the parallelogram QRCV, and by the supposition the body describes QC, in consequence of the impulse at Q, and it would have described QR if no such impulse had acted; therefore QV must represent that motion impressed at Q, which, in conjunction with the motion QR, can make a body describe QC, and QV is directed to S.

176. Now it appears from article 168, that it is a *fact*, derived from observation, that the earth's radius vector describes equal areas in equal times; and by similar observations, the same is found to be true of each of the primary planets about the sun, and of each of the satellites about its primary. Hence, it is inferred, that the primary planets all gravitate towards the sun, and that the secondary planets all gravitate towards their respective primaries.

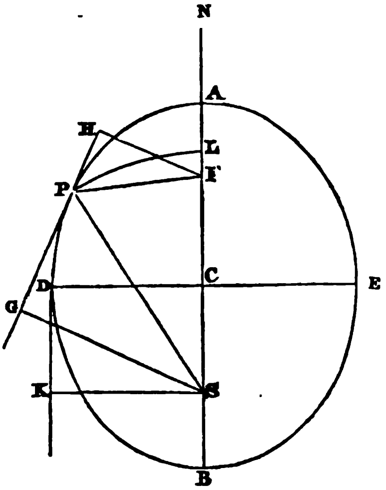
It has farther been established by observation, (Art. 162,) that the planetary orbits are *ellipses;* and hence the application of the principles of gravitation, so far as respects the sun and planets may be confined to the consideration of the motion of a body in an *elliptical orbit.*

177. *The distance of any planet from the sun at any point in its orbit, is to its distance from the superior focus, as the square of its velocity at its mean distance from the sun, is to the square of its velocity at the given point.*

Let ADBE (Fig. 37,) be the orbit of a planet, S the focus in which the sun is placed, AB the transverse and DE the conjugate

axis, C the center, and F the superior focus. Let the planet be any where at P; and draw a tangent to the orbit at P, on which from the foci let fall the perpendiculars SG, FH. Draw also DK touching the orbit in D, and let SK be perpendicular to it. Let

Fig. 37.



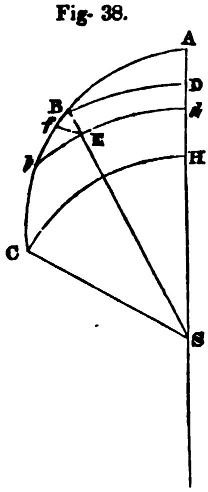
the velocity of the planet when at the mean distance at D=C, and when at P=V. Join SP, FP. Then (Art. 174,) the velocity at D is to the velocity at P, as SG to SK; that is,

$$C : V :: SG : DC, \text{ or } V = C\frac{DC}{SG}.$$

But because the triangles SGP, FHP, are equiangular, having

[redacted]

Fig. 38.



Let ABC (Fig. 38,) be a curve which a body describes about a center S to which it gravitates, while another body descends in a straight line AS to that center. Let BC be any arc of the curve ABC, and let BD, CH, be arcs of circles described from the center S, intersecting the line AS in D and H. From the center S describe the arc *bd*, indefinitely near to BD, and draw E*f* perpendicular to B*b*. Then, because the distances SD and SB are equal, the forces of gravity at D and B are also equal. Let these forces be expressed by the equal lines D*d* and BE; and let the force BE be resolved into the forces E*f* and B*f*. The force E*f*, acting at right angles to the path of the body, will not affect its velocity in that path, but will only draw it aside from a rectilinear course and make it proceed in the curve B*b*C. But the other force B*f*, acting in the direction of the course of the body, will be wholly employed in accelerating it. And because B and *b* are indefinitely near to each other, and likewise D and *d*, the accelerating force from B to *b* and from D to *d*, may be considered as acting uniformly. Therefore, the accelerations of the bodies in D and B, produced in equal times, are as the lines D*d*, B*f*; and hence, putting *d* for the velocity at *d*, and *f* for the velocity at *f*,

$d : f :: Dd$ or $BE : Bf$.(1)

And because the angle at E is a right angle,

$BE^2 = Bb \times Bf \therefore BE = \sqrt{Bb} \times \sqrt{Bf} \therefore BE \times \sqrt{Bf} = \sqrt{Bb} \times Bf$.

Hence, $BE : Bf :: \sqrt{Bb} : \sqrt{Bf}$.(2)

And, (1) and (2), $d : f :: \sqrt{Bb} : \sqrt{Bf}$.(3)

But, putting *b* for the velocity at *b*, and observing that, in falling bodies, the velocities are as the square roots of the spaces,

$b : f :: \sqrt{Bb} : \sqrt{Bf}$.(4)

Therefore, (3) and (4), $b : f :: d : f \therefore b = d$; that is, the velocity at *b* equals the velocity at *d*. And, since the same reasoning holds for successive points that may be taken at equal distances from B and D, therefore, if of two bodies, &c.*

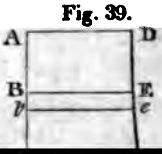
* Principia, Lib. I, Pr. 40. Stewart's Math. and Phys. Essays, Pr. 13.

179. *The law according to which the planets gravitate is such, that any body under the influence of the same force, and falling direct to the sun, will have its velocity at any point equal to a constant velocity multiplied into the square root of the distance it has fallen through, divided by the square root of the distance between the body and the sun's center.*

Suppose a planet to revolve in the elliptical orbit APB (Fig. 37); at A, the higher apsis, the velocity $V=C\left(\frac{AF}{AS}\right)^{\frac{1}{2}}$, (Art. 177); or if AN, in the axis produced $=AF$, $V=C\left(\frac{AN}{AS}\right)^{\frac{1}{2}}$. Let a body at A begin to descend towards S with this velocity, then if $SL=SP$, the velocity of the planet at P will be the same as that of the falling body at L, (Art. 178.) But the velocity of the planet at P is $C\left(\frac{PF}{PS}\right)^{\frac{1}{2}}=C\left(\frac{NL}{SL}\right)^{\frac{1}{2}}$* . But this velocity is equal to the constant velocity expressed by C, multiplied into the square root of NL, the distance fallen through,† divided by the square root of LS, the distance between the body and the sun's center.‡

180. *The force with which any planet gravitates to the sun, is inversely as the square of its distance from the sun's center.*

Fig. 39.



Let C (Fig. 39,) be the center to which the falling body gravitates, A the point from which it begins to fall, and let its velocity at any point B, be to its velocity in the point

has acquired in B.* If therefore the velocity at B be V, that at the middle point G being c, $V=c\left(\frac{AB}{BC}\right)^{\frac{1}{2}}$ by hypothesis, and therefore $2ABED=c^2.\frac{AB}{BC}$; and since $AB=AC-BC$, $2ABED=c^2.\frac{AC-BC}{BC}=c^2\left(\frac{AC}{BC}-1\right)$. For the same reason, if be be drawn indefinitely near to BE, $2AbeD=c^2\left(\frac{AC}{bC}-1\right)$, and therefore the difference of these areas, or $2BbeE$, that is, $2EB\times Bb=c^2\left(\frac{AC}{bC}-\frac{AC}{BC}\right)=c^2\frac{AC(BC-bC)}{BC\times bC}=c^2.\frac{AC\times Bb}{BC^2 \dagger}$. Wherefore, dividing by Bb, $2EB=c^2.\frac{AC}{BC^2}$; or $EB=c^2\frac{AG}{BC^2}$; now c^2 and AG are constant quantities, therefore EB varies inversely as BC^2. But EB represents the force of gravity at B, and BC the distance from the sun. Therefore, the force of gravity of a planet in different parts of its orbit, is inversely as the square of its distance from the sun.

181. The line CG is the same with the mean distance of the planet, in an orbit of which AC is the length of the transverse axis; and if the gravitation at that distance$=$F, and the mean distance itself$=a$, then since $EB=c^2\frac{AG}{BC^2}$, $F=c^2\times\frac{a}{a^2}=\frac{c^2}{a}$, or $aF=c^2$.

182. *The squares of the times of revolution of any two planets, are as the cubes of their mean distances from the sun.*

If a be the mean distance, or the semi-transverse axis, b the semi-conjugate, then πab=area of the orbit.‡ But as c is the velocity at the mean distance, or the elliptic arch which the planet moves over in a second when it is at D, (Fig. 37,) the vertex of the

* This principle is demonstrated by the aid of Fluxions as follows:

By construction, BE is proportional to the force at $B=\frac{dv}{dt}$, v being the velocity which the moving body has acquired at B, and t the time of the descent from A to B. Now Bb is the momentary increment of BA the space, and therefore $=vdt$; therefore $BE\times Bb=vdv$. And $2BE\times Bb=2vdv$. But $BE\times Bb$ is the momentary increment of the area ABED, and $2vdv$ is the momentary increment of v^2; therefore the square of the velocity of the moving body, and twice the area of ABED, increase at the same rate, and begin to exist at the same time; therefore they are equal. (See Playfair's Outlines, *Mechanics*, Art. 96.)

† bC being ultimately equal to BC. ‡ Day's Mensuration.

conjugate axis, therefore $\frac{1}{2}bc$ is the area described in that second by the radius vector; and since the area is the same for every second of the planet's revolution (Art. 172), therefore the area of the orbit divided by $\frac{1}{2}bc$ will give the number of seconds in which the revolution is completed, which $=\frac{\pi ab}{\frac{1}{2}bc}=\frac{2\pi a}{c}$; or, since $c^2=a\text{F}$, (Art. 181,) the time of a revolution $=\frac{2\pi a}{\sqrt{a\text{F}}}=2\pi\sqrt{\frac{a}{\text{F}}}$. Hence, let t, t', be the times of revolutions for two different planets, of which the mean distances are a, a', and the force of gravity at those distances F, F'. Then $t : t' :: 2\pi\sqrt{\frac{a}{\text{F}}} : 2\pi\sqrt{\frac{a'}{\text{F}'}} :: \sqrt{\frac{a}{\text{F}}} : \sqrt{\frac{a}{\text{F}'}} \therefore t^2 : t'^2 :: \frac{a}{\text{F}} : \frac{a'}{\text{F}'}$. But (Art. 180,) $\text{F} : \text{F}' :: a'^2 : a^2 \therefore t^2 : t'^2 :: \frac{a}{a'^2} : \frac{a'}{a^2}$, or $t^2 : t'^2 :: a^3 : a'^3$. That is, the squares of the times are as the cubes of the mean distances; or, since the major axes of the orbits are double the mean distances, the squares of the times are as the cubes of the major axes.

183. This is one of *Kepler's three great Laws*, which, taken in connexion, are as follows:

1. *The orbits of all the planets are ellipses, the sun occupying the common focus.* (Art. 176.)

2. *The radius vector of any planet describes areas proportional to the times.* (Art. 172.)

3. *The squares of the periodical times are as the cubes of the*

of the forces by which the planets are made to revolve in their orbits about the sun. In obedience to the first law of motion, every moving body *tends* to move in a straight line; and were not the planets deflected continually towards the sun by the force of attraction, these bodies as well as others would move forward in a rectilineal direction. We call the force by which they tend to such a direction the *projectile force*, because its effects are the same as though the body were originally projected from a certain point in a certain direction. It is an interesting problem for mechanics to solve, what was the nature of the impulse originally given to the earth, in order to impress upon it its two motions, the one around its own axis, the other around the sun? If struck in the direction of its center of gravity it might receive a forward motion, but no rotation on its axis. It must, therefore, have been impelled by a force, whose direction did not pass through its center of gravity. Bernouilli, a celebrated mathematician, has calculated that the impulse must have been given very nearly in the direction of the center, the point of projection being only the 165th part of the earth's radius from the center.* This impulse alone would cause the earth to move in a right line: gravitation towards the sun causes it to describe an orbit. Thus a top spinning on a smooth plane, as that of glass or ice, if impelled in a direction not coinciding with that of the center of gravity, may be made to imitate the two motions of the earth, especially if the experiment is tried in a concave surface like that of a large bowl. The resistance occasioned by the surface on which the top moves, and that of the air, will generally destroy the force of projection and cause the top to revolve in a smaller and smaller orbit; but the earth meets with no such resistance, and therefore makes both her days and years of the same length from age to age. A body, therefore, revolving in an orbit about a center of attraction, is constantly under the influence of two forces,—the *projectile* force, which tends to carry it forward in a straight line which is a tangent to its orbit, and the *centripetal* force, by which it tends towards the center.

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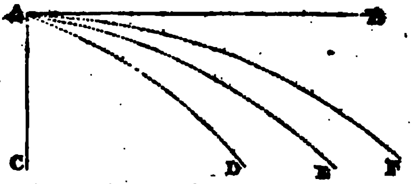
* Francœur, Uran. p. 49.

185. The most simple example we have of the combined action of these two forces is the motion of a missile thrown from the hand, or of a ball fired from a cannon. It is well known that the particular form of the curve described by the projectile, in either case, will depend upon the velocity with which it is thrown. In each case the body will begin to move in the line of direction in which it is projected, but it will soon be deflected from that line towards the earth. It will however continue nearer to the line of projection as the velocity of projection is greater. Thus let AB (Fig. 40,) perpendicular to AC represent the line of projection. The body will, in every case, commence

Fig. 40.

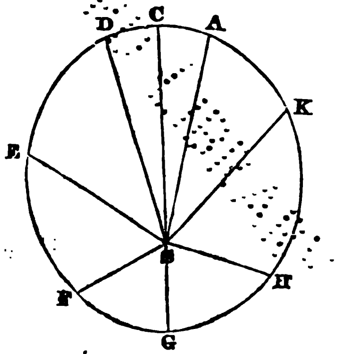


its motion in the line AB, which will therefore be the tangent to the curve it describes; but if it be thrown with a small velocity, it will soon depart from the tangent, describing the line AD; with a greater velocity it will describe a curve nearer to the tangent, as AE; and with a still greater velocity it will describe the curve AF.

186. In figure 41, suppose the planet to have passed the point C with so small a velocity, that the attraction of the sun bends its path very much, and causes it immediately to begin to approach towards the sun; the sun's attraction will increase its velocity as it moves through D, E, and F. For the sun's attractive force on the planet, when at D, is acting in the direction DS, and, on account of the small inclination of DE to DS, the force acting in the line DS helps the planet forward in the path DE, and thus increases its velocity. In like manner the velocity of the planet will be continually increasing as it passes through D, E, and F; and though the attractive force, on account of the planet's nearness, is so much increased, and tends, therefore, to make the orbit more curved, yet the velocity is also so much increased that the orbit is not more curved than before. The same

increase of velocity occasioned by the planet's approach to the sun, produces a greater increase of centrifugal force which carries it off again. We may see also why, when the planet has reached the most distant parts of its orbit, it does not entirely fly off, and never returns to the sun. For when the planet passes along H, K, A, the sun's attraction retards the planet, just as gravity retards a ball rolled up hill; and when it has reached C, its velocity is very small, and the attraction at the center of force causes a great deflection from the tangent, sufficient to give its orbit a great curvature, and the planet turns about, returns to the sun, and goes over the same orbit again.* As the planet recedes from the sun, its centrifugal force diminishes faster than the force of gravity, so that the latter finally preponderates.†

Fig. 41.



187. We may imitate the motion of a body in its orbit by suspending a small ball from the ceiling by a long string. The ball being drawn out of its place of rest, (which is directly under the point of suspension,) it will tend constantly towards the same place by a force which corresponds to the force of attraction of a central body. If an assistant stands under the point of suspension, his head occupying the place of the ball when at rest, the ball may be made to revolve about his head as the earth or any planet revolves about the sun. By projecting the ball in different directions, and with different degrees of velocity, we may make it describe different orbits, exemplifying principles which have been explained in the foregoing propositions.

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* Airy.

† The centrifugal force varies inversely as the *cube* of the distance, while the force of gravity is inversely as the *square*. The centrifugal force, therefore, increases faster than the force of gravity as a body is approaching the sun, and decreases faster as the body recedes from the sun. (See M. Stewart's Phys. and Math. Tracts, Prop. 8.)

CHAPTER IV.

PRECESSION OF THE EQUINOXES—NUTATION—ABERRATION—MOTION OF THE APSIDES—MEAN AND TRUE PLACES OF THE SUN.

188. THE PRECESSION OF THE EQUINOXES, *is a slow but continual shifting of the equinoctial points from east to west.*

Suppose that we mark the exact place in the heavens where, during the present year, the sun crosses the equator, and that this point is close to a certain star; next year the sun will cross the equator a little way westward of that star, and so every year a little farther westward, until, in a long course of ages, the place of the equinox will occupy successively every part of the ecliptic, until we come round to the same star again. As, therefore, the sun, revolving from west to east in his apparent orbit, comes round towards the point where it left the equinox, it meets the equinox before it reaches that point. The appearance is as though the equinox *goes forward* to meet the sun, and hence the phenomenon is called the *Precession of the Equinoxes*, and the fact is expressed by saying that the equinoxes retrograde on the ecliptic, until the line of the equinoxes makes a complete revolution from east to west. The equator is conceived as *sliding* westward on [redacted]

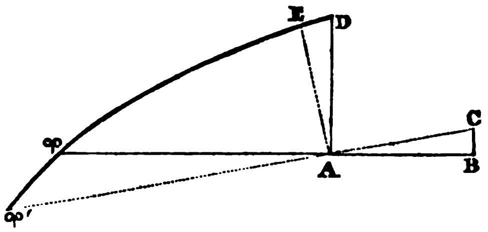
190. Suppose now we fix to the center of each of the two rings (Art. 188,) a wire representing its axis, one corresponding to the axis of the ecliptic, the other to that of the equator, the extremity of each being the pole of its circle. As the ring denoting the equator turns round on the ecliptic, which with its axis remains fixed, it is easy to conceive that the axis of the equator revolves around that of the ecliptic, and the pole of the equator around the pole of the ecliptic, and constantly at a distance equal to the inclination of the two circles. To transfer our conceptions to the celestial sphere, we may easily see that the axis of the diurnal sphere, (that of the earth produced, Art. 28,) would not have its pole constantly in the same place among the stars, but that this pole would perform a slow revolution around the pole of the ecliptic from east to west, completing the circuit in about 26,000 years. Hence the star which we now call the pole star, has not always enjoyed that distinction, nor will it always enjoy it hereafter. When the earliest catalogues of the stars were made, this star was 12° from the pole. It is now 1° 24′, and will approach still nearer; or, to speak more accurately, the pole will come still nearer to this star, after which it will leave it, and successively pass by others. In about 13,000 years, the bright star Lyra, which lies on the circle of revolution opposite to the present pole star, will be within 5° of the pole, and will constitute the Pole Star. As Lyra now passes near our zenith, the learner might suppose that the change of position of the pole among the stars, would be attended with a change of altitude of the north pole above the horizon. This mistaken idea is one of the many misapprehensions which result from the habit of considering the horizon as a fixed circle in space. However the pole might shift its position in space, we should still be at the same distance from it, and our horizon would always reach the same distance beyond it.

191. *The precession of the equinoxes is an effect of the spheroidal figure of the earth, and arises from the attraction of the sun and moon upon the excess of matter about the earth's equator.*

Were the earth a perfect sphere the attractions of the sun and moon upon the earth would be in equilibrium among themselves.

But if a globe were cut out of the earth, (taking half the polar diameter for radius,) it would leave a protuberant mass of matter in the equatorial regions, which may be considered as all collected into a ring resting on the earth. The sun being in the ecliptic, while the plane of this ring is inclined to the ecliptic 23° 28′, of course the action of the sun is oblique to the ring, and may be resolved into two forces, one in the plane of the equator, and the other perpendicular to it. The latter only can act as a disturbing force, and tending as it does to draw down the ring to the ecliptic, the ring would turn upon the line of the equinoxes as upon a hinge, and, dragging the earth along with it, the equator would ultimately coincide with the ecliptic were it not for the revolution of the earth upon its axis. This may be better understood by the aid of a diagram. Let ♈AB (Fig. 42,) represent the equator,

Fig. 42.



♈ED the ecliptic, and AD the solstitial colure. Let AB be the movement of rotation for a very short time, being of course in the order of the signs and in the direction of the equator. Let BC be [redacted]

amount of precession. The whole effect of the sun and moon being 50.″41, that of the planets is 0.31, leaving the actual amount of precession 50.″1.*

193. *The time occupied by the sun in passing from the equinoctial point round to the same point again, is called the* TROPICAL YEAR. As the sun does not perform a complete revolution in this interval but falls short of it 50.″1, the tropical year is shorter than the sidereal by 20m. 20s. in mean solar time, this being time of describing an arc of 50.″1 in the annual revolution.† The changes produced by the precession of the equinoxes in the apparent places of the circumpolar stars, have led to some interesting results in *chronology*. In consequence of the retrograde motion of the equinoctial points, the *signs* of the ecliptic (Art. 35,) do not correspond at present to the *constellations* which bear the same names, but lie about one whole sign or 30° westward of them. Thus, that division of the ecliptic which is called the sign Taurus, lies in the constellation Aries, and the sign Gemini in the constellation Taurus. Undoubtedly however when the ecliptic was thus first divided, and the divisions named, the several constellations lay in the respective divisions which bear their names. How long is it, then, since our zodiac was formed?

$$50.''1 : 1 \text{ year} :: 30^\circ (=108000'') : 2155.6 \text{ years.}$$

The result indicates that the present divisions of the zodiac, were made soon after the establishment of the Alexandrian school of astronomy. (Art. 2.)

NUTATION.

194. NUTATION *is a vibratory motion of the earth's axis, arising from periodical fluctuations in the obliquity of the ecliptic.*

If the sun and moon moved in the plane of the equator, there would be no precession, and the effect of their action in producing it varies with their distance from that plane. Twice a year, therefore, namely, at the equinoxes the effect of the sun is nothing; while at the solstices the effect of the sun is a maximum. On this account, the obliquity of the ecliptic is subject to a semiannual variation, since the sun's force which tends to produce a

* Francoeur, Uran. 162.

† 50′ 8.″3 : 24h. : : 50.″1 : 20m. 20s.

change in the obliquity is variable, while the diurnal motion of the earth which prevents the change from taking place, is constant. Hence the plane of the equator is subject to an irregular motion which is called the *Solar Nutation.* The name is derived from the oscillatory motion communicated by it to the earth's axis, while the pole of the equator is performing its revolution around the pole of the ecliptic (Art. 190.) The effect of the sun however is far less than that of the moon. By the nutation alone the pole of the earth would perform a revolution in a very small ellipse, only 18″ in diameter, the center being in the circle which the pole describes around the pole of the ecliptic; but the combined effects of precession and nutation convert the circumference of this circle into a wavy line. The motion of the equator occasioned by nutation, causes it alternately to approach to and recede from the stars, and thus to change their declinations. The solar nutation, depending on the position of the sun with respect to the equinoxes, passes through all its variations annually; but the lunar nutation depending on the position of the moon with respect to her nodes, varies through a period of about 18½ years.

ABERRATION.

195. ABERRATION *is an apparent change of place in the stars, occasioned by the joint effects of the motion of the earth in its orbit, and the progressive motion of light.*

[redacted]

to be situated at S′. The difference between its true and its apparent place, that is, the angle SES′ is the aberration, the magnitude of which is obtained from the known ratio of EA to EC, or the velocity of light to that of the earth in its orbit.

The velocity of light is 192,000 miles per second, while that of the earth in its orbit is about 19 miles per second. Representing the velocity of light by the line EA, and that of the earth by AB, then,

192,000 : 19 : : Rad. : tan. 20.″5 = the angle at E, which is the amount of aberration when the direction of the ray of light is perpendicular to the earth's motion.

The effect of aberration upon the places of the fixed stars is to carry their apparent places a little forward of their real places in the direction of the earth's motion. The effect upon each particular star will be to make it describe a small ellipse in the heavens, having for its center the point in which the star would be seen if the earth were at rest.

MOTION OF THE APSIDES.

196. The two points of the ecliptic where the earth is at the greatest and least distances from the sun respectively, do not always maintain the same places among the signs, but gradually shift their positions from west to east. If we accurately observe the place among the stars, where the earth is at the time of its perihelion the present year, we shall find that it will not be precisely at that point the next year when it arrives at its perihelion, but about 12″ (11.″66) to the east of it. And since the equinox itself, from which longitude is reckoned, moves in the opposite direction 50.″1 annually, the longitude of the perihelion increases every year 61.″76, or a little more than one minute. This fact is expressed by saying that the line of the apsides of the earth's orbit has a slow motion from west to east, and completes one entire revolution in its own plane in about 100,000 years (111,149.)

The mean longitude of the perihelion at the commencement of the present century was 99° 30′ 5″, and of course in the ninth degree of Cancer, a little past the winter solstice. In the year 1248, the perihelion was at the place of this solstice; and since the increase of longitude is 61.″76 a year, hence,

61.″76 : 1 : : 90° : 5246 = the time occupied in passing from the first of Aries to the solstice. Hence, 5246 − 1248 = 3998, which is the time before the Christian era, when the perigee was at the first of Aries. But this differs only 6 years from the time of the creation of the world, which is fixed by chronologists at 4004 years A. C. At the period of the creation, therefore, the line of the apsides of the earth's orbit, coincided with the line of the equinoxes.

197. The angular distance of a body from its aphelion is called its *Anomaly ;* and the interval between the sun's passing the point of the ecliptic corresponding to the earth's aphelion, and returning to the same point again, is called the ***anomalistic year***. This period must be a little longer than the sidereal year, since, in order to complete the anomalistic revolution, the sun must traverse an arc of 11.″66 in addition to 360°.

Now 360° : 365.256 : : 11.″66 : 4m. 44s.

198. Since the points of the annual orbit, where the sun is at the greatest and least distances from the earth, change their position with respect to the solstices, a slow change is occasioned in the duration of the respective seasons. For, let the perihelion correspond to the place of the winter solstice, as was the case in the year 1248 ; then as the sun moves more rapidly in that part of his orbit, the winter months will be shorter than the summer. But, again, let the perihelion be at the summer solstice, as it will [redacted]

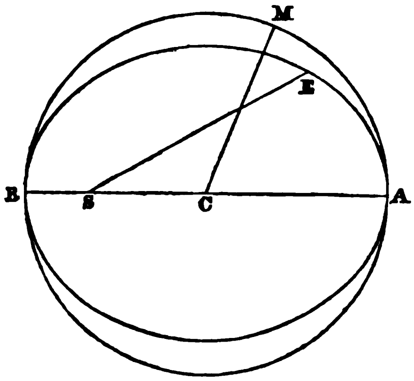
In surveying an irregular field, it is common first to strike out some regular figure, as a square or a parallelogram, by running long lines, and disregarding many small irregularities in the boundaries of the field. By this process, we obtain an approximation to the contents of the field, although we have perhaps thrown out several small portions which belong to it, and included a number of others which do not belong to it. These being separately estimated and added to or subtracted from our first computation, we obtain the true area of the field. In a similar manner, we proceed in finding the place of a heavenly body, which moves in an orbit more or less irregular. Thus we estimate the sun's distance from the vernal equinox for every day of the year at noon, on the supposition that he moves uniformly in a circular orbit: this is the sun's *mean longitude*. We then apply to this result various corrections for the irregularity of the sun's motions, and thus obtain the *true longitude*.

200. The corrections applied to the mean motions of a heavenly body, in order to obtain its true place, are called *Equations*. Thus the elliptical form of the earth's orbit, the precession of the equinoxes, and the nutation of the earth's axis, severally affect the place of the sun in his apparent orbit, for which equations are applied. In a collection of Astronomical Tables, a large part of the whole are devoted to this object. They give us the amount of the corrections to be applied under all the circumstances and constantly varying relations in which the sun, moon, and earth are situated with respect to each other. The angular distance of the earth or any planet from its aphelion, on the supposition that it moves uniformly in a circle, is called its *Mean Anomaly:* its actual distance at the same moment in its orbit is called its *True Anomaly.**

Thus in figure 44, let AEB represent the orbit of the earth having the sun in one of the foci at S. Upon AB describe the circle AMB. Let E be the place of the earth in its orbit, and M the corresponding place in the circle; then the angle MCA is the mean, and ESA the true anomaly. The difference between the

* In some astronomical treatises, the anomaly is reckoned from the perihelion.

Fig. 44.



mean and true anomaly, MCA—ESA, is called the ***Equation of the Center***, being that correction which depends on the elliptical form of the orbit, or on the distance of the center of attraction from the center of the figure, that is, on the eccentricity of the orbit. It is much the greatest of all the corrections used in finding the sun's true longitude, amounting, at its maximum, to nearly two degrees ($1^\circ\ 55'\ 26.''8$.)

[redacted]

And, since spheres are as the cubes of the diameters, their *volume* of the moon is $\frac{1}{49}$ that of the earth. Her *density* is nearly $\frac{3}{5}$ (.615) the density of the earth, and her *mass* ($=\frac{1}{49}\times.615$) is about $\frac{1}{80}$.

202. The moon shines by *reflected light* borrowed from the sun, and when full, exhibits a disk of silvery brightness, diversified by extensive portions partially shaded. By the aid of the telescope, we see undoubted signs af a varied surface, composed of extensive tracts of level country, and numerous mountains and valleys.

203. The line which separates the enlightened from the dark portions of the moon's disk, is called the *Terminator*. (See Fig. 2. *Frontispiece.*) As the terminator traverses the disk from new to full moon, it appears through the telescope exceedingly broken in some parts, but smooth in others, indicating that portions of the lunar surface are uneven while others are level. The broken regions appear brighter than the smooth tracts. The latter have been taken for seas, but it is supposed with more probability that they are extensive plains, since they are still too uneven for the perfect level assumed by bodies of water. That there are *mountains* in the moon, is known by several distinct indications. First, when the moon is increasing, certain spots are illuminated sooner than the neighboring places, appearing like bright points beyond the terminator, within the dark path of the disk. (See Fig. 2. *Frontispiece.*) Secondly, after the terminator has passed over them, they project shadows upon the illuminated part of the disk, always opposite to the sun, corresponding in shape to the form of the mountain, and undergoing changes in length from night to night, according as the sun shines upon that part of the moon more or less obliquely. Many individual mountains rise to a great height in the midst of plains, and there are several very remarkable mountainous groups, extending from a common center in long chains.

204. That there are also *valleys* in the moon, is equally evident. The valleys are known to be truly such, particularly by the man-

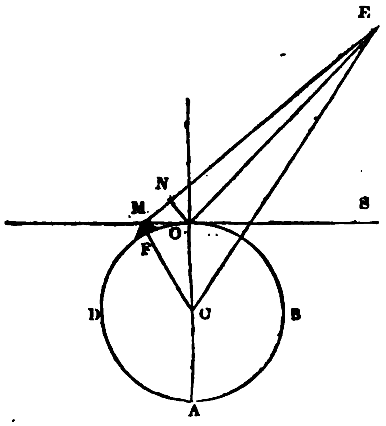
ner in which the light of the sun falls upon them, illuminating the part opposite to the sun while the part adjacent is dark, as is the case when the light of a lamp shines obliquely into a china cup. These valleys are often remarkably regular, and some of them almost perfect circles. In several instances, a circular chain of mountains surrounds an extensive valley, which appears nearly level, except that a sharp mountain sometimes rises from the center. The best time for observing these appearances is near the first quarter of the moon, when half the disk is enlightened;* but in studying the lunar geography, it is expedient to observe the moon every evening from new to full, or rather through her entire series of changes.

205. The various places on the moon's disk have received appropriate *names*. The dusky regions, being formerly supposed to be seas, were named accordingly; and other remarkable places have each two names, one derived from some well known spot on the earth, and the other from some distinguished personage. Thus the same bright spot on the surface of the moon is called *Mount Sinai* or *Tycho*, and another, *Mount Etna* or *Copernicus*. The names of individuals, however, are more used than the others. The frontispiece exhibits the telescopic appearance of the full moon. A few of the most remarkable points have the following names, corresponding to the numbers and letters on the map. (See *Frontispiece.*)

206. *The method of estimating the height of lunar mountains is as follows.*

Let ABO (Fig. 45,) be the illuminated hemisphere of the moon, SO a solar ray touching the moon in O, a point in the circle which separates the enlightened from the dark part of the moon. All the part ODA will be in darkness; but if this part contains a mountain MF, so elevated that its summit M reaches to the solar ray SOM, the point M will be enlightened. Let E be the place of the observer on the earth, the moon being at any elongation from

Fig. 45.



the sun, as measured by the angle EOS. Draw the lines EM, EO, and CM, C being the center of the moon; and let FM be the height of the mountain. Draw ON perpendicular to EM. The line EO being known, and the angle OEM being measured with a micrometer, the value of ON, the projection of the line OM, becomes known. Now $OM=\frac{ON}{\cos. MON}$; and since OEN is a very small angle, EON may be considered as a right angle; consequently, MON=MOE−90. Therefore $OM=\frac{ON}{\cos.(MOE-90)}$ $=\frac{ON}{\sin. MOE}=\frac{ON}{\sin. EOS}$. That is, the distance between the summit of the mountain and the illuminated part of the moon's disk, is equal to the projected distance as measured by the micrometer, divided by the sine of the moon's elongation from the sun.

Suppose the distance $OM = nCO$, where n represents the fraction the part OM is of CO. Then, $CM^2 = CO^2 + OM^2 = CO^2 + n^2CO^2 = CO^2(1+n^2) \therefore CM = CO(1+n^2)^{\frac{1}{2}} \therefore CM - CO$ or $FM = CO(\sqrt{1+n^2} - 1) = \frac{n^2}{2}$ CO, neglecting the higher powers of n, which would be of too little value to be worth taking into the account. The value of n has been found in one case equal to $\frac{1}{13}$ which gives the height of the mountain equal to $\frac{1}{338}$ the semi-diameter of the moon, that is, $3\frac{1}{3}$ miles.

When the moon is exactly at quadrature, then EOM becomes a right angle, and the value of OM is obtained directly from actual measurement ; and having CO and OM, we easily obtain CM and of course FM.

207. Schroeter, a German astronomer, estimated the heights of the lunar mountains by observations on their *shadows*. He made them in some cases as high as $\frac{1}{214}$ of the semi-diameter of the moon, that is, about 5 miles. The same astronomer also estimates the depths of some of the lunar valleys at more than four miles. Hence it is inferred that the moon's surface is more broken and irregular than that of the earth, its mountains being higher and its valleys deeper in proportion to the size of the moon than those of the earth.

208. Dr. Herschel is supposed also to have obtained decisive evidence of the existence of *volcanoes* in the moon, not only

[redacted]

mile in length in the moon subtends an angle at the eye of only about one second. If, therefore, works of art were to have a sufficient horizontal extent to become visible, they can hardly be supposed to attain the necessary elevation, when we reflect that the height of the great pyramid of Egypt is less than the sixth part of a mile.

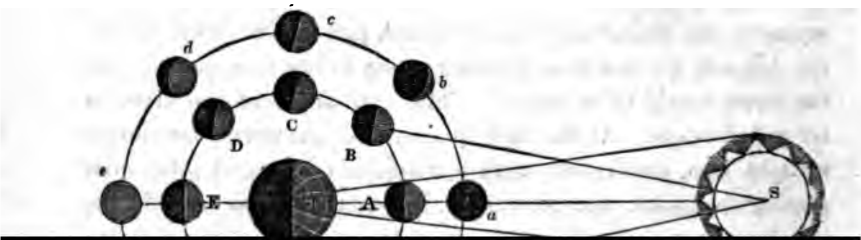
PHASES OF THE MOON.

211. The changes of the moon, commonly called her Phases, arise from different portions of her illuminated side being turned towards the earth at different times. When the moon is first seen after the setting sun, her form is that of a bright crescent, on the side of the disk next to the sun, while the other portions of the disk shine with a feeble light, reflected to the moon from the earth. Every night we observe the moon to be farther and farther eastward of the sun, and at the same time the crescent enlarges, until, when the moon has reached an elongation from the sun of 90°, half her visible disk is enlightened, and she is said to be in her *first quarter*. The terminator, or line which separates the illuminated from the dark part of the moon, is convex towards the sun from the new moon to the first quarter, and the moon is said to be *horned*. The extremities of the crescent are called *cusps*. At the first quarter, the terminator becomes a straight line, coinciding with a diameter of the disk; but after passing this point, the terminator becomes concave towards the sun, bounding that side of the moon by an elliptical curve, when the moon is said to be *gibbous*. When the moon arrives at the distance of 180° from the sun, the entire circle is illuminated, and the moon is full. She is then *in opposition* to the sun, rising about the time the sun sets. For a week after the full, the moon appears gibbous again, until, having arrived within 90° of the sun, she resumes the same form as at the first quarter, being then at her *third quarter*. From this time until new moon, she exhibits again the form of a crescent before the rising sun, until, approaching her *conjunction* with the sun, her narrow thread of light is lost in the solar blaze; and finally, at the moment of passing the sun, the dark side is wholly turned towards us and for some time we lose sight of the moon.

The two points in the orbit corresponding to new and full moon respectively, are called by the common name of *syzigies;* those which are 90° from the sun are called *quadratures;* and the points half way between the syzigies and quadratures are called *octants.* The circle which divides the enlightened from the unenlightened hemisphere of the moon, is called the *circle of illumination:* that which divides the hemisphere that is turned towards us from the hemisphere that is turned from us, is called the *circle of the disk.*

212. As the moon is an opake body of a spherical figure, and borrows her light from the sun, it is obvious that that half only which is towards the sun can be illuminated. More or less of this side is turned towards the earth, according as the moon is at a greater or less elongation from the sun. The reason of the different phases will be best understood from a diagram. Therefore let T (Fig. 46,) represent the earth, and S the sun. Let A, B, C, &c. be successive positions of the moon. At A the entire

Fig. 46.



[redacted]

REVOLUTIONS OF THE MOON.

213. *The moon revolves around the earth from west to east, making the entire circuit of the heavens in about 27⅓ days.*

The precise law of the moon's motions in her revolution around the earth, is ascertained, as in the case of the sun, (Art. 155,) by daily observations on her meridian altitude and right ascension. Thence are deduced by calculation her latitude and longitude, from which we find, that the moon describes on the celestial sphere a great circle of which the earth is the center.

The period of the moon's revolution from any point in the heavens round to the same point again, is called a *month.* A *sidereal month* is the time of the moon's passing from any star, until it returns to the same star again. A *synodical month** is the time from one conjunction or new moon to another. The synodical month is about 29½ days, or more exactly, 29d. 12h. 44m. 2s.8 = 29.53 days. The sidereal month is about two days shorter, being 27d. 7h. 43m. 11s.5 = 27.32 days. As the sun and moon are both revolving in the same direction, and the sun is moving nearly a degree a day, during the 27 days of the moon's revolution, the sun must have moved 27°. Now since the moon passes over 360° in 27.32 days, her daily motion must be 13° 17′. It must therefore evidently take about two days for the moon to overtake the sun. The difference between these two periods may, however, be determined with great exactness. The middle of an eclipse of the sun marks the exact moment of conjunction or new moon; and by dividing the interval between any two solar eclipses by the number of revolutions of the moon, or *lunations*, we obtain the precise period of the synodical month. Suppose, for example, two eclipses occur at an interval of 1,000 lunations; then the whole number of days and parts of a day that compose the interval divided by 1,000 will give the exact time of one lunation.† The time of the synodical month being ascertained, the exact period of the sidereal month may be

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* ςυν and οδος, implying that the two bodies *come together*.

† It might at first view seem necessary to know the period of one lunation before we could know the number of lunations in any given interval. This period is known very nearly from the interval between one new moon and another.

derived from it. For the arc which the moon describes in order to come into conjunction with the sun, exceeds 360° by the space which the sun has passed over since the preceding conjunction, that is, 29.53 days. Therefore,

365.24 : 360° : : 29.53 : 29°.1 = arc which the moon must describe more than 360° in order to overtake the sun. Hence,

13° 17′ : 1d. : : 29°.1 : 2.21d. = difference between the sidereal and synodical months; and 29.53 − 2.21 = 27.32, the time of the sidereal revolution.

214. ***The moon's orbit is inclined to the ecliptic in an angle of about*** 5° (5° 8′ 48″). It crosses the ecliptic in two opposite points called her *nodes*. The amount of inclination is ascertained by observations on the moon's latitude when at a maximum, that being of course the greatest distance from the ecliptic, and therefore equal to the inclination of the two circles.

215. The moon, at the same age, crosses the meridian at different altitudes at different seasons of the year. The full moon, for example, will appear much farther in the south when on the meridian at one period of the year than at another. The reason of this may be explained as follows. When the sun is in the part of the ecliptic south of the equator, the earth and of course the moon, which always keeps near to the earth, is in the part north of the equator. At such times, therefore, the new moons [redacted]

to the last quarter, is commonly above the horizon, while the sun is absent; whereas, during summer, while the sun is present, the moon is above the horizon while describing her first and last quadrants.

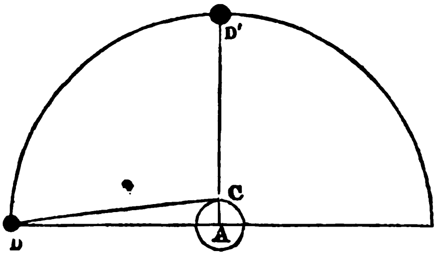
216. About the time of the autumnal equinox, the moon when near the full, rises about sunset for a number of nights in succession; and as this is, in England, the period of harvest, the phenomenon is called the *Harvest Moon*. To understand the reason of this, since the moon is never far from the ecliptic, we will suppose her progress to be in the ecliptic. If the moon moved in the equator, then, since this great circle is at right angles to the axis of the earth, all parts of it, as the earth revolves, cut the horizon at the same constant angle. But the moon's orbit, or the ecliptic, which is here taken to represent it, being oblique to the equator, cuts the horizon at different angles in different parts, as will easily be seen by reference to an artificial globe. When the first of Aries, or vernal equinox, is in the eastern horizon, it will be seen that the ecliptic, (and consequently the moon's orbit,) makes its least angle with the horizon. Now, at the autumnal equinox, the sun being in Libra, the moon at the full is in Aries, and rises when the sun sets. On the following evening, although she has advanced in her orbit about 13°, (Art. 213,) yet her progress being oblique to the horizon, and at a small angle with it, she will be found at this time but a little way below the horizon, compared with the point where she was at sunset the preceding evening. She therefore rises but little later, and so for a week only a little later each evening than she did the preceding night.

217. *The moon is about $\frac{1}{60}$ nearer to us when near the zenith than when in the horizon.*

The horizontal distance CD (Fig. 47,) is nearly equal to AD = AD′, which is greater than CD′ by AC, the semi-diameter of the earth = $\frac{1}{60}$ the distance of the moon. Accordingly, the apparent diameter of the moon, when actually measured, is about 30″ (which equals about $\frac{1}{60}$ of the whole) greater when in the zenith than in the horizon. The apparent enlargement of the full

moon when rising, is owing to the same causes as that of the sun, as explained in article 96.

Fig. 47.



218. *The moon turns on its axis in the same time in which it revolves around the earth.*

This is known by the moon's always keeping nearly the same face towards us, as is indicated by the telescope, which could not happen unless her revolution on her axis kept pace with her motion in her orbit. Thus, it will be seen by inspecting figure 31, that the earth turns different faces towards the sun at different times; and if a ball having one hemisphere white and the other black be carried around a lamp, it will easily be seen that it cannot present the same face constantly towards the lamp unless it turns once on its axis while performing its revolution. The same thing will be observed when a man walks around a tree, keeping his face constantly towards it. Since however the motion of the moon on its axis is uniform, while the motion in its orbit is une-

[redacted]

olution, more of the region around the other pole; which gives the appearance of a tilting motion to the moon's axis. This has nearly the same cause with that which occasions our change of seasons. The moon's axis being inclined to the plane of her orbit, and always remaining parallel to itself, the circle which divides the visible from the invisible part of the moon, will pass in such a way as to throw sometimes more of one pole into view, and sometimes more of the other, as would be the case with the earth if seen from the sun. (See Fig. 31.)

The moon exhibits another phenomenon of this kind called her *diurnal libration*, depending on the daily rotation of the spectator. She turns the same face towards the center of the earth only, whereas we view her from the surface. When she is on the meridian, we see her disk nearly as though we viewed it from the center of the earth, and hence in this situation it is subject to little change; but when near the horizon, our circle of vision takes in more of the upper limb than would be presented to a spectator at the center of the earth. Hence, from this cause, we see a portion of one limb while the moon is rising, which is gradually lost sight of, and we see a portion of the opposite limb as the moon declines towards the west. It will be remarked that neither of the foregoing changes implies any actual motion in the moon, but that each arises from a change of position in the spectator.

220. An inhabitant of the moon would have but one day and one night during the whole lunar month of 29½ days. One of its days, therefore, is equal to nearly 15 of ours. So protracted an exposure to the sun's rays, especially in the equatorial regions of the moon, must occasion an excessive accumulation of heat; and so long an absence of the sun must occasion a corresponding degree of cold. Each day would be a wearisome summer; each night a severe winter.* A spectator on the side of the moon which is opposite to us would never see the earth; but one on the side next to us would see the earth presenting a gradual succession of changes during his long night of 360 hours. Soon after the earth's conjunction with the sun, he would have the light of the

* Francœur, Uranog. p. 91.

earth reflected to him, presenting at first a crescent, but enlarging, as the earth approaches its opposition, to a great orb, 13 times as large as the full moon appears to us, and affording nearly 13 times as much light. Our seas, our plains, our mountains, our volcanoes, and our clouds, would produce very diversified appearances, as would the various parts of the earth brought successively into view by its diurnal rotation. The earth while in view to an inhabitant of the moon, would *remain immovably fixed in the same part of the heavens.* For being unconscious of his own motion around the earth, as we are of our motion around the sun, the earth would seem to revolve around his own planet from west to east; but, meanwhile, his rotation along with the moon on her axis, would cause the earth to have an apparent motion westward at the same rate. The two motions, therefore, would exactly balance each other, and the earth would appear all the while at rest. The earth is full to the moon when the latter is new to us; and universally the two phases are complementary to each other.*

221. It has been observed already, (Art. 214,) that the moon's orbit crosses the ecliptic in two opposite points called the *nodes.* That which the moon crosses from south to north, is called the *ascending node*; that which the moon crosses from north to south, the *descending node.*

From the manner in which the figure representing the earth's orbit and that of the moon, is commonly drawn, the learner is sometimes puzzled to see how the orbit of the moon can cut the [redacted]

convey an erroneous idea; for the moon, as well as the earth, revolves around the sun under the influence of two forces, and would continue her motion around the sun, were the earth removed out of the way. Indeed, the moon is attracted towards the sun $2\frac{1}{4}$ times more than towards the earth,* and would abandon the earth were not the latter also carried along with her by the same forces. So far as the sun acts equally on both bodies, their motion with respect to each other would not be disturbed. Because the gravity of the moon towards the sun is found to be greater, at the conjunction, than her gravity towards the earth, some have apprehended that, if the doctrine of universal gravitation is true, the moon ought necessarily to abandon the earth. In order to understand the reason why it does not do thus, we must reflect, that when a body is revolving in its orbit under the action of the projectile force and gravity, whatever diminishes the force of gravity while that of projection remains the same, causes the body to recede from the center; and whatever increases the amount of gravity carries the body towards the center. Now, when the moon is in conjunction, her gravity towards the earth acts in opposition to that towards the sun, while her velocity remains too great to carry her, with what force remains, in a circle about the sun, and she therefore recedes from the sun, and commences her revolution around the earth. On arriving at the opposition, the gravity of the earth conspires with that of the sun, and the moon's projectile force being less than that required to make her revolve in a circular orbit, when attracted towards the sun by the sum of these forces, she accordingly begins to approach the sun and descends again to the conjunction.†

223. The attraction of the sun, however, being every where greater than that of the earth, the actual path of the moon around

* It is shown by writers on Mechanics, that the forces with which bodies revolving in circular orbits tend towards their centers, are as the radii of their orbits divided by the squares of their periodical times. Hence, supposing the orbits of the earth and the moon to be circular, (and their slight eccentricity will not much affect the result,) we have

$$G : G' :: \frac{400}{(365.25)^2} : \frac{1}{(27.32)^2} :: 2.2 : 1.$$

† M'Laurin's Discoveries of Newton, B. IV, ch. 5.

the sun is every where concave towards the latter. Still the elliptical path of the moon around the earth, is to be conceived of in the same way as though both bodies were at rest with respect to the sun. Thus, while a steamboat is passing swiftly around an island, and a man is walking slowly around a post in the cabin, the line which he describes in space between the forward motion of the boat and his circular motion around the post, may be every where concave towards the island, while his path around the post will still be the same as though both were at rest. A nail in the rim of a coach wheel, will turn around the axis of the wheel, when the coach has a forward motion in the same manner as when the coach is at rest, although the line actually described by the nail will be the resultant of both motions, and very different from either.

CHAPTER VI.

LUNAR IRREGULARITIES.

224. We have hitherto regarded the moon as describing a great circle on the face of the sky, such being the visible orbit as seen by projection. But, on more exact investigation, it is found that [redacted]

and intelligible as far as it goes, may be acquired by first gaining a distinct idea of the mutual actions of the sun, the moon, and the earth.

225. *The irregularities of the moon's motions, are due chiefly to the disturbing influence of the sun, which operates in two ways; first, by acting unequally on the earth and moon, and, secondly, by acting obliquely on the moon, on account of the inclination of her orbit to the ecliptic.**

If the sun acted equally on the earth and moon, and always in parallel lines, this action would serve only to restrain them in their annual motions round the sun, and would not affect their actions on each other, or their motions about their common center of gravity. In that case, if they were allowed to fall directly towards the sun, they would fall equally, and their respective situations would not be affected by their descending equally towards it. We might then conceive them as in a plane, every part of which being equally acted on by the sun, the whole plane would descend towards the sun, but the respective motions of the earth and the moon in this plane, would be the same as if it were quiescent. Supposing then this plane and all in it, to have an annual motion imprinted on it, it would move regularly round the sun, while the earth and moon would move in it with respect to each other, as if the plane were at rest, without any irregularities. But because the moon is nearer the sun in one half of her orbit than the earth is, and in the other half of her orbit is at a greater distance than the earth from the sun, while the power of gravity is always greater at a less distance; it follows, that in one half of her orbit the moon is more attracted than the earth towards the sun, and in the other half less attracted than the earth. The *excess* of the attraction, in the first case, and the *defect* in the second constitutes a disturbing force, to which we may add another, namely, that arising from the *oblique action* of the solar force, since this action is not directed in parallel lines, but in lines that meet in the center of the sun.

* M'Laurin's Discoveries of Newton, B. IV, ch. 4. La Place's Syst. du Monde, B. IV, ch. 5.

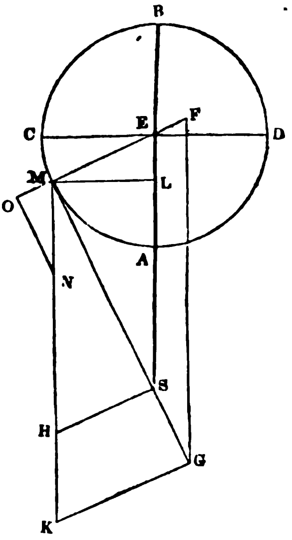
226. To see the effects of this process, let us suppose that the projectile motions of the earth and moon were destroyed, and that they were allowed to fall freely towards the sun. If the moon was in conjunction with the sun, or in that part of her orbit which is nearest to him, the moon would be more attracted than the earth, and fall with greater velocity towards the sun; so that the distance of the moon from the earth would be increased in the fall. If the moon was in opposition, or in the part of her orbit which is farthest from the sun, she would be less attracted than the earth by the sun, and would fall with a less velocity towards the sun, and would be left behind; so that the distance of the moon from the earth would be increased in this case also. If the moon was in one of the quarters, then the earth and moon being both attracted towards the center of the sun, they would both descend directly towards that center, and by approaching it, they would necessarily at the same time approach each other, and in this case their distance from each other would be diminished. Now whenever the action of the sun would increase their distance, if they were allowed to fall towards the sun, then the sun's action, by endeavoring to separate them, diminishes their gravity to each other; whenever the sun's action would diminish the distance, then it increases their mutual gravitation. Hence, in the conjunction and opposition, that is, *in the syzigies*, their gravity towards each other is diminished by the action of the sun, while in the quadratures it is increased. But it must be remembered that it is not the total action of the sun

fall farther at the end of an arc below the tangent drawn at the other end of it. Her motion will be thus accelerated, and it will continue to be accelerated until she arrives at the ensuing conjunction, because the direction of the sun's action upon her, during that time, makes an acute angle with the direction of her motion. (See Fig. 41.) At the conjunction, her gravity towards the earth being diminished by the action of the sun, her orbit will then be less curved, and she will be carried farther from the earth as she moves to the next quarter; and because the action of the sun makes there an obtuse angle with the direction of her motion, she will be retarded in the same degree as she was accelerated before.

228. After this *general* explanation of the mode in which the sun acts as a disturbing force on the motions of the moon, the learner will be prepared to understand the mathematical development of the same doctrine.

Therefore, let ADBC (Fig. 48,) be the orbit, nearly circular, in which the moon M revolves in the direction CADB, round the earth E. Let S be the sun, and let SE the radius of the earth's orbit, be taken to represent the force with which the earth gravitates to the sun. Then (Art. 180,) $\frac{1}{SE^2} : \frac{1}{SM^2} :: SE : \frac{SE^3}{SM^2}$ = the force by which the sun draws the moon in the direction MS. Take $MG=\frac{SE^3}{SM^2}$, and let the parallelogram KF be described, having MG for its diagonal, and having its sides parallel to EM and ES. The force MG may be resolved into two, MF and MK, of which MF, directed towards E, the center of the earth, increases the gravity of the moon to the earth, and does not hinder the areas described by the radius vector from being proportional to the

Fig. 48.



times. The other force MK draws the moon in the direction of the line joining the centers of the sun and earth. It is, however, only the excess of this force, above the force represented by SE, or that which draws the earth to the sun, which disturbs the relative position of the moon and earth. This is evident, for if KM were just equal to ES, no disturbance of the moon relative to the sun could arise from it. If then ES be taken from MK, the difference HK is the whole force in the direction parallel to SE, by which the sun disturbs the relative position of the moon and earth. Now, if in MK, MN be taken equal to HK, and if NO be drawn perpendicular to the radius vector EM produced, the force MN may be resolved into two, MO and ON, the first lessening the gravity of the moon to the earth; and the second, being parallel to the tangent of the moon's orbit in M, accelerates the moon's motion from C to A, and retards it from A to D, and so alternately in the other two quadrants. Thus the whole solar force directed to the center of the earth, is composed of the two parts MF and MO, which are sometimes opposed to one another, but which never affect the uniform description of the areas about E. Near the quadratures the force MO vanishes, and the force MF, which increases the gravity of the moon to the earth, coincides with CE or DE. As the moon approaches the conjunction at A, the force MO prevails over MF, and lessens the gravity of the moon to the earth. In the opposite point of the orbit, when the moon is in opposition at B, the force with which the [redacted]

229. With these general principles in view, we may now proceed to investigate the figure of the moon's orbit, and the irregularities to which the motions of this body are subject.

230. *The figure of the moon's orbit is an ellipse, having the earth in one of the foci.*

The elliptical figure of the moon's orbit, is revealed to us by observations on her changes in apparent diameter, and in her horizontal parallax. First, we may measure from day to day the apparent diameter of the moon. Its variations being inversely as the distances (Art. 163,) they give us at once the *relative* distance of each point of observation from the focus. Secondly, the variations on the moon's horizontal parallax, which also are inversely as the distances, (Art. 82,) lead to the same results. Observations on the angular velocities, combined with the changes in the lengths of the radius vector, afford the means of laying down a plot of the lunar orbit, as in the case of the sun, represented in figure 32. The orbit is shown to be nearly an ellipse, because it is found to have the properties of an ellipse.

The moon's greatest and least apparent diameters are respectively 33′.518 and 29′.365, while her corresponding changes of parallax are 61′.4 and 53′.8. The two ratios ought to be equal, and we shall find such to be the fact very nearly; for,

61.4 : 53.8 : : 33.518 : 29.369.

The greatest and least distances of the moon from the earth, derived from the parallaxes, are 63.8419 and 55.9164, or nearly 64 and 56, the radius of the earth being taken for unity. Hence, taking the arithmetical mean, which is 59.879, we find that the mean distance of the moon from the earth is very nearly 60 times the radius of the earth. The point in the moon's orbit nearest the earth, is called her *perigee;* the point farthest from the earth, her *apogee.*

The greatest and least apparent diameters of the *sun* are respectively 32.583, and 31.517, which numbers express also the ratio of the greatest and least distances of the earth from the sun. By comparing this ratio with that of the distances of the moon, it will be seen that the latter vary much more than the former, and consequently that the lunar orbit is much more eccentric

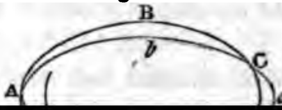
than the solar. The eccentricity of the moon's orbit is in fact 0.0548, (the semi-major axis being as usual taken for unity)$=\frac{1}{18}$ of its mean distance from the earth, while that of the earth is only .01685$=\frac{1}{59}$ of its mean distance from the sun.

231. *The moon's nodes constantly shift their positions in the ecliptic from east to west, at the rate of* 19° 35′ *per annum, returning to the same points in* 18.6 *years.*

Suppose the great circle of the ecliptic marked out on the face of the sky in a distinct line, and let us observe, at any given time, the exact point where the moon crosses this line, which we will suppose to be close to a certain star; then, on its next return to that part of the heavens, we shall find that it crosses the ecliptic sensibly to the westward of that star, and so on, farther and farther to the westward every time it crosses the ecliptic at either node. This fact is expressed by saying that *the nodes retrograde on the ecliptic*, and that the line which joins them, or the line of the nodes, revolves from east to west.

232. This shifting of the moon's nodes implies that the lunar orbit is not a curve returning into itself, but that it more resembles a spiral like the curve represented in figure 49, where *abc* represents the ecliptic, and ABC the lunar orbit, having its nodes at C and E, instead of A and *a*; consequently, the nodes shift backwards through

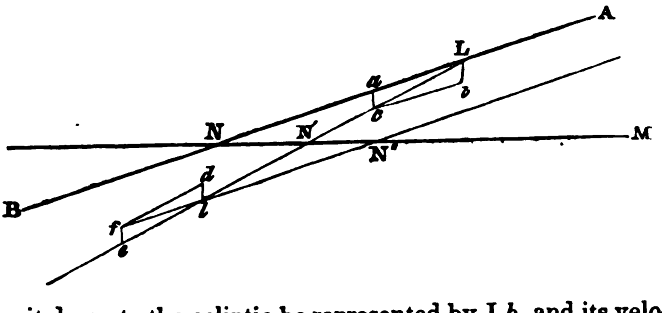
Fig. 49.



[redacted]

earth. Hence the moon meets the plane of the ecliptic sooner than it would have done if that force had not acted. At every half revolution, therefore, the point in which the moon meets the ecliptic, shifts in a direction contrary to that of the moon's motion, or contrary to the order of the signs. If the earth and sun were at rest, the effect of the deflecting force just described, would be to produce a retrograde motion of the line of the nodes till that line was brought to pass through the sun, and of consequence, the plane of the moon's orbit to do the same, after which they would both remain in their position, there being no longer any force tending to produce change in either. But the motion of the earth carries the line of the nodes out of this position, and produces, by that means, its continual retrogradation. The same force produces a small variation in the inclination of the moon's orbit, giving it an alternate increase and decrease within very narrow limits.* These points will be easily understood by the aid of a diagram. Therefore, let MN (Fig. 50,) be the ecliptic, ANB the orbit of the moon, the moon being in L, and N its descending node. Let the disturbing force of the sun which tends to

Fig. 50.



bring it down to the ecliptic be represented by L*b*, and its velocity in its orbit by L*a*. Under the action of these two forces, the moon will describe the diagonal L*c* of the parallelogram *ba*, and its orbit will be changed from AN to LN′; the node N passes to N′; and the exterior angle at N′ of the triangle LNN′ being greater than the interior and opposite angle at N, the inclination of the orbit is increased at the node. After the moon has passed the ecliptic to the south side to *l*, the disturbing force *ld* produces a

* Playfair.

new change of the orbit N′*le* to N″*lf*, and the inclination is diminished as at N″. In general, while the moon is receding from one of the nodes, its inclination is diminishing; while it is approaching a node, the inclination is increasing.*

233. The period occupied by the sun in passing from one of the moon's nodes until it comes round to the same node again, is called *the synodical revolution of the node.* This period is shorter than the sidereal year, being only about 346½ days. For since the node shifts its place to the westward 19° 35′ per annum, the sun, in his annual revolution, comes to it so much before he completes his entire circuit; and since the sun moves about a degree a day, the synodical revolution of the node is 365 − 19 = 346, or more exactly, 346.619851. The time from one new moon, or from one full moon, to another, is 29.5305887 days. Now 19 synodical revolutions of the nodes contain very nearly 223 of these periods.

$$\text{For } 346.619851 \times 19 = 6585.78,$$
$$\text{And } 29.5305887 \times 223 = 6585.32.$$

Hence, if the sun and moon were to leave the moon's node together, after the sun had been round to the same node 19 times, the moon would have performed very nearly 223 synodical revolutions, and would, therefore, at the end of this period meet at the same node, to repeat the same circuit. And since eclipses of the sun and moon depend upon the relative position of the sun, [redacted]

ing cycle; and, since the regulation of games, feasts, and fasts, has been made very extensively according to new or full moons, hence this lunar cycle has been much used both in ancient and modern times. The Athenians adopted it 433 years before the Christian era, for the regulation of their calendar, and had it inscribed in letters of gold on the walls of the temple of Minerva. Hence the term *Golden Number*, which denotes the year of the lunar cycle.

235. *The line of the apsides of the moon's orbit revolves from west to east through her whole orbit in about nine years.*

If, in any revolution of the moon, we should accurately mark the place in the heavens where the moon comes to its perigee, (Art. 230,) we should find, that at the next revolution, it would come to its perigee at a point a little farther eastward than before, and so on at every revolution, until, after 9 years, it would come to its perigee at nearly the same point as at first. This fact is expressed by saying that the perigee, and of course the apogee, revolves, and that the line which joins these two points, or the line of the apsides, also revolves.

The place of the perigee may be found by observing when the moon has the greatest apparent diameter. But as the magnitude of the moon varies slowly at this point, a better method of ascertaining the position of the apsides, is to take two points in the orbit where the variations in apparent diameter are most rapid, and to find where they are equal on opposite sides of the orbit. The middle point between the two will give the place of the perigee.

The angular distance of the moon from her perigee in any part of her revolution, is called the *Moon's Anomaly.*

236. The change of place in the apsides of the moon's orbit, like the shifting of the nodes, is caused by the disturbing influence of the sun. If when the moon sets out from its perigee, it were urged by no other force than that of projection, combined with its gravitation towards the earth, it would describe a symmetrical curve (Art. 186,) coming to its apogee at the distance of 180°. But as the mean disturbing force in the direction of the radius

vector tends, on the whole, to diminish the gravitation of the moon to the earth, the portion of the path described in an instant will be less deflected from her tangent, or less curved, than if this force did not exist. Hence the path of the moon will not intersect the radius vector at right angles; that is, she will not arrive at her apogee until after passing more than 180° from her perigee, by which means these points will constantly shift their positions from west to east.* The motion of the apsides is found to be 3° 1′ 20″ for every sidereal revolution of the moon.

237. On account of the greater eccentricity of the moon's orbit above that of the sun, the *Equation of the Center*, or that correction which is applied to the moon's mean anomaly to find her true anomaly (Art. 200,) is much greater than that of the sun, being when greatest more than six degrees, (6° 17′ 12″.7,) while that of the sun is less than two degrees, (1° 55′ 26″.8.)

238. Next to the equation of the center, the greatest correction to be applied to the moon's longitude, is that which belongs to the *Evection*. The evection is *a change of form in the lunar orbit*, by which its eccentricity is sometimes increased, and sometimes diminished. It depends on the position of the line of the apsides with respect to the line of the syzigies.

This irregularity, and its connexion with the place of the perigee with respect to the place of conjunction or opposition, was

[redacted]

place by deducting the equation of the center from the mean anomaly (See Art. 200,) seven days after conjunction, the computed longitude will be greater than that determined by actual observation, by about 80 minutes; but if the same estimate is made when the line of the apsides is in quadrature, the computed longitude will be less than the observed, by the same quantity. These results plainly show a connexion between the velocity of the moon's motions and the position of the line of the apsides with respect to the line of the syzigies.

239. Now any cause which, at the perigee, should have the effect to increase the moon's gravitation towards the earth beyond its mean, and, at the apogee, to diminish the moon's gravitation towards the earth, would augment the difference between the gravitation at the perigee and at the apogee, and consequently increase the eccentricity of the orbit. Again, any cause which at the perigee should have the effect to lessen the moon's gravitation towards the earth, and, at the apogee, to increase it, would lessen the difference between the two, and consequently diminish the eccentricity of the orbit, or bring it nearer to a circle. Let us see if the disturbing force of the sun produces these effects. The sun's disturbing force, as we have seen in article 228, admits of two resolutions, one in the direction of the radius vector, (OM, Fig. 48,) the other (ON) in the direction of a tangent to the orbit. First, let AB be the line of the apsides in syzigy, A being the place of the perigee. The sun's disturbing force OM is greatest in the direction of the line of the syzigies; yet being proportional to the distance of the moon from the earth, it is at a minimum when acting at the perigee, and at a maximum when acting at the apogee. Hence its effect is to draw away the moon from the earth less than usual at the perigee, and of course to make her gravitation towards the earth greater than usual, while at the apogee its effect is to diminish the tendency of the moon to the earth more than usual, and thus to increase the disproportion between the two distances of the moon from the focus at these two points, and of course to increase the eccentricity of the orbit. The moon, therefore, if moving towards the perigee, is brought to the line of the apsides in a point between its mean place and

the earth; or if moving towards the apogee, she reaches the line of the apsides in a point more remote from the earth than its mean place.

Secondly, let CD be the line of the apsides, in quadrature, C being the place of the perigee. The effect of the sun's disturbing force is to increase the tendency of the moon towards the earth when in quadrature. If, however, the moon is then at her perigee, such increase will be less than usual, and if at her apogee, it will be more than usual; hence its effect will be to lessen the disproportion between the two distances of the moon from the forces at these two points; and of course to diminish the eccentricity of the orbit. The moon, therefore, if moving towards the perigee, meets the line of the apsides in a point more remote from the earth than the mean place of the perigee; and if moving towards the apogee, in a point between the earth and the mean place of the apogee.*

240. A third inequality in the lunar motions, is the *Variation*. By comparing the moon's place as computed from her mean motion corrected for the equation of the center and for evection, with her place as determined by observation, Tycho Brahe discovered that the computed and observed places did not always agree. They agreed only in the syzigies, and disagreed most at a point half way between, that is, at the *octants*. Here, at the maximum, it amounted to more than half a degree (35′ 41.″6.) It appeared evident from examining the daily observations while

moon's orbit. The former, as already explained, produces the Evection: the latter produces the Variation. This latter force will accelerate the moon's velocity, in every point of the quadrant which the moon describes in moving from quadrature to conjunction, or from C to A, (Fig. p. 127,) but at an unequal rate, the acceleration being greatest at the octant, and nothing but at the quadrature and the conjunction; and when the moon is past conjunction, the tangential force will change its direction and retard the moon's motion. All these points will be understood by inspection of figure 48.

241. A fourth lunar inequality is the *Annual Equation.* This depends on the distance of the earth (and of course the moon) from the sun. Since the disturbing influence of the sun has a greater effect in proportion as the sun is nearer,* consequently all the inequalities depending on this influence must vary at different seasons of the year. Hence, the amount of this effect due to any particular time of the year is called the Annual Equation.

242. The foregoing are the largest of the inequalities of the moon's motions, and may serve as *specimens* of the corrections that are to be applied to the mean place of the moon in order to find her true place. These were first discovered by actual observation; but a far greater number, though most of them are exceedingly minute, have been made known by the investigations of Physical Astronomy, in following out all the consequences of universal gravitation. In the best tables, about 30 equations are applied to the mean motions of the moon. That is, we first compute the place of the moon on the supposition that she moves uniformly in a circle. This gives us her *mean* place. We then proceed, by the aid of the Lunar Tables, to apply the different corrections, such as the equation of the center, evection, variation, the annual equation, and so on, to the number of 28. Numerous as these corrections appear, yet La Place informs us, that the whole number belonging to the moon's longitude is no less than 60; and that to give the tables all the requisite degree of precision, additional investigations will be necessary, as extensive at least as

* Varying reciprocally as the *cube* of the sun's distance from the earth.

those already made.* The best tables in use in the time of Tycho Brahe, gave the moon's place only by a distant approximation. The tables in use in the time of Newton, (Halley's tables,) approximated within 7 minutes. Tables at present in use give the moon's place to 5 seconds. These additional degrees of accuracy have been attained only by immense labor, and by the united efforts of Physical Astronomy, and the most refined observations.

243. The inequalities of the moon's motions are divided into periodical and secular. *Periodical* inequalities are those which are completed in comparatively short periods, like evection and variation: *Secular* inequalities are those which are completed only in very long periods, such as centuries or ages. Hence the corresponding terms *periodical equations*, and *secular equations.* As an example of a secular inequality, we may mention the *acceleration of the moon's mean motion.* It is discovered, that the moon actually revolves around the earth in less time now than she did in ancient times. The difference however is exceedingly small, being only about 10″ in a century, but increases from century to century as the square of the number of centuries. This remarkable fact was discovered by Dr. Halley.† In a lunar eclipse the moon's longitude differs from that of the sun, at the middle of the eclipse, by exactly 180°; and since the sun's longitude at any given time of the year is known, if we can learn the day and hour when an eclipse occurs, we shall of course know [redacted]

This phenomenon at first led astronomers to apprehend that the moon encountered a resisting medium, which, by destroying at every revolution a small portion of her projectile force, would have the effect to bring her nearer and nearer to the earth and thus to augment her velocity. But in 1786, La Place demonstrated that this acceleration is one of the legitimate effects of the sun's disturbing force, and is so connected with changes in the eccentricity of the earth's orbit, that the moon will continue to be accelerated while that eccentricity diminishes, but when the eccentricity has reached its minimum (as it will do after many ages) and begins to increase, then the moon's motion will begin to be retarded, and thus her mean motions will oscillate forever about a mean value.

244. The lunar inequalities which have been considered are such only as effect the moon's longitude; but the sun's disturbing force also causes inequalities in the moon's *latitude* and *parallax*. Those of latitude alone require no less than twelve equations. Since the moon revolves in an orbit inclined to the ecliptic, it is easy to see that the oblique action of the sun must admit of a resolution into two forces, one of which being perpendicular to the moon's orbit must effect changes in her latitude. Since also several of the inequalities already noticed involve changes in the length of the radius vector, it is obvious that the moon's parallax must be subject to corresponding perturbations.

CHAPTER VII.

ECLIPSES.

245. An *eclipse of the moon* happens, when the moon in its revolution about the earth, falls into the earth's shadow. An *eclipse of the sun* happens, when the moon, coming between the earth and the sun, covers either a part or the whole of the solar disk. An eclipse of the sun can occur only at the time of con-

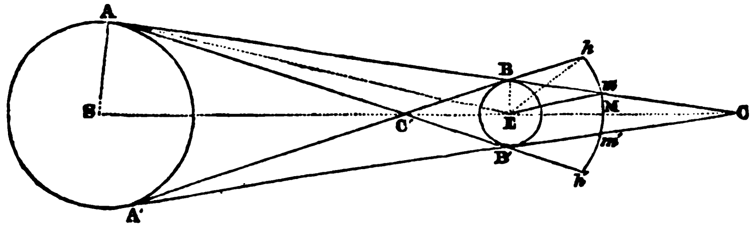
junction, or new moon; and an eclipse of the moon, only at the time of opposition, or full moon. Were the moon's orbit in the same plane with that of the earth, or did it coincide with the ecliptic, then an eclipse of the sun would take place at every conjunction, and an eclipse of the moon at every opposition; for as the sun and earth both lie in the ecliptic, the shadow of the earth must also extend in the same plane, being of course always directly opposite to the sun; and since, as we shall soon see, the length of this shadow is much greater than the distance of the moon from the earth, the moon, if it revolved in the plane of the ecliptic, must pass through the shadow at every full moon. For similar reasons, the moon would occasion an eclipse of the sun, partial or total, in some portions of the earth at every new moon. But the lunar orbit is inclined to the ecliptic about 5°, so that the center of the moon, when she is farthest from her node, is 5° from the axis of the earth's shadow (which is always in the ecliptic;) and, as we shall show presently, the greatest distance to which the shadow extends on each side of the ecliptic, that is, the greatest semi-diameter of the shadow, where the moon passes through it, is only about $\frac{3}{4}$ of a degree, while the semi-diameter of the moon's disk is only about $\frac{1}{4}$ of a degree; hence the two semi-diameters, namely, that of the moon and the earth's shadow, cannot overlap one another, unless, at the time of new or full moon, the sun is at or very near the moon's node. In the course of the sun's apparent revolution around the earth once a year, he is succes- [redacted]

246. If the sun were of the same size with the earth, the shadow of the earth would be cylindrical and infinite in length, since the tangents drawn from the sun to the earth (which form the boundaries of the shadow) would be parallel to each other; but as the sun is a vastly larger body than the earth, the tangents converge and meet in a point at some distance behind the earth, forming a cone of which the earth is the base, and whose vertex (and of course its axis) lies in the ecliptic. A little reflection will also show us, that the form and dimensions of the shadow must be affected by several circumstances; that the shadow must be of the greatest length and breadth when the sun is farthest from the earth; that its figure will be slightly modified by the spheroidal figure of the earth; and that the moon, being, at the time of its opposition, sometimes nearer to the earth, and sometimes farther from it, will accordingly traverse it at points where its breadth varies more or less.

247. *The semi-angle of the cone of the earth's shadow, is equal to the sun's apparent semi-diameter, minus his horizontal parallax.*

Let AS (Fig. 51,) be the semi-diameter of the sun, BE that of the earth, and EC the axis of the earth's shadow. Then the semi-angle of the cone of the earth's shadow ECB=AES−EAB,

Fig. 51.



of which AES is the sun's semi-diameter and EAB his horizontal parallax; and as both these quantities are known, hence the angle at the vertex of the shadow becomes known. Putting δ for the sun's semi-diameter, and p for his horizontal parallax, we have the semi-angle of the earth's shadow ECB$=\delta-p$.

248. *At the mean distance of the earth from the sun, the length of the earth's shadow is about* 860,000 *miles, or more than three times the distance of the moon from the earth.*

In the right angled triangle ECB, the angle ECB being known, and the side EB, we can find the side EC. For sin. $\delta - p$: EB :: R : EC $= \frac{\text{EB}}{\sin.\ \delta - p}$. This value will vary with the sun's semi-diameter, being greater as that is less. Its mean value being 16′ 1″.5 and the sun's horizontal parallax being 8″.6, $\delta - p =$ 15′ 52″.9, and EB = 3956.2. Hence,

Sin. 15′ 53″ : Rad. : : 3956.2 : 856,275.

Since the distance of the moon from the earth is 238.545 miles, the shadow extends about 3.6 times as far as the moon, and consequently, the moon passes the shadow towards its broadest part, where its breadth is much more than sufficient to cover the moon's disk.

249. *The average breadth of the earth's shadow where it eclipses the moon, is almost three times the moon's diameter.*

Let mm' (Fig. 51,) represent a section of the earth's shadow where the moon passes through it, M being the center of the circular section. Then the angle MEm will be the angular breadth of half the shadow. But,

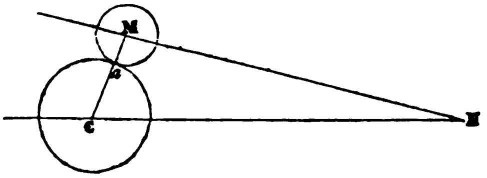
MEm = BmE − BCE ; that is, since BmE is the moon's horizontal parallax (Art. 82,) and BCE equals the sun's semi-diame- [redacted]

the *Solar Ecliptic Limit.* The Limits are respectively the farthest possible distances from the node at which eclipses can take place.

251. *The Lunar Ecliptic Limit is nearly* 12 *degrees.*

Let CN (Fig. 52,) be the sun's path, MN the moon's, and N the node. Let C*a* be the semi-diameter of the earth's shadow, and M*a* the semi-diameter of the moon. Since C*a* and M*a* are known

Fig. 52.



quantities, their sum CM is also known. The angle at N is known, being the inclination of the lunar orbit to the ecliptic. Hence, in the spherical triangle MCN, right angled at M,* by Napier's theorem, (p. 60.)

$$\text{Rad.} \times \sin.\ \text{CM} = \sin.\ \text{CN} \times \sin.\ \text{MNC}.$$

The greatest apparent semi-diameter of the earth's shadow where the moon crosses it, computed by article 249, is 45′ 52″, and the moon's greatest apparent semi-diameter, is 16′ 45″.5, which together, give MC equal to 62′ 37″.5. Taking the inclination of the moon's orbit, or the angle MNC (what it generally is in these circumstances) at 5° 17′, and we have Rad. × sin. 62′ 37″.5 = sin. CN × sin. 5° 17′, or $\sin.\ \text{CN} = \frac{\text{Rad} \times \sin.62' \, 37''.5}{\sin.\ 5^\circ \, 17'}$ $= 11^\circ \, 25' \, 40''$.† This is the greatest distance of the moon from her node (in longitude) at which an eclipse of the moon can take place. By varying the value of CM, corresponding to variations in the distances of the sun and moon from the earth, it is found that if NC is less than 9°, there *must* be an eclipse; but between this and the limit, the case is doubtful.

When the moon's disk only comes in contact with the earth's shadow, as in figure 52, the phenomenon is called an *appulse;*

* The line CM is to be regarded as the *projection* of the line which connects the centers of the moon and section of the earth's shadow, as seen from the earth.

† Woodhouse's Astronomy, p. 718.

when only a part of the disk enters the shadow, the eclipse is said to be *partial*, and *total* if the whole of the disk enters the shadow. The eclipse is called *central* when the moon's center coincides with the axis of the shadow, which happens when the moon at the time of the eclipse is exactly at her node.

252. Before the moon enters the earth's shadow, the earth begins to intercept from it portions of the sun's light, gradually increasing until the moon reaches the shadow. This partial light is called the moon's *Penumbra*. Its limits are ascertained by drawing the tangents AC′B′, and A′C′B. Throughout the space included between these tangents more or less of the sun's light is intercepted from the moon by the interposition of the earth; for it is evident, that as the moon moves towards the shadow, she would gradually lose the view of the sun, until, on entering the shadow, the sun would be entirely hidden from her.

253. *The semi-angle of the Penumbra equals the sun's semi-diameter and horizontal parallax, or* $\delta + p$.

The angle hC′M (Fig. 51,) = AC′S = AES + B′AE. But AES is the sun's semi-diameter, and B′AE is the sun's horizontal parallax, both of which quantities are known.

254. *The semi-angle of a section of the Penumbra, where the moon crosses it, equals the moon's horizontal parallax, plus the* [redacted]

counted for by supposing that a portion of the solar rays which graze the earth's surface are absorbed and extinguished by the lower strata of the atmosphere. This amounts to the same thing as though the earth were larger than it is, in which case the moon's horizontal parallax would be increased; and accordingly, in order that theory and observation may coincide, it is found necessary to increase the parallax by $\frac{1}{60}$.

256. In a total eclipse of the moon, its disk is still visible, shining with a dull red light. This light cannot be derived directly from the sun, since the view of the sun is completely hidden from the moon; nor by reflexion from the earth, since the illuminated side of the earth is wholly turned from the moon; but it is owing to refraction by the earth's atmosphere, by which a few scattered rays of the sun are bent round into the earth's shadow and conveyed to the moon, sufficient in number to afford the feeble light in question.

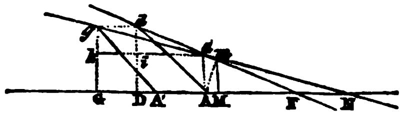
257. In *calculating an eclipse of the moon*, we first learn from the tables in what month the sun, at the time of full moon in that month, is near the moon's node, or within the lunar ecliptic limit. This it must evidently be easy to determine, since the tables enable us to find both the longitudes of the nodes, and the longitudes of the sun and moon, for every day of the year. Consequently, we can find when the sun has nearly the same longitude as one of the nodes, and also the precise moment when the longitude of the moon is 180° from that of the sun, for this is the time of opposition, or of the middle of the eclipse. Having the time of the middle of the eclipse, and the breadth of the shadow, (Art. 249,) and knowing, from the tables, the rate at which the moon moves per hour faster than the shadow, we can find how long it will take her to traverse half the breadth of the shadow; and this time subtracted from the time of the middle of the eclipse, will give the beginning, and added to the time of the middle will give the end of the eclipse. Or if instead of the breadth of the shadow, we employ the breadth of the penumbra (Art. 253,) we may find, in the same manner, when the moon enters and when she leaves the penumbra. We see,

therefore, how by having a few things known by observation, such as the sun and moon's semi-diameters, and their horizontal parallaxes, we rise, by the aid of trigonometry, to the knowledge of various particulars respecting the length and breadth of the shadow and of the penumbra. These being known, we next have recourse to the tables which contain all the necessary particulars respecting the motions of the sun and moon, together with equations or corrections, to be applied for all their irregularities. Hence it is comparatively an easy task to calculate with great accuracy an eclipse of the moon.

258. Let us then see how we may find the exact time of the beginning, end, duration, and magnitude, of a lunar eclipse.

Let NG (Fig. 53,) be the ecliptic, and N*ag* the moon's orbit, the sun being in A when the moon is in opposition at *a*; let N be the ascending node, and A*a* the moon's latitude at the instant

Fig. 53.

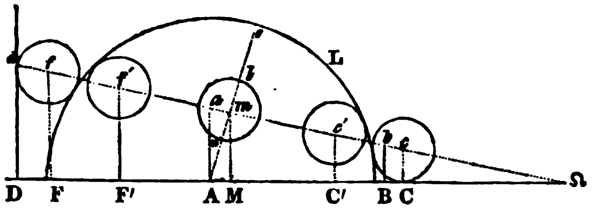


of opposition. An hour afterwards the sun will have passed to A′, and the moon to *g*, when the difference of longitude of the two bodise will be *g*A′. Then *gh* is the moon's hourly motion in latitude, and *ah* her hourly motion in longitude. As the character and form of the eclipse will depend solely upon the distances

[redacted]

Draw CD (Fig. 54,) to represent the ecliptic, and let A be the place of the sun. As the tables give the computation of the moon's latitude at every instant, consequently, we may take from them the latitude corresponding to the instant of opposition, and to one hour later; and we may take also the sun's and moon's hourly motions in longitude. Take AD, AB, each equal to the

Fig. 54.



relative motion, and Aa = the latitude in opposition, Dd = the latitude one hour afterwards; join da and produce the line da both ways, and it will represent the moon's relative orbit. Draw Bb at right angles to CD and it will be the latitude an hour before opposition. At the time of the eclipse, the apparent distance of the center of the shadow from the moon is very small; consequently, CD, cd, Dd, &c. may be regarded as straight lines. During the short interval between the beginning and end of an eclipse, the motion of the sun, and consequently that of the center of the shadow, may likewise be regarded as uniform.

259. The various particulars that enter into the calculation of an eclipse are called its *Elements;* and as our object is here merely *to explain the method* of calculating an eclipse of the moon, (reserving the actual computation for the fourth part of this work,) we may take the elements at their mean value. Thus, we will consider cd as inclined to CD $5^\circ\ 9'$, the moon's horizontal parallax as $58'$, its semi-diameter as $16'$, and that of the earth's shadow as $42'$. The line Am perpendicular to cd gives the point m for the place of the moon at the middle of the eclipse, for this line bisects the chord, which represents the path of the moon through the shadow; and mM, perpendicular to CD, gives AM for the time of the middle of the eclipse before opposition, the number of minutes before opposition being the same part of an hour that

AM is of AB.* From the center A, with a radius equal to that of the earth's shadow (42′) describe the semi-circle BLF, and it will represent the projection of the shadow traversed by the moon. With a radius equal to the semi-diameter of the shadow and that of the moon ($=42'+16'=58'$) and with the center A, mark the two points c and f on the relative orbit, and they will be the places of the center of the moon at the beginning and end of the eclipse. The perpendiculars cC, fF, give the times AC and AF of the commencement and the end of the eclipse, and CM, or MF gives half the duration. From the centers c and f with a radius equal to the semi-diameter of the moon (16′) describe circles, and they will each touch the shadow, (Euc. 3.12.) indicating the position of the moon at the beginning and end of the eclipse. If the same circle described from m is wholly within the shadow, the eclipse will be *total;* if it is only partly within the shadow, the eclipse will be *partial.* With the center A, and radius equal to the semi-diameter of the shadow minus that of the moon ($42'-16'=26'$) mark the two points c', f', which will give the places of the center of the moon, at the beginning and end of total darkness, and MC′, MF′ will give the corresponding times before and after the middle of the eclipse. Their sum will be the duration of total darkness.

260. If the foregoing projection be accurately made from a scale, the required particulars of the eclipse may be ascertained by measuring on the same scale, the lines which respectively rep-

[redacted]

(Fig. 54,) having the side A*m* and the angle A*m*M ($=a$A*m*) we can find AM = the arc of relative longitude described by the moon from the time of the middle of the eclipse to the time of opposition; and knowing the moon's hourly motion in longitude, we can convert AM into time, and this subtracted from the time of opposition gives us the *time of the middle of the eclipse.*

Secondly, since we know the length of the line A*c** and can easily find the angle *c*AC, we can thus obtain the side AC; and AC − AM = MC, which arc, converted into time by comparing it with the moon's hourly motion in longitude, gives us, when subtracted from the time of the middle of the eclipse, *the time of the beginning of the eclipse*, or when added to that of the middle, *the time of the end of the eclipse.* The sum of the two equals the *whole duration.*

Thirdly, by a similar method we calculate the value of MC′, which converted into time, and subtracted from the time of the middle of the eclipse, gives the *commencement of total darkness*, or when added gives the *end of total darkness.* Their sum is the *duration of total darkness.*

Fourthly, the *quantity of the eclipse* is determined by supposing the diameter of the moon divided into twelve equal parts called *Digits*, and finding how many such parts lie within the shadow, at the time when the centers of the moon and the shadow are nearest to each other. Even when the moon lies wholly within the shadow, the quantity of the eclipse is still expressed by the number of digits contained in that part of the line which joins the center of the shadow and the center of the moon, which is intercepted between the edge of the shadow and the inner edge of the moon. Thus in figure 54, the number of digits eclipsed, equals $\frac{no}{\frac{1}{12}nl}=\frac{Ao-An}{\frac{1}{12}nl}=\frac{Ao-(Am-nm)}{\frac{1}{12}nl}$, an expression containing only known quantities.

261. The foregoing will serve as an explanation of the *general principles*, on which proceeds the calculation of a lunar eclipse. The actual methods practiced employ many expedients to facilitate the process, and to insure the greatest possible accuracy, the

* This line is not represented in the figure, but may be easily imagined.

nature of which will be explained and exemplified in the fourth part of this work.

262. The leading particulars respecting an ECLIPSE OF THE SUN, are ascertained very nearly like those of a lunar eclipse. The shadow of the moon travels over a portion of the earth, as the shadow of a small cloud, seen from an eminence in a clear day, rides along over hills and plains. Let us imagine ourselves standing on the moon; then we shall see the earth partially eclipsed by the shadow of the moon, in the same manner as we now see the moon eclipsed by the earth's shadow; and we might proceed to find the length of the shadow, its breadth where it eclipses the earth, the breadth of the penumbra, and its duration and quantity, in the same way as we have ascertained these particulars for an eclipse of the moon.

But, although the general characters of a solar eclipse might be investigated on these principles, so far as respects the earth at large, yet as the appearances of the same eclipse of the sun are very different at different places on the earth's surface, it is necessary to calculate its peculiar aspects for each place separately, a circumstance which makes the calculation of a solar eclipse much more complicated and tedious than of an eclipse of the moon. The moon, when she enters the shadow of the earth, is deprived of the light of the part immersed, and that part appears black alike to all places when the moon is above the horizon.

[redacted]

miles, and the shadow moves of course at the same rate, or 2280 miles per hour, traversing the entire disk of the earth in less than four hours. This is the velocity of the shadow when it passes *perpendicularly* over the earth; when the direction of the axis of the shadow is oblique to the earth's surface the velocity is increased in proportion of radius to the sine of obliquity; for having a greater space to pass over in the same time, its velocity must of course be greater. When the conjunction takes place exactly at the node, the axis of the moon's shadow lies in the ecliptic, and the shadow traverses the earth perpendicularly to its surface; but when the conjunction occurs on either side of the node, but within the solar ecliptic limits, the shadow falls obliquely on the earth. The moon's shadow being a cone, the oblique section of it made by the earth, is an ellipse.

Let us endeavor to form a just conception of the manner in which these three bodies, the sun, the earth, and the moon, are situated with respect to each other at the time of a solar eclipse. First, suppose the conjunction to take place at the node. Then the straight line which connects the centers of the sun and the earth, also passes through the center of the moon, and coincides with the axis of its shadow; and, since the earth is bisected by the plane of the ecliptic, the shadow would traverse the earth in the direction of the terrestrial ecliptic, from west to east, passing over the middle regions of the earth. Here the diurnal motion of the earth being in the same direction with the shadow, but with a less velocity, the shadow will appear to move with a speed equal only to the difference between the two. Secondly, suppose the moon is on the north side of the ecliptic at the time of conjunction, and moving towards her descending node, and that the conjunction takes place just within the solar ecliptic limit, say 16° from the node. The shadow will now not fall in the plane of the ecliptic, but a little northward of it, so as just to graze the earth near the pole of the ecliptic. The nearer the conjunction comes to the node, the farther the shadow will fall from the pole of the ecliptic towards the equatorial regions. In certain cases, the shadow strikes beyond the pole of the earth; and then its easterly motion being opposite to the diurnal motion of the places which it traverses, consequently its velocity is greatly increased, being equal to the sum of both.

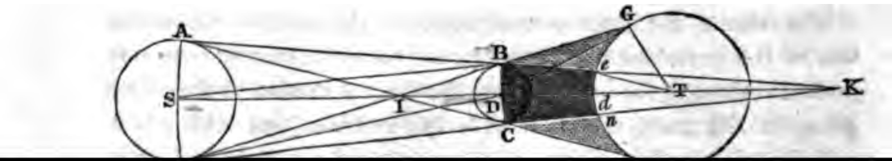
264. After these general considerations, we will now examine more particularly the method of investigating the elements of a solar eclipse.

The *length of the moon's shadow*, is the first object of inquiry. The moon as well as the earth, is at different distances from the sun at different times, and hence the length of her shadow varies, being always greatest when she is farthest from the sun. Also, since her distance from the earth varies, the section of the moon's shadow made by the earth, is greater in proportion as the moon is nearer the earth. The greatest eclipses of the sun, therefore, happen when the sun is in apogee,* and the moon in perigee.

265. *When the moon is at her mean distance from the earth, and from the sun, her shadow nearly reaches the earth's surface.*

Let S (Fig. 55,) represent the sun, D the moon, and T the earth. Then, the semi-angle of the cone of the moon's shadow, DKR, will, as in the case of the earth, (Art. 247,) equal SDR – DRK, of which SDR is the sun's apparent semi-diameter, as seen from the moon, and DRK, is the sun's horizontal parallax at the moon. Since, on account of the great distance of the

Fig. 55.



[redacted]

Now, SDR : STR :: ST : SD* :: 400 : 399 ; hence SDR $=\frac{400}{399}$ STR $=1.0025$ STR ; and the sun's mean semi-diameter STR being 16.025, hence SDR $=1.0025 \times 16.025=16.065=$ $16' \, 3''.9$.

Again, since parallax is inversely as the distance, the sun's horizontal parallax at the moon, is on account of her being nearer the sun $\frac{1}{400}$ greater than at the earth ; but on account of her inferior size it is $\frac{7912}{2160}$ less than at the earth. Hence, increasing the sun's horizontal parallax at the earth by the former fraction, and diminishing it by the latter, we have $\frac{400}{399} \times \frac{2160}{7912} \times 9''=2''.5=$ sun's horizontal parallax at the moon. Therefore, the semi-angle of the cone of the moon's shadow, which, as appears above, equals SDR $-$ DRK, equals $16' \, 3''.9 - 2''.5=16' \, 1''.4$, which so nearly equals the sun's apparent semi-diameter, as seen from the earth, that we may adopt the latter as the value of the semi-angle of the shadow. Hence, sin. $16' \, 1''.5$: 1080(BD)::Rad. : DK$=$ 231690. But the mean distance of the moon from the surface of the earth is $238545-3956=234589$, which exceeds a little the mean length of the shadow as above.

But when the moon is nearest the earth her distance from the center of the earth is only 221148 miles ; and when the earth is farthest from the sun, the sun's apparent semi-diameter is only $15' \, 45''.5$. By employing this number in the foregoing estimate, we shall find the length of the shadow 235630 miles ; and $235630-221148=14482$, the distance which the moon's shadow may reach beyond the center of the earth.

266. *The diameter of the moon's shadow where it traverses the earth, is, at its maximum, about* 170 *miles.*†

In the triangle eTK, the angle at K$=15' \, 45''.5$ (Art. 265,) the side T$e$$=3956$, and TK$=14482$.

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* The apparent magnitude of an object being reciprocally as its distance from the eye. See Note, p. 85.

† This supposes the conjunction to take place at the node, and the shadow to strike the earth perpendicularly to its surface ; where it strikes it obliquely, the section may be greater than this.

Or, 3956 : 14482 : : sin. 15′ 45.″5 : sin. 57′ 41″.5.
And 57′ 41.″5 + 15′ 45.″5 = 1° 13′ 27″ = *d*T*e*, on the arc *de*.
And 2*de* = 2° 26′ 54″ = *en*.
Hence 360 : 2.45 (= 2° 26′ 54″) : : 24899* : 170 (nearly).

267. *The greatest portion of the earth's surface ever covered by the moon's penumbra, is about* 4393 *miles.*

The semi-angle of the penumbra BID = BSD + SBR, of which BSD the sun's horizontal parallax at the moon = 2″.5, and SBR the sun's apparent semi-diameter = 16′ 3″.9, and hence BID is known. The moon's apparent semi-diameter BGD = 16′ 45″.5. Therefore GDT is known, as likewise DT and TG. Hence the angle GT*d* may be found, and the arc *d*G and its double GH, which equals the angular breadth of the penumbra. It may be converted into miles by stating a proportion as in article 266. On making the calculation it will be found to be 4393 miles.

268. The apparent diameter of the moon is sometimes larger than that of the sun, sometimes smaller, and sometimes exactly equal to it. Suppose an observer placed on the right line which joins the centers of the sun and moon; if the apparent diameter of the moon is greater than that of the sun, the eclipse will be total. If the two diameters are equal, the moon's shadow just reaches the earth, and the sun is hidden but for a moment from

[redacted]

cause alone, of two observers at a distance from each other, one might see an eclipse which was not visible to the other.* If the horizontal diameter of the moon differs but little from the apparent diameter of the sun, the case might occur where the eclipse would be annular over the places where it was observed morning and evening, but total where it was observed at midday.

The earth in its diurnal revolution and the moon's shadow both move from west to east, but the shadow moves faster than the earth; hence the moon overtakes the sun on its western limb and crosses it from west to east. The excess of the apparent diameter of the moon above that of the sun in a total eclipse is so small, that total darkness seldom continues longer than four minutes, and can never continue so long as eight minutes. An annular eclipse may last 12m. 24s.

Since the sun's ecliptic limits are more than 17° and the moon's less than 12°, eclipses of the sun are more frequent than those of the moon. Yet lunar eclipses being visible to every part of the terrestrial hemisphere opposite to the sun, while those of the sun are visible only to the small portion of the hemisphere on which the moon's shadow falls, it happens that for any particular place on the earth, lunar eclipses are more frequently visible than solar. In any year, the number of eclipses of both luminaries cannot be less than two nor more than seven: the most usual number is four, and it is very rare to have more than six. A total eclipse of the moon frequently happens at the next full moon after an eclipse of the sun. For since, in an eclipse of the sun, the sun is at or near one of the moon's nodes, the earth's shadow must be at or near the other node, and may not have passed so far from the node as the lunar ecliptic limits, before the moon overtakes it.

270. It has been observed already, that were the spectator on the moon instead of on the earth, he would see the earth eclipsed by the moon, and the calculation of the eclipse would be very similar to that of a lunar eclipse; but to an observer on the earth the eclipse does not of course begin when the earth first enters the moon's shadow, and it is necessary to determine not only

* Biot, Ast. Phys. p. 401.

what portion of the earth's surface will be covered by the moon's shadow, but likewise the path described by its center relative to various places on the surface of the earth. This is known when the latitude and longitude of the center of the shadow on the earth, is determined for each instant. The latitude and longitude of the moon are found on the supposition that the spectator views it from the center of the earth, whereas his position on the surface changes, in consequence of parallax, both the latitude and longitude, and the amount of these changes must be accurately estimated, before the appearance of the eclipse at any particular place can be fully determined.

The details of the method of calculating a solar eclipse cannot be understood in any way so well, as by actually performing the process according to a given example. Such details therefore are reserved for a subsequent part of this work.

271. In total eclipses of the sun, there has sometimes been observed a remarkable radiance of light from the margin of the sun. This has been ascribed to an illumination of the solar atmosphere, but it is with more probability owing to the zodiacal light (Art. 152,) which at that time is projected around the sun, and which is of such dimensions as to extend far beyond the solar orb.*

A total eclipse of the sun is one of the most sublime and impressive pheuomena of nature. Among barbarous nations it is ever contemplated with fear and astonishment, while amoug cultivated [redacted]

as the period of total obscuration approached, a gloom pervaded all nature. When the sun was wholly lost sight of, planets and stars came into view; a fearful pall hung upon the sky, unlike both to night and to twilight; and, the temperature of the air rapidly declining, a sudden chill came over the earth. Even the animal tribes exhibited tokens of fear and agitation.

From 1831 to 1838 was a period remarkable for great eclipses of the sun, in which time there were no less than five of the most remarkable character. The next total eclipse of the sun, visible in the United States, will occur on the 7th of August, 1869.

CHAPTER VIII.

LONGITUDE—TIDES.

272. As eclipses of the sun afford one of the most approved methods of finding the longitudes of places, our attention is naturally turned next towards that subject.

The ancients studied astronomy in order that they might read their destinies in the stars: the moderns, that they may securely navigate the ocean. A large portion of the refined labors of modern astronomy, has been directed towards perfecting the astronomical tables with the view of finding the longitude at sea,—an object manifestly worthy of the highest efforts of science, considering the vast amount of property and of human life involved in navigation.

273. *The difference of longitude between two places, may be found by any method by which we can ascertain the difference of their local times, at the same instant of absolute time.*

As the earth turns on its axis from west to east, any place that lies eastward of another will come sooner under the sun, or will have the sun earlier on the meridian, and consequently, in respect to the hour of the day, will be in advance of the other at the

rate of one hour for every 15°, or four minutes of time for each degree. Thus, to a place 15° east of Greenwich, it is 1 o'clock, P. M. when it is noon at Greenwich; and to a place 15° west of that meridian, it is 11 o'clock, A. M. at the same instant. Hence, the difference of time at any two places, indicates their difference of longitude.

274. The easiest method of finding the longitude is by means of an accurate time piece, or *chronometer*. Let us set out from London with a chronometer accurately adjusted to Greenwich time, and travel eastward to a certain place, where the time is accurately kept, or may be ascertained by observation. We find, for example, that it is 1 o'clock by our chronometer, when it is 2 o'clock and 30 minutes at the place of observation. Hence, the longitude is $15 \times 1.5 = 22\frac{1}{2}°$ E. Had we travelled westward until our chronometer was an hour and a half in advance of the time at the place of observation, (that is, so much later in the day,) our longitude would have been $22\frac{1}{2}°$ W. But it would not be necessary to repair to London in order to set our chronometer to Greenwich time. This might be done at any observatory, or any place whose longitude had been accurately determined. For example, the time at New York is 4h. 56m. $4^s.5$ behind that of Greenwich. If, therefore, we set our chronometer so much before the true time at New York, it will indicate the time at Greenwich. Moreover, on arriving at different places, any where on [redacted]

quired, especially where the instrument is conveyed over land, although the uncertainty attendant on one instrument may be nearly obviated by employing several and taking their mean results.*

275. *Eclipses of the sun and moon* are sometimes used for determining the longitude. The exact instant of immersion or of emersion, or any other definite moment of the eclipse which presents itself to two distant observers, affords the means of comparing their difference of time, and hence of determining their difference of longitude. Since the entrance of the moon into the earth's shadow, in a lunar eclipse, is seen at the same instant of absolute time at all places where the eclipse is visible, (Art. 262,) this observation would be a very suitable one for finding the longitude were it not that, on account of the increasing darkness of the penumbra near the boundaries of the shadow, it is difficult to determine the precise instant when the moon enters the shadow. By taking observations on the immersions of known spots on the lunar disk, a mean result may be obtained which will give the longitude with tolerable accuracy. In an eclipse of the sun, the instants of immersion and emersion may be observed with greater accuracy, although, since these do not take place at the same instant of absolute time, the calculation of the longitude from observations on a solar eclipse are complicated and laborious.

A method very similar to the foregoing, by observations on eclipses of Jupiter's satellites, and on occultations of stars, will be mentioned hereafter.

276. *The Lunar method of finding the longitude,* at sea, is in many respects preferable to every other. It consists in measuring (with a sextant) the angular distance between the moon and the sun, or between the moon and a star, and then turning to the Nautical Almanac,† and finding what time it was at Greenwich when

* Woodhouse, p. 838.

† The *Nautical Almanac,* is a book published annually by the British Board of Longitude, containing various tables and astronomical information for the use of navigators. The *American Almanac* also contains a variety of astronomical information, peculiarly interesting to the people of the United States, in connexion with a vast amount of statistical matter. It is well deserving a place in the library of the student.

that distance was the same. The moon moves so rapidly, that this distance will not be the same except at very nearly the same instant of absolute time. For example, at 9 o'clock, A. M., at a certain place, we find the angular distance of the moon and the sun to be 72°; and, on looking into the Nautical Almanac, we find that at the time when this distance was the same for the meridian of Greenwich was 1 o'clock, P. M.; hence we infer that the longitude of the place is four hours, or 60° west.

The Nautical Almanac contains the true angular distance of of the moon from the sun, from the four large planets, (Venus, Mars, Jupiter, and Saturn,) and from nine bright fixed stars, for the beginning of every third hour of mean time for the meridian of Greenwich; and the mean time corresponding to any intermediate hour, may be found by proportional parts.*

277. It would be a very simple operation to determine the longitude by Lunar Distances, if the process as described in the preceding article were all that is necessary; but the various circumstances of parallax, refraction, and dip of the horizon, would differ more or less at the two places, even were the bodies whose distances were taken in view from both, which is not necessarily the case. The observations, therefore, at each meridian, require to be reduced to the center of the earth, being cleared of the effects of parallax and refraction. Hence, three observers are necessary in order to take a lunar distance in the most exact [redacted]

hours, or one minute of space in two minutes of time. Therefore, if we make an error of one minute in observing the distance, we make an error of two minutes in time, or 30 miles of longitude at the equator. A single observation with the best sextants, may be liable to an error of more than half a minute; but the accuracy of the result may be much increased by a mean of several observations taken to the east and west of the moon. The imperfection of lunar tables was until recently considered as an objection to this method. Until within a few years, the best lunar tables were frequently erroneous to the amount of one minute, occasioning an error of 30 miles. The error of the best tables now in use will rarely exceed 7 or 8 seconds.*

TIDES.

279. The tides are an alternate rising and falling of the waters of the ocean, at regular intervals. They have a maximum and a minimum twice a day, twice a month, and twice a year. Of the daily tide, the maximum is called *High tide*, and the minimum *Low tide*. The maximum for the month is called *Spring tide*, and the minimum *Neap tide*. The rising of the tide is called *Flood* and its falling *Ebb* tide.

Similar tides, whether high or low, occur on opposite sides of the earth at once. Thus at the same time it is high tide at any given place, it is also high tide on the inferior meridian, and the same is true of the low tides.

The interval between two successive high tides is 12h. 25m.; or, if the same tide be considered as returning to the meridian, after having gone around the globe, its return is about 50 minutes later than it occurred on the preceding day. In this respect, as well as in various others, it corresponds very nearly to the motions of the moon.

The average height for the whole globe is about 2½ feet; or, if the earth were covered uniformly with a stratum of water, the difference between the two diameters of the oval would be 5 feet, or more exactly 5 feet and 8 inches; but its natural height at various places is very various, sometimes rising to 60 or 70 feet,

* Brinkley's Elements of Astronomy, p. 241.

and sometimes being scarcely perceptible. At the same place also, the phenomena of the tides are very different at different times.

Inland lakes and seas, even those of the largest class, as Lake Superior, or the Caspian, have no perceptible tide.

280. *Tides are caused by the unequal attraction of the sun and moon upon different parts of the earth.*

Suppose the projectile force by which the earth is carried forward in her orbit, to be suspended, and the earth to fall towards one of these bodies, the moon, for example, in consequence of their mutual attraction. Then, if all parts of the earth fell equally towards the moon, no derangement of its different parts would result, any more than of the particles of a drop of water in its descent to the ground. But if one part fell faster than another, the different portions would evidently be separated from each other. Now this is precisely what takes place with respect to the earth in its fall towards the moon. The portions of the earth in the hemisphere next to the moon, on account of being nearer to the center of attraction, fall faster than those in the opposite hemisphere, and consequently leave them behind. The solid earth, on account of its cohesion, cannot obey this impulse, since all its different portions constitute one mass, which is acted on in the same manner as though it were all collected in the center; but the waters on the surface, moving freely under this impulse, endeavor to desert the solid mass and fall towards the

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that of the remoter hemisphere. Now the force of attraction exerted by the moon, acts in the same manner as though the solid mass were all concentrated in C, and the waters of each hemisphere at A and B respectively; and (the moon being supposed above E) it is evident that A will tend to leave C, and C to leave B behind. The same must evidently be true of the respective portions of matter, of which these points are the centers of gravity. The waters of the globe will thus be reduced to an oval shape, being elongated in the direction of that meridian which is under the moon, and flattened in the intermediate parts, and most of all at points ninety degrees distant from that meridian.

Were it not, therefore, for impediments which prevent the force from producing its full effects, we might expect to see the great *tide wave*, as the elevated crest is called, always directly beneath the moon, attending it regularly around the globe. But the inertia of the waters prevents their instantly obeying the moon's attraction, and the friction of the waters on the bottom of the ocean, still farther retards its progress. It is not therefore until several hours (differing at different places) after the moon has passed the meridian of a place, that it is high tide at that place.

282. The *sun* has a similar action to the moon, but only *one third* as great. On account of the great mass of the sun compared with that of the moon, we might suppose that his action in raising the tides would be greater than that of the moon; but the nearness of the moon to the earth more than compensates for the sun's greater quantity of matter. Let us, however, form a just conception of the advantage which the moon derives from her proximity. It is not that her actual amount of attraction is thus rendered greater than that of the sun; but it is that her attraction for the *different parts* of the earth is very unequal, while that of the sun is nearly uniform. It is the *inequality* of this action, and not the absolute force, that produces the tides. The diameter of the earth is $\frac{1}{30}$ of the distance of the moon, while it is less than $\frac{1}{10000}$ of the distance of the sun.

283. Having now learned the general cause of the tides, we will next attend to the explanation of *particular phenomena*.

The *Spring tides*, or those which rise to an unusual height twice a month, are produced by the sun and moon's acting together; and the *Neap tides*, or those which are unusually low twice a month, are produced by the sun and moon's acting in opposition to each other. The Spring tides occur at the syzigies: the Neap tides at the quadratures. At the time of new moon, the sun and moon both being on the same side of the earth, and acting upon it in the same line, their actions conspire, and the sun may be considered as adding so much to the force of the moon. We have already explained how the moon contributes to raise a tide on the opposite side of the earth. But the sun as well as the moon raises its own tide-wave, which, at new moon, coincides with the lunar tide-wave. At full moon, also, the two luminaries conspire in the same way to raise the tide; for we must recollect that each body contributes to raise the tide on the opposite side of the earth as well as on the side nearest to it. At both the conjunctions and oppositions, therefore, that is, at the syzigies, we have unusually high tides. But here also the maximum effect is not at the moment of the syzigies, but 36 hours afterwards.

At the quadratures, the solar wave is lowest when the lunar wave is highest; hence the low tide produced by the sun is subtracted from high water and produces the Neap tides. Moreover, at the quadratures the solar wave is highest when the lunar wave is lowest, and hence is to be added to the height of low water at [redacted]

285. The *declinations of the sun and moon* have a considerable influence on the height of the tide. When the moon for example, has no declination, or is in the equator, as in figure 57,* the rotation of the earth on its axis NS will make the two tides exactly equal on every part of the earth. Thus a place which is carried through the parallel TT′ will have the height of one tide T2 and the other tide T′3. The tides are in this case greatest at the equator, and diminish gradually to the poles, where it will be low water during the whole day. When the moon is on the north side of the equator, as in figure 58, at her greatest northern

Fig. 57.

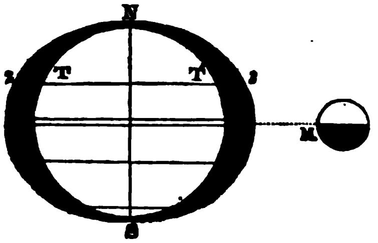
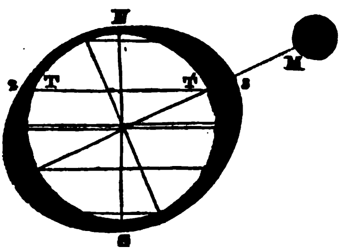


Fig. 58.



declination, a place describing the parallel TT′ will have T′3 for the height of the tide when the moon is on the superior meridian, and T2 for the height when the moon is on the inferior meridian. Therefore, all places north of the equator will have the highest tide when the moon is above the horizon, and the lowest when she is below it; the difference of the tides diminishing towards the equator, where they are equal. In like manner, places south of the equator have the highest tides when the moon is below the horizon, and the lowest when she is above it. When the moon is at her greatest declination, the highest tides will take place towards the tropics. The circumstances are all reversed when the moon is south of the equator.†

* Diagrams like these are apt to mislead the learner, by exhibiting the protuberance occasioned by the tides as much greater than the reality. We must recollect that it amounts, at the highest, to only a very few feet in eight thousand miles. Were the diagram, therefore, drawn in just proportions, the alterations of figure produced by the tides would be wholly insensible.

† Edinb. Encyc. Art. *Astronomy*, p. 623.

286. The motion of the tide-wave, it should be remarked, is not a *progressive* motion, but a mere undulation, and is to be carefully distinguished from the currents to which it gives rise. If the ocean completely covered the earth, the sun and moon being in the equator, the tide-wave would travel at the same rate as the earth on its axis. Indeed, the correct way of conceiving of the tide-wave, is to consider the moon at rest, and the earth in its rotation from west to east as bringing successive portions of water under the moon, which portions being elevated successively at the same rate as the earth revolves on its axis, have a relative motion westward in the same degree.

287. The tides of *rivers, narrow bays, and shores far from the main body of the ocean*, are not produced in those places by the direct action of the sun and moon, but are subordinate waves propagated from the great tide-wave.

Lines drawn through all the adjacent parts of any tract of water, which have high water at the same time, are called *cotidal lines.** We may, for instance, draw a line through all places in the Atlantic Ocean which have high tide in a given day at 1 o'clock, and another through all places which have high tide at 2 o'clock. The cotidal line for any hour may be considered as representing the summit or ridge of the tide-wave at that time; and could the spectator, detached from the earth, perceive the summit of the wave, he would see it travelling round the earth in the open [redacted]

low places, and the cotidal lines will be irregular. The direction also of the derivative tide, may be totally different from that of the primitive. Thus, (Fig. 59,) if the great tide-wave, moving from east to west, be represented by the lines 1, 2, 3, 4, the derivative tide which is propagated up a river or bay, will be represented by the cotidal lines 3, 4, 5, 6, 7. Advancing faster in the channel than next the banks, the tides will lag behind towards the shores, and the cotidal lines will take the form of curves as represented in the diagram.

Fig. 59.

289. On account of the retarding influence of shoals, and an uneven, indented coast, the tide-wave travels more slowly along the shores of an island than in the neighboring sea, assuming convex figures at a little distance from the island and on opposite sides of it. These convex lines sometimes meet and become blended in such a manner as to create singular anomalies in a sea much broken by islands, as well as on coasts indented with numerous bays and rivers.* Peculiar phenomena are also produced, when the tide flows in at opposite extremities of a reef or island, as into the two opposite ends of Long Island Sound. In certain cases a tide-wave is forced into a narrow arm of the sea, and produces very remarkable tides. The tides of the Bay of Fundy (the highest in the world) sometimes rise to the height of 60 or 70 feet; and the tides of the river Severn, near Bristol in England, rise to the height of 40 feet.

290. The *Unit of Altitude* of any place, is the height of the maximum tide after the syzigies, (Art. 283,) being usually about 36 hours after the new or full moon. But as the amount of this tide would be affected by the distance of the sun and moon from

* See an excellent representation and description of these different phenomena by Professor Whewell, Phil. Trans. 1833, p. 153.

the earth, (Art. 284,) and by their declinations, (Art. 285,) these distances are taken at their mean value, and the luminaries are supposed to be in the equator; the observations being so reduced as to conform to these circumstances. The unit of altitude can be ascertained by observation only. The actual rise of the tide depends much on the strength and direction of the wind. When high winds conspire with a high flood tide, as is frequently the case near the equinoxes, the tide often rises to a very unusual height. We subjoin from the American Almanac a few examples of the unit of altitude for different places.

	Feet.
Cumberland, head of the Bay of Fundy,	71
Boston, ...	11½
New Haven, ...	8
New York, ...	5
Charleston, S. C., . . .	6

291. The *Establishment* of any port is the mean interval between noon and the time of high water, on the day of new or full moon. As the interval for any given place is always nearly the same, it becomes a criterion of the retardation of the tides at that place. On account of the importance to navigation of a correct knowledge of the tides, the British Board of Admiralty, at the suggestion of the Royal Society, recently issued orders to their agents in various important naval stations, to have accurate

[redacted]

293. The largest *lakes* and *inland seas* have no perceptible tides. This is asserted by all writers respecting the Caspian and Euxine, and the same is found to be true of the largest of the North American lakes, Lake Superior.*

Although these several tracts of water appear large when taken by themselves, yet they occupy but small portions of the surface of the globe, as will appear evident from the delineation of them on an artificial globe. Now we must recollect that the primitive tides are produced by the *unequal* action of the sun and moon upon the different parts of the earth; and that it is only at points whose distance from each other bears a considerable ratio to the whole distance of the sun or the moon, that the inequality of action becomes manifest. The space required is larger than either of these tracts of water. It is obvious also that they have no opportunity to be subject to a derivative tide.

294. To apply the theory of universal gravitation to all the varying circumstances that influence the tides, becomes a matter of such intricacy, that La Place pronounces "the problem of the tides" the most difficult problem of celestial mechanics.

295. The *Atmosphere* that envelops the earth, must evidently be subject to the action of the same forces as the covering of waters, and hence we might expect a rise and fall of the barometer, indicating an atmospheric tide corresponding to the tide of the ocean. La Place has calculated the amount of this aerial tide. It is too inconsiderable to be detected by changes in the barometer, unless by the most refined observations. Hence it is concluded, that the fluctuations produced by this cause are too slight to affect meteorological phenomena in any appreciable degree.†

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* See Experiments of Gov. Cass, Am. Jour. Science.

† Bowditch's La Place, II, 797.

CHAPTER IX.

OF THE PLANETS—THE INFERIOR PLANETS, MERCURY AND VENUS.

296. THE name planet signifies a *wanderer*,* and is applied to this class of bodies because they shift their positions in the heavens, whereas the fixed stars constantly maintain the same places with respect to each other. The planets known from a high antiquity, are Mercury, Venus, Earth, Mars, Jupiter, and Saturn. To these, in 1781, was added Uranus,† (or *Herschel*, as it is sometimes called from the name of its discoverer,) and, as late as the commencement of the present century, four more were added, namely, Ceres, Pallas, Juno, and Vesta. These bodies are designated by the following characters:

1. Mercury	☿	7. Ceres	⚳
2. Venus	♀	8. Pallas	⚴
3. Earth	⊕	9. Jupiter	♃
4. Mars	♂	10. Saturn	♄
5. Vesta	⚶	11. Uranus	♅
6. Juno	⚵		

The foregoing are called the *primary* planets. Several of these have one or more attendants, or satellites, which revolve around them, as they revolve around the sun. The earth has one satel-

[redacted]

most of the other planets pursue very nearly the same path with the earth, in their annual revolution around the sun. The new planets, however, make wider excursions from the plane of the ecliptic, amounting, in the case of Pallas, to $34\frac{1}{2}°$.

298. Mercury and Venus are called *inferior* planets, because they have their orbits nearer to the sun than that of the earth; while all the others, being more distant from the sun than the earth, are called *superior* planets. The planets present great diversities among themselves in respect to distance from the sun, magnitude, time of revolution, and density. They differ also in regard to satellites, of which, as we have seen, three have respectively four, six, and seven, while more than half have none at all. It will aid the memory, and render our view of the planetary system more clear and comprehensive, if we classify, as far as possible, the various particulars comprehended under the foregoing heads.

299. DISTANCES FROM THE SUN.*

1. Mercury,	37,000,000	0.3870981
2. Venus,	68,000,000	0.7233316
3. Earth,	95,000,000	1.0000000
4. Mars,	142,000,000	1.5236923
5. Vesta,	225,000,000	2.3678700
6. Juno,	} 261,000,000	2.6690090
7. Ceres,		2.7672450
8. Pallas,		2.7728860
9. Jupiter,	485,000,000	5.2027760
10. Saturn,	890,000,000	9.5387861
11. Uranus,	1800,000,000	19.1823900

The *dimensions* of the planetary system are seen from this table to be vast, comprehending a circular space thirty six hundred millions of miles in diameter. A railway car, travelling constantly

* The distance in miles, as expressed in the first column, in round numbers, is to be treasured up in the memory, while the second column expresses the relative distances, that of the earth being 1, from which a more exact determination may be made, when required, the earth's distance being taken at 94,885,491. (Baily.)

at the rate of 20 miles an hour, would require more than 20,000 years to cross the orbit of Uranus.

It may aid the memory to remark, that in regard to the planets nearest the sun, the distances increase in an arithmetical ratio, while those most remote increase in a geometrical ratio. Thus, if we add 30 to the distance of Mercury, it gives us nearly that of Venus; 30 more gives that of the Earth; while Saturn is nearly twice the distance of Jupiter, and Uranus twice the distance of Saturn. Between the orbits of Mars and Jupiter, a great chasm appeared, which broke the continuity of the series; but the discovery of the new planets has filled the void. A more exact law of the series was discovered a few years since by Mr. Bode of Berlin. It is as follows: if we represent the distance of Mercury by 4, and increase each term by the product of 3 into a certain power of 2, we shall obtain the distances of each of the planets in succession. Thus,

Mercury, . . .	4	$= 4$
Venus, . . .	$4+3.2^0$	$= 7$
Earth, . . .	$4+3.2^1$	$= 10$
Mars, . . .	$4+3.2^2$	$= 16$
Ceres, . . .	$4+3.2^3$	$= 28$
Jupiter, . . .	$4+3.2^4$	$= 52$
Saturn, . . .	$4+3.2^5$	$= 100$
Uranus, . . .	$4+3.2^6$	$= 196$

For example, by this law, the distances of the Earth and Jupiter [redacted]

300. MAGNITUDES.

	Diam. in Miles.	Mean apparent Diam.	Volume.
Mercury, . . .	3140	6″.9	$\frac{1}{17}$
Venus, ...	7700	16″.9	$\frac{9}{10}$
Earth, ...	7912		1
Mars, ...	4200	6″.3	$\frac{1}{7}$
Ceres, ...	160	0″.5	
Jupiter, ...	89000	36″.7	1281
Saturn, ...	79000	16″.2	995
Uranus, ...	35000	4″.0	80

We remark here a great diversity in regard to magnitude, a diversity which does not appear to be subject to any definite law. While Venus, an inferior planet, is $\frac{9}{10}$ as large as the earth, Mars, a superior planet is only $\frac{1}{7}$, while Jupiter is 1281 times as large. Although several of the planets, when nearest to us, appear brilliant and large when compared with the fixed stars, yet the angle which they subtend is very small, that of Venus, the greatest of all, never exceeding about 1′, or more exactly 61″.2, and that of Jupiter being when greatest only about $\frac{3}{4}$ of a minute.

The distance of one of the near planets, as Venus or Mars, may be determined from its parallax; and the distance being known, its real diameter can be estimated from its apparent diameter, in the same manner as we estimate the diameter of the sun. (Art. 145.)

301. PERIODIC TIMES.

	Revolution in its orbit.		Mean daily motion.
Mercury,	3 months, or	88 days,	4° 5′ 32″.6
Venus,	7½ " "	224 "	1° 36′ 7″.8
Earth,	1 year, "	365 "	0° 59′ 8″.3
Mars,	2 " "	687 "	0° 31′ 26″.7
Ceres,	4 " "	1681 "	0° 12′ 50″.9
Jupiter,	12 " "	4332 "	0° 4′ 59″.3
Saturn,	29 " "	10759 "	0° 2′ 0″.6
Uranus,	84 " "	30686 "	0° 0′ 42″.4

From this view, it appears that the planets nearest the sun move most rapidly. Thus Mercury performs nearly 350 revolu-

tions while Uranus performs one. This is evidently not owing merely to the greater dimensions of the orbit of Uranus, for the length of its orbit is not 50 times that of the orbit of Mercury, while the time employed in describing it is 350 times that of Mercury. Indeed this ought to follow from Kepler's law that the squares of the periodical times are as the *cubes* of the distances, from which it is manifest that the times of revolution increase faster than the dimensions of the orbit. Accordingly, the apparent progress of the most distant planets is exceedingly slow, the daily rate of Uranus being only 42″.4 per day; so that for weeks and months, and even years, this planet but slightly changes its place among the stars.

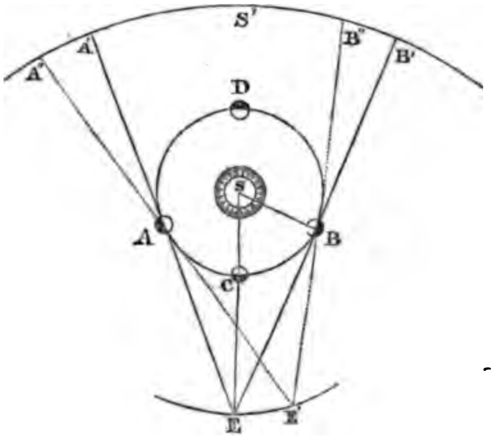
THE INFERIOR PLANETS, MERCURY AND VENUS.

302. The inferior planets, Mercury and Venus, having their orbits so far within that of the earth, appear to us as attendants upon the sun. Mercury never appears farther from the sun than 29° (28° 48′) and seldom so far; and Venus never more than about 47° (47° 12′). Both planets, therefore, appear either in the west soon after sunset, or in the east a little before sunrise. In high latitudes, where the twilight is prolonged, Mercury can seldom be seen with the naked eye, and then only at the periods of its greatest elongation.* The reason of this will readily appear from the following diagram.

Let S (Fig. 60,) represent the sun, ADB the orbit of Mercury,

[redacted]

Fig. 60.



conjunctions, the inferior, and the superior. The *inferior conjunction* is its position when in conjunction on the same side of the sun with the earth, as at C in the figure: the *superior conjunction* is its position when on the side of the sun most distant from the earth, as at D.

304. The period occupied by a planet between two successive conjunctions with the earth, is called its *synodical revolution.* Both the planet and the earth being in motion, the time of the synodical revolution exceeds that of the sidereal revolution of Mercury or Venus; for when the planet comes round to the place where it before overtook the earth, it does not find the earth at that point, but far in advance of it. Thus, let Mercury come into inferior conjunction with the earth at C, (Fig. 60.) In about 88 days, the planet will come round to the same point again; but meanwhile the earth has moved forward through the arc EE′, and will continue to move while the planet is moving more rapidly to overtake her, the case being analogous to that of the hour and second hand of a clock.

Having the *sidereal* period of a planet, (which may always be accurately determined by observation,) we may ascertain its *synodical* period as follows. Let T denote the sidereal period of the earth, and T′ that of the planet. Since, in the time T the

earth describes a complete revolution, $T : T' :: 1 : \frac{T'}{T}$ = the part of the circumference described by the earth in the time T'. But during the same time the planet describes a whole circumference. Therefore, $1 - \frac{T'}{T}$ is what the planet gains on the earth in one revolution. In order to a new conjunction the planet must gain an entire circumference; therefore, denoting the synodical period by S, the gain in one revolution will be to the time in which it is acquired, as a whole circumference is to the time in which that is gained, which is the synodical period. That is,

$$1 - \frac{T'}{T} : T' :: 1 : S = \frac{TT'}{T - T'}.$$

From this formula we may find the synodical period of Mercury or Venus by substituting the numbers denoted by the letters. Thus, $\frac{365.256 \times 87.969}{277.287} = 115.877$, which is the synodical period of Mercury.

By a similar computation, the synodical revolution of Venus will be found to be about 584 days.

305. *The motion of an inferior planet is direct in passing through its superior conjunction, and retrograde in passing through its inferior conjunction.* Thus Venus, while going from B through D to A, (Fig. 60,) moves in the order of the signs, or [redacted]

at A′ but at A″, being accelerated by the arc A′A″ in consequence of the earth's motion. On the other hand, when the planet is passing through its inferior conjunction ACB, it appears to move backwards in the heavens from A′ to B′ if the earth is at rest, but from A′ to B″ if the earth has in the mean time moved from E to E′, being retarded by the arc B′B″. Although the motions of the earth have the effect to accelerate the planet in the superior conjunction, and to retard it in the inferior, yet, on account of the greater distance, the apparent motion of the planet is much slower in the superior than in the inferior conjunction.

306. *When passing from the superior to the inferior conjunction, or from the inferior to the superior conjunction, through the greatest elongations, the inferior planets are stationary.*

If the earth were at rest, the stationary points would be at the greatest elongations as at A and B, for then the planet would be moving directly towards or from the earth, and would be seen for some time in the same place in the heavens; but the earth itself is moving nearly at right angles to the line of the planet's motion, that is, the line which is drawn from the earth to the planet through the point of greatest elongation; hence a direct motion is given to the planet by this cause. When the planet, however, has passed this line, by its superior velocity it soon overcomes this tendency of the earth to give it a relative motion eastward, and becomes retrograde as it approaches the inferior conjunction. Its stationary point obviously lies between its place of greatest elongation, and the place where its motion becomes retrograde. Mercury is stationary at an elongation of from 15° to 20° from the sun; and Venus at about 29°.*

307. *Mercury and Venus exhibit to the telescope phases similar to those of the moon.*

When on the side of their inferior conjunctions, these planets appear horned, like the moon in her first and last quarters; and when on the side of their superior conjunctions they appear gibbous. At the moment of superior conjunction, the whole enlightened orb of the planet is turned towards the earth, and the

* Herschel, p. 242.—Woodhouse, 557.

appearance would be that of the full moon, but the planet is too near the sun to be commonly visible.

These different phases show that these bodies are opake, and shine only as they reflect to us the light of the sun; and the same remark applies to all the planets.

308. *The distance of an inferior planet from the sun, may be found by observations at the time of its greatest elongation.*

Thus if E be the place of the earth, and B that of Venus at the time of her greatest elongation, the angle SBE will be known, being a right angle. Also the angle SEB is known from observation. Hence the *ratio* of SB to SE becomes known; or, since SE is given, being the distance of the earth from the sun, SB the radius of the orbit of the planet is determined. If the orbits were both circles, this method would be very exact; but being elliptical, we obtain the mean value of the radius SB by observing its greatest elongation in different parts of its orbit.*

309. *The orbit of Mercury is the most eccentric, and the most inclined of all the planets;*† *while that of Venus varies but little from a circle, and lies much nearer to the ecliptic.*

The eccentricity of the orbit of Mercury is nearly $\frac{1}{5}$ its semi-major axis, while that of Venus is only $\frac{1}{143}$; the inclination of Mercury's orbit is 7°, while that of Venus is less than $3\frac{1}{2}$°‡. Mercury, on account of his different distances from the earth, varies

311. ***An inferior planet is brightest at a certain point between its greatest elongation and inferior conjunction.***

Its maximum brilliancy would happen at the inferior conjunction, (being then nearest to us,) if it shined by its own light; but in that position, its dark side is turned towards us. Still, its maximum cannot be when most of the illuminated side is towards us; for then, being at the superior conjunction, it is at its greatest distance from us. The maximum must therefore be somewhere between the two. Venus gives her greatest light when about 40° from the sun.

312. *Mercury and Venus both revolve on their axes, in nearly the same time with the earth.*

The diurnal period of Mercury is 24h. 5m. 28s., and that of Venus 23h. 21m. 7s. The revolutions on their axes have been determined by means of some spot or mark seen by the telescope, as the revolution of the sun on his axis is ascertained by means of his spots.

313. Venus is regarded as the most beautiful of the planets, and is well known as the *morning and evening star*. The most ancient nations did not indeed recognize the evening and morning star as one and the same body, but supposed they were different planets, and accordingly gave them different names, calling the morning star Lucifer, and the evening star Hesperus. At her period of greatest splendor, Venus casts a shadow, and is sometimes visible in broad daylight. Her light is then estimated as equal to that of twenty stars of the first magnitude.* At her period of greatest elongation, Venus is visible from three to four hours after the setting or before the rising of the sun.

314. *Every eight years, Venus forms her conjunctions with the sun in the same part of the heavens.*

For, since the synodical period of Venus is 584 days, and her sidereal period 224.7,

224.7 : 360° : : 584 : 935.6 = the arc of longitude described by Venus between the first and second conjunctions. Deducting

* Francœur, Uranography, p. 125.

720°, or two entire circumferences, the remainder, 215°.6, shows how far the place of the second conjunction is in advance of the first. Hence, in five synodical revolutions, or 2920 days, the same point must have advanced $215°.6 \times 5 = 1078°$, which is nearly three entire circumferences, so that at the end of five synodical revolutions, occupying 2920 days, or 8 years, the conjunction of Venus takes place nearly in the same place in the heavens as at first.

Whatever appearances of this planet, therefore, arise from its positions with respect to the earth and the sun, they are repeated every eight years in nearly the same form.

TRANSITS OF THE INFERIOR PLANETS.

315. *The Transit of Mercury or Venus, is its passage across the sun's disk, as the moon passes over it in a solar eclipse.*

As a transit takes place only when the planet is in inferior conjunction, at which time her motion is retrograde (Art. 305,) it is always from left to right, and the planet is seen projected on the solar disk in a black round spot. Were the orbits of the inferior planets coincident with the plane of the earth's orbit a transit would occur to some part of the earth at every inferior conjunction. But the orbit of Venus makes an angle of $3\frac{1}{2}°$ with the ecliptic, and Mercury an angle of 7°; and, moreover, the apparent diameter of each of these bodies is very small, both

period (Art. 304,) is $S=\frac{T\times T'}{T-T'}$, where S denotes the period, T the sidereal revolution of the earth, and T′ that of the planet. If we now represent by *m* the number of synodical revolutions of the earth in the required period, and by *n* the number of revolutions of the planet in the same time, then, since the number of revolutions in each case is inversely as the time of one, we have, $T : \frac{T\times T'}{T-T} :: n : m \therefore mT=\frac{nTT'}{T-T} \therefore \frac{m}{n}=\frac{T'}{T-T'}$. But *m* is the number of revolutions which the earth must perform in order that the two bodies, after having once met at the planet's nodes, may meet again at the same place. In the case of Mercury, whose sidereal period is 87.969 days, while that of the earth is 365.256 days, $\frac{m}{n}=\frac{87969}{277287}$; that is, after the earth has revolved 87969 times, (or after this number of years,) Mercury will have revolved just 277287, and the two bodies will be together again at the place where they started. But as periods of such enormous length do not fall within the observation of man, let us search for smaller numbers having nearly the same ratio. Now,

$$87969 : 365256 :: 1 : 4\tfrac{1}{6} \text{ (nearly.)}$$

This shows that in one year Mercury will have made 4 revolutions and $\frac{1}{6}$ of another; so that, when the sun returns to the same node, Mercury will be more than 60° in advance of it; consequently, no transit can take place after an interval of one year. But, by making trial of 2, 3, 4, &c. years, we shall find a nearer approximation at the end of 6 years; for,

$87969 : 365265 :: 6 : 25-\frac{1}{11}$. In 6 years, therefore, Mercury will fall short of reaching the node by only $\frac{1}{11}$ of a revolution, or about 33°. In 13 years the chance of meeting will be much greater, for in this period the earth will have made 13 and Mercury 54 revolutions. The numbers 33 and 137, 46 and 191, afford a still nearer approximation.*

317. In a similar manner, transits of Venus are probable after 8, 227, 235, and 243 years. Since Venus returns to her conjunction at nearly the same point of her orbit, after 8 years, (Art. 314,)

* This series may readily be obtained by the method of *Continued Fractions*. See Davies's Bourdon's Algebra.

it frequently happens that a transit takes place after an interval of 8 years. But at that time Venus is so far from her node, that her latitude amounts to from 20′ to 24′. Still she may possibly come within the sun's disk as she passes by him; for suppose at the preceding transit her latitude was 10′ on one side of the node and is now 10′ on the other side, this being less than the sun's semi-diameter, a transit may occur 8 years after another. Thus transits of Venus took place in 1761 and 1769. But in 16 years the latitude changes from 40′ to 48′, and Venus could not reach any part of the solar disk in her inferior conjunction.

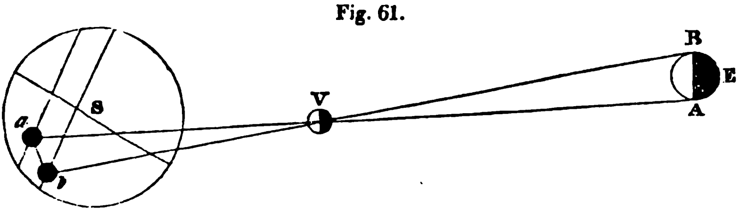
From the above series we should infer that another transit could not take place under 227 years; but since there are two nodes, the chance is doubled, so that a transit may occur at the other node in half that interval, or in about 113 years. If, at the occurrence of the first transit, Venus had passed her node, the next transit at the other node will happen 8 years before the 113 are completed; or if she had not reached the node, it will happen 8 years later. Hence, after two transits have occurred within 8 years, another cannot be expected before 105, 113, or 121 years. Thus, the next transit will happen in 1874=1769+105; also in 1782=1874+8.

318. The great interest attached by astronomers to a transit of Venus, arises from its furnishing the most accurate means in our power of determining the *sun's horizontal parallax*,—an element [redacted]

319. If the sun and Venus were equally distant from us, they would be equally affected by parallax as viewed by spectators in different parts of the earth, and hence their *relative* situation would not be altered by it; but since Venus, at the inferior conjunction, is only about one third as far off as the sun, her parallax is proportionally greater, and therefore spectators at distant points will see Venus projected on different parts of the solar disk, and as the planet traverses the disk, she will appear to describe chords of different lengths, by means of which the duration of the transit may be estimated at different places. The difference in the duration of the transit does not amount to many minutes; but to make it as large as possible very distant places are selected for observation. Thus in the transit of 1769, among the places selected, two of the most favorable were Wardhuz in Lapland, and Oteheite, one of the South Sea Islands.

The principle on which the sun's horizontal parallax is estimated from the transit of Venus, may be illustrated as follows: Let E (Fig. 61,) be the earth, V Venus, and S the sun. Suppose A, B, two spectators at opposite extremities of that diameter of the earth which is perpendicular to the ecliptic. The spectator at A will see Venus on the sun's disk at *a*, and the spectator at

Fig. 61.



B will see Venus at *b*; and since AV and BV may be considered as equal to each other, as also V*b* and V*a*, therefore the triangles AVB and V*ab* are similar to each other, and AV : V*a*: : AB : *ab*. But the ratio of AV to V*a* is known, (Art. 308); hence, the ratio of AB to *ab* is known, and when the angular value of *ab* as seen from the earth, is found, that of AB becomes known, as seen from the sun;* and half AB, or the semi-diameter of the earth as seen

* If, for example, *ab* is $2\frac{1}{2}$ times AB, (which is nearly the fact,) then if AB were on the sun instead of on the earth, it would subtend an angle at the eye equal to $\frac{1}{2.5}$ of *ab*. But if viewed from the sun, the distance being the same, its apparent diameter must be the same.

from the sun, is the sun's horizontal parallax. To find the apparent diameter of *ab*, we have only to find the breadth of the space between the two chords. Now, we can ascertain the value of each chord by the time occupied in describing it, since the motions of Venus and those of the sun are accurately known from the tables. Each chord being double the sine of half the arc cut off by it, therefore the sine of half the arc and of course the versed sine becomes known, and the difference of the two versed sines is the breadth of the zone in question. There are many circumstances to be taken into the account in estimating, from observations of this kind, the sun's horizontal parallax; but the foregoing explanation may be sufficient to give the learner an idea of the *general principles* of this method. The appearance of Venus on the sun's disk, being that of a well defined black spot, and the exactness with which the moment of external or internal contact may be determined, are circumstances favorable to the exactness of the result; and astronomers repose so much confidence in the estimation of the sun's horizontal parallax as derived from the observations on the transit of 1769, that this important element is thought to be ascertained within $\frac{1}{10}$ of a second. The general result of all these observations give the sun's horizontal parallax 8.″6, or more exactly, 8.″5776.*

320. During the transits of Venus over the sun's disk in 1761 and 1769, a sort of penumbral light was observed around the [redacted]

mosphere. Similar proofs of the existence of an atmosphere around this planet, are derived from appearances of twilight.

The elder astronomers imagined they had discovered a *satellite* accompanying Venus in her transit. If Venus had in reality any satellite, the fact would be obvious at her transits, as the satellite would be projected near the primary on the sun's disk; but later astronomers have searched in vain for any appearances of the kind, and the inference is that former astronomers were deceived by some optical illusion.

Astronomers have detected very high *mountains* on Venus, sometimes reaching to the elevation of 22 miles; and it is remarkable that the highest mountains in Venus, in Mercury, in the moon, and in the earth, are always in the southern hemisphere.

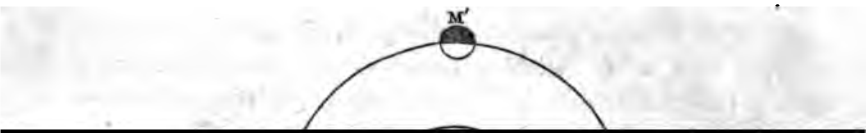
CHAPTER X.

OF THE SUPERIOR PLANETS—MARS, JUPITER, SATURN, AND URANUS.

321. The Superior planets are distinguished from the Inferior, by being seen at all distances from the sun from 0° to 180°. Having their orbits exterior to that of the earth, they of course never come between us and the sun, that is, they never have any inferior conjunction like Mercury and Venus, but they are sometimes seen in superior conjunction, and sometimes in opposition. Nor do they, like the inferior planets, exhibit to the telescope different phases, but, with a single exception, they always present the side that is turned towards the earth fully enlightened. This is owing to their great distance from the earth; for were the spectator to stand upon the sun he would of course always have the illuminated side of each of the planets turned towards him; but, so distant are all the superior planets except Mars, that they are viewed by us very nearly in the same manner as they would be if we actually stood on the sun.

322. MARS is a small planet, his diameter being only about half that of the earth, or 4100 miles. He also, at times, comes nearer to the earth than any other planet except Venus. His *mean* distance from the sun is 142,000,000 miles; but his orbit is so eccentric that his distance varies much in different parts of his revolution. Mars is always very near the ecliptic, never varying from it 2°. He is distinguished from all the planets by his deep red color, and fiery aspect; but his brightness and apparent magnitude vary much at different times, being sometimes nearer to us than at others, by the whole diameter of the earth's orbit, that is, by about 190,000,000 of miles. When Mars is on the same side of the sun with the earth, or at his opposition, he comes within 47,000,000 miles of the earth, and rising about the time the sun sets surprises us by his magnitude and splendor; but when he passes to the other side of the sun to his superior conjunction, he dwindles to the appearance of a small star, being then 237,000,000 miles from us. Thus, let M (Fig. 62,) represent Mars in opposition, and M′ in the superior conjunction. It is obvious that in the former situation, the planet must be nearer to the earth than in the latter by the whole diameter of the earth's orbit.

Fig. 62.



[redacted]

tion is turned towards the earth, such a portion of the remoter part of it being concealed from our view as to render the form more or less gibbous.

324. When viewed with a powerful telescope, the surface of Mars appears diversified with numerous varieties of light and shade. The region around the poles is marked by white spots, which vary their appearance with the changes of seasons in the planet. Hence Dr. Herschel conjectured that they were owing to ice and snow, which alternately accumulates and melts, according to the position of each pole with respect to the sun.* It has been common to ascribe the ruddy light of this planet to an extensive and dense atmosphere, which was said to be distinctly indicated, by the gradual diminution of light observed in a star as it approached very near to the planet in undergoing an occultation; but more recent observations afford no such evidence of an atmosphere.†

By observations on the spots we learn that Mars revolves on his axis in very nearly the same time with the earth, (24h. 39m. 21s.3); and that the inclination of his axis to that of the ecliptic is also nearly the same, being 30° 18′ 10″.8.

325. As the diurnal rotation of Mars is nearly the same as that of the earth, we might expect a similar flattening at the poles, giving to the planet a spheroidal figure. Indeed the compression or ellipticity of Mars greatly exceeds that of the earth, being no less than $\frac{1}{16}$ of the equatorial diameter, while that of the earth is only $\frac{1}{300}$, (Art. 138.) This remarkable flattening of the poles of Mars has been supposed to arise from a great variation of density in the planet in different parts.‡

326. JUPITER is distinguished from all the other planets by his vast *magnitude*. His diameter is 86,000 miles, and his volume 1280 times that of the earth. His figure is strikingly spheroidal, the equatorial being larger than the polar diameter in the proportion of 107 to 100. (See Frontispiece, Fig. 4.) Such a figure might

* Phil. Trans. 1784. † Sir James South, Phil. Trans. 1833.

‡ Ed. Encyc. Art. *Astronomy*.

naturally be expected from the rapidity of his diurnal rotation, which is accomplished in about 10 hours. A place on the equator of Jupiter must turn 27 times as fast as on the terrestrial equator. The distance of Jupiter from the sun is nearly 490,000,000 miles, and his revolution around the sun occupies nearly 12 years.

327. The view of Jupiter through a good telescope, is one of the most magnificent and interesting spectacles in astronomy. The disk expands into a large and bright orb like the full moon; the spheroidal figure which theory assigns to revolving spheres, is here palpably exhibited to the eye; across the disk, arranged in parallel stripes, are discerned several dusky bands, called *belts;* and four bright satellites, always in attendance, but ever varying their positions, compose a splendid retinue. Indeed, astronomers gaze with peculiar interest on Jupiter and his moons as affording a miniature representation of the whole solar system, repeating on a smaller scale, the same revolutions, and exemplifying, in a manner more within the compass of our observation, the same laws as regulate the entire assemblage of sun and planets. (See Fig. 63.)

328. The *Belts of Jupiter*, are variable in their number and dimensions. With the smaller telescopes, only one or two are seen across the equatorial regions; but with more powerful instruments, the number is increased, covering a great part of the whole disk. Different opinions have been entertained by astronomers respecting the cause of the belts; but they have generally

[redacted]

us to discern them. Indeed one or two of them have been occasionally seen with the naked eye. In the largest telescopes, they severally appear as bright as Sirius. With such an instrument the view of Jupiter with his moons and belts is truly a magnificent spectacle, a world within itself. As the orbits of the satellites do not deviate far from the plane of the ecliptic, and but little from the equator of the planet, they are usually seen in nearly a straight line with each other extending across the central part of the disk. (See Frontispiece.)

330. Jupiter's satellites are distinguished from one another by the denominations of *first*, *second*, *third*, and *fourth*, according to their relative distances from Jupiter, the first being that which is nearest to him. Their apparent motion is oscillatory, like that of a pendulum, going alternately from their greatest elongation on one side to their greatest elongation on the other, sometimes in a straight line, and sometimes in an elliptical curve, according to the different points of view in which we observe them from the earth. They are sometimes stationary; their motion is alternately direct and retrograde; and, in short, they exhibit in miniature all the phenomena of the planetary system. Various particulars of the system are exhibited in the following table. The distances are given in radii of the primary.

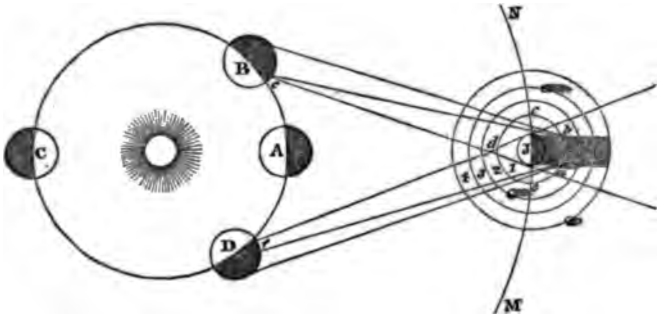
Satellite.	Diameter.	Mean Distance.	Sidereal Revolution.
1	2508	6.04853	1d. 18h. 28m.
2	2068	9.62347	3 13 14
3	3377	15.35024	7 3 43
4	2890	26.99835	16 16 32

Hence it appears, first, that Jupiter's satellites are all somewhat larger than the moon, but that the second satellite is the smallest, and the third the largest of the whole, but the diameter of the latter is only about $\frac{1}{26}$ part of that of the primary; secondly, that the distance of the innermost satellite from the planet is three times his diameter, while that of the outermost satellite is nearly fourteen times his diameter; thirdly, that the first satellite completes its revolution around the primary in one day and three fourths, while the fourth satellite requires nearly sixteen and three fourths days.

331. The orbits of the satellites are nearly or quite circular, and deviate but little from the plane of the planet's equator, and of course are but slightly inclined to the plane of his orbit. They are, therefore, in a similar situation with respect to Jupiter as the moon would be with respect to the earth if her orbit nearly coincided with the ecliptic, in which case she would undergo an eclipse at every opposition.

332. The eclipses of Jupiter's satellites, in their general conception, are perfectly analogous to those of the moon, but in their detail they differ in several particulars. Owing to the much greater distance of Jupiter from the sun, and its greater magnitude, the cone of its shadow is much longer and larger than that of the earth, (Art. 246.) On this account, as well as on account of the little inclination of their orbits to that of their primary, the three inner satellites of Jupiter pass through the shadow, and are totally eclipsed at every revolution. The fourth satellite, owing to the greater inclination of its orbit, sometimes though rarely escapes eclipse, and sometimes merely grazes the limits of the shadow or suffers a partial eclipse.* These eclipses, moreover, are not seen, as is the case with those of the moon, from the center of their motion, but from a remote station, and one whose situation with respect to the line of the shadow is variable. This, of course, makes no difference in the *times* of the eclipses, but a very great one in their visibility, and in their apparent situations [redacted]

Fig. 63.



C and A, that is, while the earth is passing from the position where it has the planet in superior conjunction, to that where it has the planet in opposition; for while the earth is in this situation, the planet conceals from its view the *emersion*, as is evident from the direction of the visual ray *fd*. For a similar reason the emersion only is visible while the earth passes from A to C, or from the opposition to the superior conjunction. In other words, when the earth is to the *westward* of Jupiter, only the immersions of a satellite are visible; when the earth is to the *eastward* of Jupiter, only the emersions are visible. This, however, is strictly true only of the first satellite; for the third and fourth, and sometimes even the second, owing to their greater distances from Jupiter, occasionally disappear and reappear on the same side of the disk. The reason why they should reappear on the *same* side of the disk, will be understood from the figure. Conceive the whole system of Jupiter and his satellites as projected on the more distant concave sphere, by lines drawn, like *fd*, from the observer on the earth through the planet and each of the satellites; then it is evident that the remoter parts of the shadow where the interior satellites traverse it, will fall to the westward of the planet, and of course these satellites as they emerge from the shadow will be projected to a point on the same side of the disk as the point of their emersion. The same mode of reasoning will show that when the earth is to the eastward of the planet, the immersions and emersions of the outermost satellites will be both seen on the east side of the disk. When the earth is in either of the positions C or A, that is, at the superior conjunction or opposition of the planet, both the immersions and emersions take place behind the planet, and the eclipses occur close to the disk.

334. When one of the satellites is passing between Jupiter and the sun, it casts a shadow upon its primary, which is seen by the telescope travelling across the disk of Jupiter, as the shadow of the moon would be seen to traverse the earth by a spectator favorably situated in space. When the earth is to the westward of Jupiter, as at D, the shadow reaches the disk of the planet, or is seen on the disk, before the satellite itself reaches it. For the satellite will not enter on the disk, until it comes up to the line *fd* at *d*, a point which it reaches later than its shadow reaches the same line. After the earth has passed the opposition, as at B, then the satellite will reach the visual ray *cd* at *d* sooner than the shadow, and of course be sooner projected on the disk. In the transits of Jupiter's satellites, which with very powerful telescopes may be observed with great precision, the satellite itself is sometimes seen on the disk as a *bright* spot, if it chances to be projected upon one of the belts. Occasionally, also, it is seen as a *dark* spot, of smaller dimensions than the shadow. This curious fact has led to the conclusion, that certain of the satellites have sometimes on their own bodies or in their atmospheres, obscure spots of great extent.*

335. A very singular relation subsists between the mean motions of the three first satellites of Jupiter. If the mean angular velocity of the first satellite be added to twice that of the third, the sum will be equal to three times that of the second. Hence,

nomena are such as would present themselves to a spectator on Jupiter, and not to a spectator on the earth.

336. The eclipses of Jupiter's satellites have been studied with great attention by astronomers, on account of their affording one of the easiest methods of determining the *longitude*. On this subject Sir J. Herschel remarks:* The discovery of Jupiter's satellites by Galileo, which was one of the first fruits of the invention of the telescope, forms one of the most memorable epochs in the history of astronomy. The first astronomical solution of the great problem of "the longitude,"—the most important problem for the interests of mankind that has ever been brought under the dominion of strict scientific principles, dates immediately from their discovery. The final and conclusive establishment of the Copernican system of astronomy, may also be considered as referable to the discovery and study of this exquisite miniature system, in which the laws of the planetary motions, as ascertained by Kepler, and especially that which connects their periods and distances, were speedily traced, and found to be satisfactorily maintained.

337. The entrance of one of Jupiter's satellites into the shadow of the primary being seen like the entrance of the moon into the earth's shadow, at the same moment of absolute time, at all places where the planet is visible, and being wholly independent of parallax; being, moreover, predicted beforehand with great accuracy for the instant of its occurrence at Greenwich, and given in the Nautical Almanac, this would seem to be one of those events (Art. 273,) which are peculiarly adapted for finding the longitude. It must be remarked, however, that the extinction of light in the satellite at its immersion, and the recovery of its light at its emersion, are not instantaneous but gradual; for the satellite, like the moon, occupies some time in entering into the shadow or in emerging from it, which occasions a progressive diminution or increase of light. The better the light afforded by the telescope with which the observation is made, the later the satellite will be seen at its immersion, and the sooner

* Elements of Ast. p. 279.

at its emersion.* In noting the eclipses even of the first satellite, the time must be considered as uncertain to the amount of 20 or 30 seconds; and those of the other satellites involve still greater uncertainty. Two observers, in the same room, observing with different telescopes the same eclipse, will frequently disagree in noting its time to the amount of 15 or 20 seconds; and the difference will be always the same way.†

Better methods, therefore, of finding the longitude are now employed, although the facility with which the necessary observations can be made, and the little calculation required, still render this method eligible in many cases where extreme accuracy is not important. As a telescope is essential for observing an eclipse of one of the satellites, it is obvious that this method cannot be practiced at sea.

338. The grand discovery of the *progressive motion of light*, was first made by observations on the eclipses of Jupiter's satellites. In the year 1675, it was remarked by Roemer, a Danish astronomer, on comparing together observations of these eclipses during many successive years, that they take place sooner by about sixteen minutes (16m. 26s.6) when the earth is on the same side of the sun with the planet, than when she is on the opposite side. This difference he ascribed to the progressive motion of light, which takes that time to pass through the diameter of the earth's orbit, making the velocity of light about 192,000 [redacted]

attended by seven satellites. But a still more wonderful appendage is its *Ring*, a broad wheel encompassing the planet at a great distance from it. We have already intimated that Saturn's system is on a grand scale. As, however, Saturn is distant from us nearly 900,000,000 miles, we are unable to obtain the same clear and striking views of his phenomena as we do of the phenomena of Jupiter, although they really present a more wonderful mechanism.

340. Saturn's ring, when viewed with telescopes of a high power, is found to consist of two concentric rings,* separated from each other by a dark space. (See Frontispiece.) Although this division of the rings appears to us, on account of our immense distance, as only a fine line, yet it is in reality an interval of not less than about 1800 miles. The dimensions of the whole system are in round numbers, as follows :†

	Miles.
Diameter of the planet, ...	79,000
From the surface of the planet to the inner ring,	20,000
Breadth of the inner ring, ...	17,000
Interval between the rings, ...	1,800
Breadth of the outer ring, ...	10,500
Extreme dimensions from outside to outside,	176,000

The figure represents Saturn as it appears to a powerful telescope, surrounded by its rings, and having its body striped with dark belts, somewhat similar but broader and less strongly marked than those of Jupiter, and owing doubtless to a similar cause. That the ring is a solid opake substance, is shown by its throwing its shadow on the body of the planet on the side nearest the sun and on the other side receiving that of the body. From the parallelism of the belts with the plane of the ring, it may be conjectured that the axis of rotation of the planet is perpendicular to that plane; and this conjecture is confirmed by the occasional appearance of extensive dusky spots on its surface, which when watched indicate a rotation parallel to the ring in 10h. 29m. 17s. This motion, it will be remarked, is nearly the

* It is said that several additional divisions of the ring have been detected.—(Kater, Ast. Trans. IV. 383.)

† Prof. Struve, Mem. Ast. Soc., 3. 301.

same with the diurnal motion of Jupiter, subjecting places on the equator of the planet to a very swift revolution, and occasioning a high degree of compression at the poles, the equatorial being to the polar diameter in the high ratio of 11 to 10. But it is remarkable that the globe of Saturn appears to be flattened at the equator as well as at the poles. The polar compression extends to a great distance over the surface of the planet, and the greatest diameter is that of the parallel of 43° of latitude. The disk of Saturn, therefore, resembles a square of which the four corners have been rounded off.* It requires a telescope of high magnifying powers and a strong light to give a full and striking view of Saturn with his rings and belts and satellites; for we must bear in mind that at the distance of Saturn one second of angular measurement corresponds to 4,000 miles, a space equal to the semi-diameter of our globe. But with a telescope of moderate powers, the leading phenomena of the ring itself may be observed.

341. *Saturn's ring, in its revolution around the sun, always remains parallel to itself.*

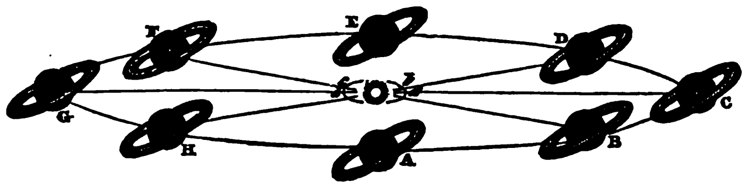
If we hold opposite to the eye a circular ring or disk like a piece of coin, it will appear as a complete circle when it is at right angles to the axis of vision, but when oblique to that axis it will be projected into an ellipse more and more flattened as its obliquity is increased, until, when its plane coincides with the axis of vision, it is projected into a straight line. Let us place on the table a lamp to represent the sun, and holding the ring at a cer-

in respect to so distant a body as Saturn, very near the sun, those appearances are presented to us in nearly the same manner as though we viewed them from the sun. Accordingly, we sometimes see Saturn's ring under the form of a broad ellipse, which grows continually more and more acute until it passes into a line, and we either lose sight of it altogether, or with the aid of the most powerful telescopes, we see it as a fine thread of light drawn across the disk and projecting out from it on each side. As the whole revolution occupies 30 years, and the edge is presented to the sun twice in the revolution, this last phenomenon, namely, the disappearance of the ring, takes place every 15 years.

342. The learner may perhaps gain a clearer idea of the foregoing appearances from the following diagram:

Let A, B, C, &c. represent successive positions of Saturn and his ring in different parts of his orbit, while *abc* represents the orbit of the earth.* Were the ring when at C and G perpendicular to the line CG, it would be seen by a spectator situated at *a* or *d* a perfect circle, but being inclined to the line of vision 28° 4′, it is projected into an ellipse. This ellipse contracts in breadth

Fig. 64.



as the ring passes towards its nodes at A and E, where it dwindles into a straight line. From E to G the ring opens again, becomes broadest at G, and again contracts till it becomes a straight line at A, and from this point expands till it recovers its original breadth at C. These successive appearances are all exhibited to a telescope of moderate powers. The ring is extremely *thin*, since the smallest satellite, when projected on it, more than covers it. The thickness is estimated at 100 miles.

* It may be remarked by the learner, that these orbits are made so elliptical, not to represent the eccentricity of either the earth's or Saturn's orbit, but merely as the projection of circles seen very obliquely.

343. *Saturn's ring shines wholly by reflected light derived from the sun.* This is evident from the fact, that that side only which is turned towards the sun is enlightened; and it is remarkable, that the illumination of the ring is greater than that of the planet itself, but the outer ring is less bright than the inner. Although, as we have already remarked, we view Saturn's ring nearly as though we saw it from the sun, yet the plane of the ring produced may pass between the earth and the sun, in which case also the ring becomes invisible, the illuminated side being wholly turned from us. Thus, when the ring is approaching its node at E, a spectator at *b* would have the dark side of the ring presented to him. The ring was invisible in 1833, and will be invisible again in 1847. At present (1839) it is the northern side of the ring that is seen, but in 1855 the southern side will come into view.

It appears, therefore, that there are three causes for the disappearance of Saturn's ring; first, when the edge of the ring is presented to the sun; secondly, when the edge is presented to the earth; and thirdly, when the unilluminated side is towards the earth.

344. *Saturn's ring revolves in its own plane in about* 10½ *hours*, (10h. 32m. $15^s.4$). La Place inferred this from the doctrine of universal gravitation. He proved that such a rotation was necessary, otherwise the matter of which the ring is composed would be precipitated upon its primary. He showed that in order [redacted]

minute orbit. This fact, unimportant as it may seem, is of the utmost consequence to the stability of the system of rings. Supposing them mathematically perfect in their circular form, and exactly concentric with the planet, it is demonstrable that they would form (in spite of their centrifugal force) a system in a state of *unstable equilibrium*, which the slightest external power would subvert—not by causing a rupture in the substance of the rings—but by precipitating them *unbroken* on the surface of the planet.* The ring may be supposed of an unequal breadth in its different parts, and as consisting of irregular solids, whose common center of gravity does not coincide with the center of the figure. Were it not for this distribution of matter, its equilibrium would be destroyed by the slightest force, such as the attraction of a satellite, and the ring would finally precipitate itself upon the planet.†

As the smallest difference of velocity between the planet and its rings must infallibly precipitate the rings upon the planet, never more to separate, it follows either that their motions in their common orbit round the sun, must have been adjusted to each other by an external power, with the minutest precision, or that the rings must have been formed about the planet while subject to their common orbitual motion, and under the full and free influence of all the acting forces.

The rings of Saturn must present a magnificent spectacle from those regions of the planet which lie on their enlightened sides, appearing as vast arches spanning the sky from horizon to horizon, and holding an invariable situation among the stars. On the other hand, in the region beneath the dark side, a solar eclipse of 15 years in duration, under their shadow, must afford (to our ideas) an inhospitable abode to animated beings, but ill compensated by the full light of its satellites. But we shall do wrong to judge of the fitness or unfitness of their condition from what we see around us, when, perhaps, the very combinations which convey to our minds only images of horror, may be in reality theatres of the most striking and glorious displays of beneficent contrivance.‡

* Sir J. Herschel. † La Place. ‡ Sir J. Herschel.

346. Saturn is attended by *seven satellites.* Although bodies of considerable size, their great distance prevents their being visible to any telescopes but such as afford a strong light and high magnifying powers. The outermost satellite is distant from the planet more than 30 times the planet's diameter, and is by far the largest of the whole. It is the only one of the series whose theory has been investigated further than suffices to verify Kepler's law of the periodic times, which is found to hold good here as well as in the system of Jupiter. It exhibits, like the satellites of Jupiter, periodic variations of light, which prove its revolution on its axis in the time of a sidereal revolution about Saturn. The next satellite in order, proceeding inwards, is tolerably conspicuous; the three next are very minute, and require pretty powerful telescopes to see them; while the two interior satellites, which just skirt the edge of the ring, and move exactly in its plane, have never been discovered but with the most powerful telescopes which human art has yet constructed, and then only under peculiar circumstances. At the time of the disappearance of the rings (to ordinary telescopes) they were seen by Sir William Herschel with his great telescope, projected along the edge of the ring, and threading like beads the thin fibre of light to which the ring is then reduced. Owing to the obliquity of the ring, and of the orbits of the satellites to that of their primary, there are no eclipses of the satellites, the two interior ones excepted, until near the time when the ring is seen edgewise.*

[redacted]

The sun himself when seen from Uranus dwindles almost to a star, subtending as it does an angle of only 1′ 40″; so that the surface of the sun would appear there 400 times less than it does to us.

This planet was discovered by Sir William Herschel on the 13th of March, 1781. His attention was attracted to it by the largeness of its disk in the telescope; and finding that it shifted its place among the stars, he at first took it for a comet, but soon perceived that its orbit was not eccentric like the orbits of comets, but nearly circular like those of the planets. It was then recognized as a new member of the planetary system, a conclusion which has been justified by all succeeding observations.

348. Uranus is attended by *six satellites.* So minute objects are they that they can be seen only by powerful telescopes. Indeed the existence of more than two is still considered as somewhat doubtful. These two, however, offer remarkable, and indeed quite unexpected and unexampled peculiarities. Contrary to the unbroken analogy of the whole planetary system, *the planes of their orbits are nearly perpendicular to the ecliptic,* being inclined no less than 78° 58′ to that plane, and in these orbits *their motions are retrograde;* that is, instead of advancing from west to east around their primary, as is the case with all the other planets and satellites, they move in the opposite direction.* With this exception, all the motions of the planets, whether around their own axes, or around the sun, are from west to east.

OF THE NEW PLANETS, CERES, PALLAS, JUNO, AND VESTA.

349. The commencement of the present century was rendered memorable in the annals of astronomy, by the discovery of four new planets between Mars and Jupiter. Kepler, from some analogy which he found to subsist among the distances of the planets from the sun, had long before suspected the existence of one at this distance; and his conjecture was rendered more probable by the discovery of Uranus, which follows the analogy of the other planets. So strongly, indeed, were astronomers im-

* Sir J. Herschel.

pressed with the idea that a planet would be found between Mars and Jupiter, that, in the hope of discovering it, an association was formed on the continent of Europe of twenty four observers, who divided the sky into as many zones, one of which was allotted to each member of the association. The discovery of the first of these bodies was however made accidentally by Piazzi, an astronomer of Palermo, on the first of January, 1801. It was shortly afterwards lost sight of on account of its proximity to the sun, and was not seen again until the close of the year, when it was re-discovered in Germany. Piazzi called it *Ceres* in honor of the tutelary goddess of Sicily, and her emblem, the sickle ⚳, has been adopted as its appropriate symbol.

The difficulty of finding Ceres induced Dr. Olbers, of Bremen, to examine with particular care all the small stars that lie near her path, as seen from the earth; and while prosecuting these observations, in March, 1802, he discovered another similar body, very nearly at the same distance from the sun, and resembling the former in many other particulars. The discoverer gave to this second planet the name of *Pallas*, choosing for its symbol the lance ⚴, the characteristic of Minerva.

350. The most surprising circumstance connected with the discovery of Pallas, was the existence of two planets at nearly the same distance from the sun, and apparently having a common node. On account of this singularity, Dr. Olbers was led to conjecture [redacted]

might be sought for in those parts with greater chance of success than in a wider zone, embracing the entire limits of these orbits. Accordingly, in 1804, near one of the nodes of Ceres and Pallas, a third planet was discovered. This was called *Juno*, and the character ⚵ was adopted for its symbol, representing the starry sceptre of the queen of Olympus. Pursuing the same researches, in 1807, a fourth planet was discovered, to which was given the name of *Vesta*, and for its symbol the character ⚶ was chosen, an altar surmounted with a censer holding the sacred fire.

After this historical sketch, it will be sufficient to classify under a few heads the most interesting particulars relating to the New Planets.

351. The *average distance* of these bodies from the sun is 261,000,000 miles; and it is remarkable that their orbits are very near together. Taking the distance of the earth from the sun for unity, their respective distances are 2.77, 2.77, 2.67, 2.37.

As they are found to be governed, like the other members of the solar system, by Kepler's law, that regulates the distances and times of revolution, their *periodical times* are of course pretty nearly equal, averaging about 4½ years.

In respect to the *inclination of their orbits*, there is considerable diversity. The orbit of Vesta is inclined to the ecliptic only about 7°, while that of Pallas is more than 34°. They all therefore have a higher inclination than the orbits of the old planets, and of course make excursions from the ecliptic beyond the limits of the Zodiac.

The *eccentricity of their orbits* is also, in general, greater than that of the old planets; and the eccentricities of the orbits of Pallas and Juno exceed that of the orbit of Mercury.

Their *small size* constitutes one of their most remarkable peculiarities. The difficulty of estimating the apparent diameter of bodies at once so very small and so far off, would lead us to expect different results in the actual estimates. Accordingly, while Dr. Herschel estimates the diameter of Pallas at only 80 miles, Schroeter places it as high as 2,000 miles, or about the size of the moon. The volume of Vesta is estimated at only one fifteen thousandth part of the earth's, and her surface is only

about equal to that of the kingdom of Spain.* These little bodies are surrounded by *atmospheres* of great extent, some of which are uncommonly luminous, and others appear to consist of nebulous matter. These planets in general shine with a more vivid light than might be expected from their great distance and diminutive size.

CHAPTER XI.

MOTIONS OF THE PLANETARY SYSTEM.

352. We have waited until the learner may be supposed to be familiar with the contemplation of the heavenly bodies, individually, before inviting his attention to a systematic view of the planets, and of their motions around the sun. The time has now arrived for entering more advantageously upon this subject, than could have been done at an earlier period.

There are two methods of arriving at a knowledge of the motions of the heavenly bodies. One is to begin with the *apparent*, and from these to deduce the *real* motions; the other is, to begin with considering things as they really are in nature, and then to inquire why they appear as they do. The latter of these methods

[redacted]

We shall find it no easy matter to form a correct notion of infinite space; but let us fix our attention, for some time, upon extension alone, devoid of every thing material, without light or life, and without bounds. Of such a space we could not predicate the ideas of up or down, east, west, north, or south, but all reference to our own horizon (which habit is the most difficult of all to eradicate from the mind) must be completely set aside. Into such a void we would introduce the SUN. We would contemplate this body alone, in the midst of boundless space, and continue to fix the attention upon this object, until we had fully settled its relations to the surrounding void. The ideas of up and down would now present themselves, but as yet there would be nothing to suggest any notion of the cardinal points. We suppose ourselves next to be placed on the surface of the sun, and the firmament of stars to be lighted up. The slow revolution of the sun on his axis, would be indicated by a corresponding movement of the stars in the opposite direction; and in a period equal to more than 27 of our days, the spectator would see the heavens perform a complete revolution around the sun, as he now sees them revolve around the earth once in 24 hours. The point of the firmament where no motion appeared, would indicate the position of one of the poles, which being called North, the other cardinal points would be immediately suggested.

Thus prepared, we may now enter upon the consideration of the *planetary motions*.

354. Standing on the sun, we see all the planets moving slowly around the celestial sphere, nearly in the same great high way, and in the same direction from west to east. They move, however, with very unequal velocities. Mercury makes very perceptible progress from night to night, like the moon revolving about the earth, his daily progress eastward being about one third as great as that of the moon, since he completes his entire revolution in about three months. If we watch the course of this planet from night to night, we observe it, in its revolution, to cross the ecliptic in two opposite points of the heavens, and wander about 7° from that plane at its greatest distance from it. Knowing the position of the orbit of Mercury with respect to

the ecliptic. We may now, in imagination, represent that orbit by a great circle passing through the center of the planet and the center of the sun, and cutting the plane of the ecliptic in two opposite points at an angle of 7°. We may imagine the intersection of these two great circles with the celestial vault to be marked out in plain and palpable lines on the face of the sky; but we must bear in mind that these orbits are mere *mathematical planes*, having no permanent existence in nature, any more than the path of an eagle flying through the sky: and if we conceive of their orbits as marked on the celestial vault, we must be careful to attach to the representation the same notion as to a thread or wire carried round to trace out the course pursued by a horse in a race-ground.*

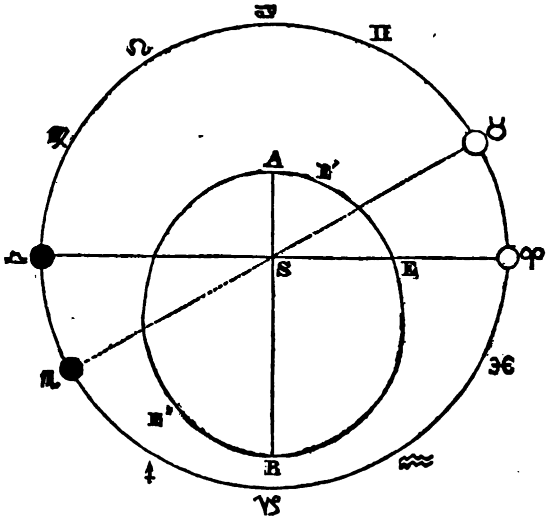
The *planes* of both the ecliptic and the orbit of Mercury, may be conceived of as indefinitely extended to a great distance until they meet the sphere of the stars; but the *lines* which the earth and Mercury describe in those planes, that is, their orbits, may be conceived of as comparatively *near to the sun*. Could we now for a moment be permitted to imagine that the planes of the ecliptic, and of the orbit of Mercury, were made of thin plates of glass, and that the paths of the respective planets were marked out on their planes in distinct lines, we should perceive the orbit of the earth to be almost a perfect circle, while that of Mercury would appear distinctly elliptical. But having once made use of a palpable surface and visible lines to aid us in giving position

other planets. Standing on the sun we should see each of the planets pursuing a similar course to that of Mercury, all moving from west to east, with motions differing from each other chiefly in two respects, namely, in their velocities, and in the distances to which they ever recede from the ecliptic.

The earth revolves about the sun very much like Venus, and to a spectator on the sun, the motions of these two planets would exhibit much the same appearances. We have supposed the observer to select the plane of the earth's orbit as his standard of reference, and to see how each of the other orbits is related to it; but such a selection of the ecliptic is entirely arbitrary; the spectator on the sun, who views the motions of the planets as they actually exist in nature, would make no such distinction between the different orbits, but merely inquire how they were mutually related to each other. Taking, however, the ecliptic as the plane to which all the others are referred, we do not, as in the case of the other planets, inquire how its plane is *inclined*, nor what are its *nodes*, since it has neither inclination nor node.

356. Such, in general, are the *real* motions of the planets, and such the appearances which the planetary system would exhibit to a spectator at the center of motion. But in order to represent correctly the positions of the planetary orbits, at any given time, *three* things must be regarded,—the *Inclination* of the orbit to the ecliptic—the position of *the line of the Nodes*—and the position of *the line of the Apsides*. In our common diagrams, the orbits are incorrectly represented, being all in the same plane, as in the following diagram, where AEB (Fig. 65,) represents the orbit of Mercury as lying in the same plane with the ecliptic. To exhibit its position justly (AB being taken as the line of the nodes) it should be elevated on one side about 7° and depressed by the same number of degrees on the other side, turning on the line AB as on a hinge. But even then the representation may be incorrect in other respects, for we have taken it for granted that the line of the nodes coincides with the line of the apsides, or that the orbit of Mercury cuts the ecliptic in the line AB. Whereas, it may lie in any given position with respect to the line of the apsides depending on the longitude of the nodes. If, for exam-

Fig. 65.



ple, the line of the nodes had chanced to pass through Taurus and Scorpio instead of Cancer and Capricorn, then it would have been represented by the line ♉ ♏ instead of ♋♑, and the plane when elevated or depressed with respect to the plane of the equator, would be turned on this line in our figure.* Moreover, our diagram represents the line of the apsides as passing through Cancer and Capricorn, whereas it may have any other position among the signs, according to the longitudes of the perigee and

[redacted]

feet in diameter. If we preserve the same proportions in regard to distance, we must place Mercury 250 feet, and Uranus 12,500 feet, or more than two miles from the sun. The mind of the student of astronomy must, therefore, raise itself from such imperfect representations of celestial phenomena as are afforded by artificial mechanism, and, transferring his contemplations to the celestial regions themselves, he must conceive of the sun and planets as bodies that bear an insignificant ratio to the immense spaces in which they circulate, resembling more a few little birds flying in the open sky, than they do the crowded machinery of an orrery.

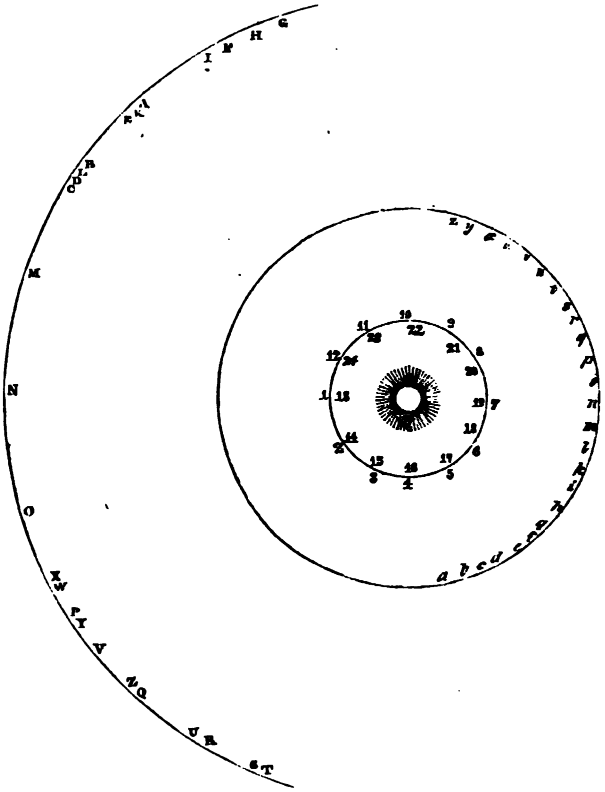
358. Having acquired as correct an idea as we are able of the planetary system, and of the positions of the orbits with respect to the ecliptic, let us next inquire into the nature and causes of the *apparent motions.*

The apparent motions of the planets are exceedingly unlike the real motions, a fact which is owing to two causes ; first, *we view them out of the center of their orbits ;* secondly, *we are ourselves in motion.* From the first cause, the apparent places of the planets are greatly changed by perspective ; and from the second cause, we attribute to the planets changes of place which arise from our own motions of which we are unconscious.

359. The situation of a heavenly body as seen from the center of the sun, is called its *heliocentric* place ; as seen from the center of the earth, its *geocentric* place. The geocentric motions of the planets must, according to what has just been said, be far more irregular and complicated than the heliocentric, as will be evident from the following diagram, which represents the geocentric motions of Mercury for two entire revolutions, embracing a period of nearly six moths.

Let S (Fig. 66,) represent the sun, 1, 2, 3, &c. the orbit of Mercury, *a*, *b*, *c*, &c. that of the earth, and GT the concave sphere of the heavens. The orbit of Mercury is divided into 12 equal parts, each of which he describes in 7½ days, and a portion of the earth's orbit described by that body in the time that Mercury describes the two complete revolutions, is divided into 24 equal

Fig. 66.



parts. Let us now suppose that Mercury is at the point 1 in his

[redacted]

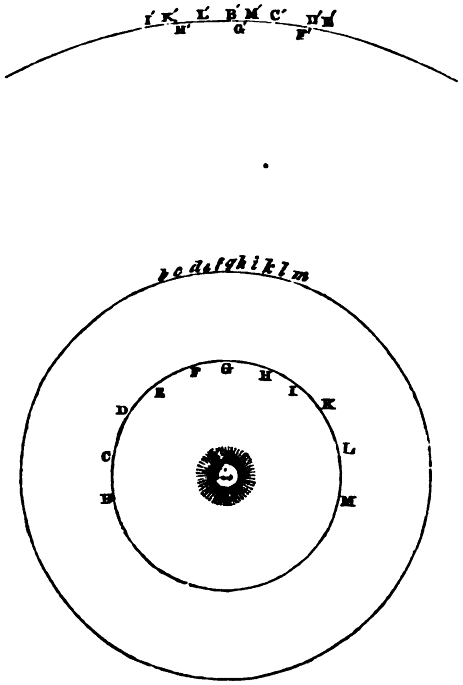
rapidly, since her period of revolution approaches much more nearly to that of the earth.

261. The apparent motions of the *superior* planets, are, like those of Mercury and Venus, alternately direct, stationary, and retrograde. In this case, however, the earth moves faster than the planet, and the planet has its opposition but no inferior conjunction, whereas an inferior planet has its inferior conjunction, but no opposition. These differences render the apparent motions of the superior planets in some respects unlike those of Mercury and Venus. When a superior planet is in conjunction, its motion is direct, because, as in the case of Venus in her *superior* conjunction, (See Fig. 60,) the only effect of the earth's motion is to accelerate it; but when the planet is in opposition, the earth is moving past it with a greater velocity, and makes the planet seem to move backwards, like the apparent backward motion of a vessel when we overtake it and pass rapidly by it in a steamboat.

362. But the various motions of a superior planet will be best understood from a diagram. Hence, let S (Fig. 67,) be the sun; B, C, D, E, the orbit of the earth; *b*, *c*, *d*, &c. the orbit of a superior planet, as Jupiter for example; and I′E′ a portion of the concave sphere of the heavens. Let *bm* be the arc described by Jupiter in the time the earth describes the arc BM; let *bc*, *cd*, and *de*, &c. be described by Jupiter while the earth describes BC, CD, and DE. Now when the earth is at B and Jupiter at *b*, he will appear in the heavens at B′. When the earth reaches C, the planet reaches *c* and will be seen at C′, his motions having been direct from west to east. While the earth moves from C to D and from D to E, Jupiter has moved from *c* to *d*, and from *d* to *e*, and will appear to have advanced among the stars from C′ to D′, and from D′ to E′, his motion being still direct, but slower than before, as he has passed over only the space D′E′ in the same time that he before moved through the greater spaces B′C′ and C′D′.

During the motion of the earth from E to F, and of Jupiter from *e* to *f*, the earth passes by Jupiter; and not being conscious of our own motion, Jupiter seems to us to have moved backward

Fig. 67.



from E′ to F′. At E′ where the direct motion was changed to a retrograde, he would appear to be stationary. Upon the arrival of the earth at G, and of Jupiter at *g*, in opposition to the sun, [redacted]

CHAPTER XII.

DETERMINATION OF THE PLANETARY ORBITS—KEPLER'S DISCOVERIES—ELEMENTS OF THE ORBIT OF A PLANET—QUANTITY OF MATTER IN THE SUN AND PLANETS—STABILITY OF THE SOLAR SYSTEM.

363. In chapter II, we have shown that the figure of the earth's orbit is an ellipse, having the sun in one of its foci, and that the earth's radius vector describes equal areas in equal times ; and in Chapter III, we have remarked that these are only particular examples under the law of Universal Gravitation, as is also the additional fact, that the squares of the periodical times of the planets are as the cubes of the major axes of their orbits. We may now learn, more particularly, the process by which the illustrious Kepler was conducted to the discovery of these grand laws of the planetary system.

364. Ptolemy, while he held that the orbits of the planets were perfect circles in which the planets revolved uniformly about the earth, was nevertheless obliged to suppose that the earth was situated out of the center of the circles, and that at the same distance on the other side of the center was situated the point (*punctum æquans*) about which the angular motion of the body was equable and uniform. In regard to the orbit of the sun, however, the earth was held to occupy the exact center. On nearly the same suppositions, Tycho Brahe had made a great number of very accurate observations on the planetary motions, which served Kepler as standards of comparison for results, which he deduced from calculations founded on the application of geometrical reasoning to hypotheses of his own.

Kepler first applied himself to investigate the orbit of Mars, the motions of which planet appeared more irregular than those of any other, except Mercury, which, being seldom seen, had then been very little studied. According to the views of Ptole-

my and Tycho, he at first supposed the orbit to be circular, and the planet to move uniformly about a point at a certain distance from the sun. He made seventy suppositions before he obtained one that agreed with observation, the calculation of which was extremely long and tedious, occupying him more than five years.* The supposition of an equable motion in a circle, however varied, could not be made to conform to the observations of Tycho, whereas the supposition that the orbit was of an oval figure, depressed at the sides, but coinciding with a circle at the perihelion, agreed very nearly with observation. Such a figure naturally suggested the idea of an ellipse, and reasoning on the known properties of the ellipse, and comparing the results of calculation with actual observation, the agreement was such as to leave no doubt that the orbit of Mars is an ellipse, having the sun in one of the foci. He immediately conjectured that the same is true of the orbits of all the other planets, and a similar comparison of this hypothesis with observation, confirmed its truth. Hence he established the first great law, *that the planets revolve about the sun in ellipses, having the sun in one of the foci.*

365. Kepler also discovered from observation, that the velocities of the planets when in their apsides, are inversely as their distances from the sun, whence it follows that they describe, in these points, equal areas about the sun in equal times. Although he could not prove, from observation, that the same was true in [redacted]

them, having a strong passion for finding analogies in nature. He saw that the more distant a planet was from the sun, the slower it moved; so that the periodic times of the more distant planets would be increased on two accounts, first, because they move over a greater space, and secondly, because their motions in their orbits is actually slower than the motions of the planets nearer the sun. Saturn, for example, is 9½ times further from the sun than the earth is, and the circle described by Saturn is greater than that of the earth in the same ratio; and since the earth revolves around the sun in one year, were their velocities equal, the periodic time of Saturn would be 9½ years, whereas it is nearly 30 years. Hence it was evident, that the periodic times of the planets increase in a greater ratio than their distances, but in a less ratio than the squares of their distances, for on that supposition the periodic time of Saturn would be about 90¼ years. Kepler then took the squares of the times and compared them with the cubes of the distances, and found an exact agreement between them. Thus he discovered the famous law, *that the squares of the periodic times of all the planets, are as the cubes of their mean distances from the sun.**

This law is strictly true only in relation to planets whose quantity of matter in comparison with that of the central body is inappreciable. When this is not the case, the periodic time is shortened in the ratio of the square root of the sun's mass divided by the sun's plus the planet's mass $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$. The mass of most of the planets is so small compared with the sun's, that this modification of the law is unnecessary except where extreme accuracy is required.

ELEMENTS OF THE PLANETARY ORBITS.

367. The particulars necessary to be known in order to determine the precise situation of a planet at any instant, are called the *Elements of its Orbit.* They are seven in number, of which the first two determine the absolute situation of the orbit, and the

* Vince's Complete System, I, 98.

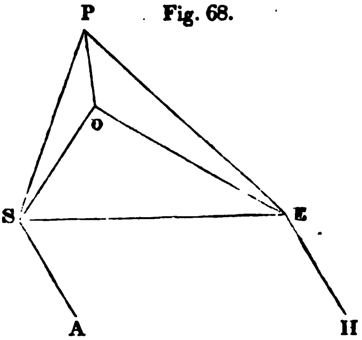
other five relate to the motion of the planet in its orbit. These elements are,

(1.) *The position of the line of the nodes.*
(2.) *The inclination to the ecliptic.*
(3.) *The periodic time.*
(4.) *The mean distance from the sun, or semi-axis major.*
(5.) *The eccentricity.*
(6.) *The place of the perihelion.*
(7.) *The place of the planet in its orbit at a particular epoch.*

368. It may at first view be supposed that we can proceed to find the elements of the orbit of a planet in the same manner as we did those of the solar or lunar orbit, namely, by observations on the right ascension and declination of the body, converted into latitudes and longitudes by means of spherical trigonometry, (See page 59.) But in the case of the moon, we are situated in the center of her motions, and the apparent coincide with the real motions; and, in respect to the sun, our observations on his *apparent* motions give us the earth's *real* motions, allowing 180° difference in longitude. But, as we have already seen, the motions of the planets appear exceedingly different to us, from what they would if seen from the center of their motions. It is necessary therefore to deduce from observations made on the earth the corresponding results as they would be if viewed from the center of the sun; that is, in the language of astronomers, hav- [redacted]

Let S and E (Fig. 68,) be the sun and earth, P the planet, PO a line drawn from P perpendicular to the ecliptic, SA the direction of Aries, and EH parallel to SA, and therefore (on account of the immense distance of the fixed stars) also in the direction of Aries. Then OEH, being the apparent distance of the planet from Aries in the direction of the ecliptic, is the geocentric longitude, and OEP, being the apparent distance of the planet from the ecliptic taken on a secondary to the ecliptic, is the geocentric latitude. It is obvious also that the angles OSA and PSO are

Fig. 68.



the heliocentric longitude and latitude. The planet's angular distance from the sun, PES, is also known from observation. Hence, in the triangle SEP, we know SP and SE and the angle SEP, from which we can find PE; and knowing PE and the angle PEO, we can find OE, since OEP is a right angled triangle. Hence in the triangle SEO, ES and EO, and the angle SEO (=OEH − SEH=difference of longitude of the planet and the sun) are known, and hence we can obtain OSE, (Art. 135,) which added to the sun's longitude ESA, gives us OSA the planet's *heliocentric longitude.*

Also, because PS : Rad. : : OP : Sin. PSO

$\therefore$ PS × Sin. PSO = OP × Rad.

But EP : Rad. : : OP : Sin. OEP

$\therefore$ EP × Sin. OEP = OP × Rad.

$\therefore$ PS × Sin. PSO = EP × Sin. OEP

$\therefore$ PS : EP : : Sin. OEP : Sin. PSO.

The first three terms of this proposition being known, the last is found which is the *heliocentric latitude.**

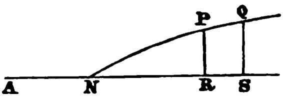
* Brinkley's Elements of Astronomy, p. 164.

370. **Having now learned how observations made at the earth may be converted into corresponding observations made at the sun, we may proceed to explain the mode of finding the several elements before enumerated; although our limits will not permit us to enter further into detail on this subject, than to explain the leading principles on which each of these elements is determined.***

371. First, to determine the *position of the Nodes*, and the *Inclination of the Orbit.*

These two elements, whieh determine the orbit, (Art. 368,) may be derived from two heliocentric longitudes and latitudes. Let AR and AS (Fig. 69,) be two heliocentric longitudes, PR and QS the heliocentric latitudes, and N the ascending node. Then, by Napier's theorem, (Art. 132.)

Fig. 69.



$$\frac{\text{Sin. NR}(=\text{AR}-\text{AN})}{\text{tan. PR}}=\text{cot. PNR}=\frac{\text{sin. NS}\ (=\text{AS}-\text{AN})}{\text{tan. QS}}$$

$$\therefore \frac{\text{Sin. AR}\times\text{cos. AN}-\text{cos. AR}\times\text{sin. AN}\dagger}{\text{tan. PR}}=$$

$$\frac{\text{sin. AS}\times\text{cos. AN}-\text{cos. AS}\times\text{sin. AN}}{\text{tan. QS}}.$$

$$\text{But tan. AN}=\frac{\text{sin. AN}}{\text{cos. AN}}=\frac{\text{Sin. AR}\times\text{tan. QS}-\text{sin. AS}\times\text{tan. PR}}{\text{Cos. AR}\times\text{tan. QS}-\text{cos. AS}\times\text{tan. PR}}.$$

But AN is the *longitude of the ascending node;* and its value [redacted]

turns to the same node. We may know when a planet is at the node because then its latitude is nothing. If, from a series of observations on the right ascension and declination of a planet, we deduce the latitudes, and find that one of the observations gives the latitude 0, we infer that the planet was at that moment at the node. But if, as commonly happens, no observation gives exactly 0, then we take two latitudes that are nearest to 0, but on opposite sides of the ecliptic, one south and the other north, and as the sum of the arcs of latitude is to the whole interval, so is one of the arcs to the corresponding time in which it was described, which time being added to the first observation, or subtracted from the second, will give the precise moment when the planet was at the node.

By repeated observations it is found, that the nodes of the planets have a very slow retrograde motion.

373. If the orbit of a planet cut the ecliptic at right angles, then small differences of latitude would be appreciable; but in fact the planetary orbits are in general but little inclined to the ecliptic, and some of them lie almost in the same plane with it. Hence arises a difficulty in ascertaining the exact time when a planet reaches its node. Among the most valuable observations for determining the elements of a planet's orbit, are those made when a superior planet is in or near *its opposition* to the sun, for then the heliocentric and geocentric longitudes are the same. When a number of oppositions are observed, the planet's motion in longitude as would be observed from the sun will be known. The inferior planets, also, when in superior conjunction, have their geocentric and heliocentric longitudes the same. When in inferior conjunction, these longitudes differ 180°; but the inferior planets can seldom be observed in superior conjunction, on account of their proximity to the sun, nor in inferior conjunction except in their transits, which occur too rarely to admit of observations sufficiently numerous. Therefore, we cannot so readily ascertain by simple observation, the motions of the inferior planets seen from the sun, as we can those of the superior.*

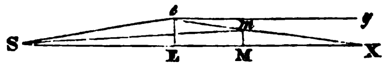
* Brinkley, p. 167.

Hence, in order to obtain accurately the periodic time of a planet, we find the interval elapsed between two oppositions separated by a long interval, when the planet was nearly in the same part of the Zodiac. From the periodic time, as determined approximately by other methods, it may be found when the planet has the same heliocentric longitude as at the first observation. Hence the time of a complete number of revolutions will be known, and thence the time of one revolution. The greater the interval of time between the two oppositions, the more accurately the periodic time will be obtained, because the errors of observation will be divided between a great number of periods; therefore by using very accurate observations, much precision may be attained. For example, the planet Saturn was observed in the year 228 B. C. March 2, (according to our reckoning of time,) to be near a certain star called γ Virginis, and it was at the same time nearly in opposition to the sun. The same planet was again observed in opposition to the sun, and having nearly the same longitude in Feb. 1714. The exact difference between these dates was 1943y. 118d. 21h. 15m. It is known from other sources, that the time of a revolution is 29½ years nearly, and hence it was found that in the above period there were 66 revolutions of Saturn; and dividing the interval by this number, we obtain 29.444 years, which is nearly the periodic time of Saturn according to the most accurate determination.

[redacted]

orbit, and M*m* a corresponding portion of that of a superior planet, described on the day of opposition, about the sun S, on which day the three bodies lie in one straight line SEMX. Then the angle ES*e* and MS*m*, representing the respective angular velocities

Fig. 70.



ties of the two bodies are known. Now if *em* be joined, and prolonged to meet SM continued in X, the angle *e*X*e*, which is equal to the alternate angle X*ey*, being equal to the retrogradation of the planet in the same time (being known from observation) is also given. E*e*, therefore, and the angle EX*e* being given in the right angled triangle E*e*X, the side EX is easily calculated, and thus SX becomes known. Consequently, in the triangle S*m*X, we have given the side SX, and the two angles *m*SX and *m*XS, whence the other sides S*m* and *m*X are easily determined. Now S*m* is the radius of the orbit of the superior planet required, which in this calculation is supposed circular as well as that of the earth,—a supposition not exact, but sufficiently so to afford a satisfactory approximation to the dimensions of its orbit, and which, if the process be often repeated, in every variety of situation at which the opposition can occur, will ultimately afford an average or mean value of its distance fully to be depended on.*

375. *The transverse or major axes of the planetary orbits remain always the same.* Amidst all the perturbations to which other elements of the orbit are subject, the line of the apsides is of the same invariable length. It is no matter in what *direction* the planet may be moving at that moment. Various circumstances will influence the eccentricity and the position of the ellipse, but none of them affects its length.

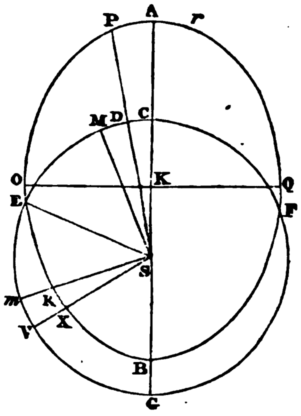
376. Fourthly, to determine *the place of the perihelion—the epoch of passing the perihelion—and the eccentricity.*

* Sir J. Herschel.

There are various methods of finding the eccentricity of a planet's orbit and the place of the perihelion, and of course the position of the line of the apsides. One is derived from the *greatest equation of the center*, (Art. 200.) The greatest equation is the greatest difference that occurs between the mean and the true motion of a body revolving in an ellipse. It will be necessary first to explain the manner in which the greatest equation is found.

Let AEBF (Fig. 71,) be the orbit of the planet, having the sun in the focus at S. In an ellipse, the square root of the product of the semi axes gives the radius of a circle of the same area as the ellipse.* Therefore with the center S, at the distance $SE = \sqrt{\overline{AK} \times \overline{OK}}$, describe the circle CEGF, then will the area of this circle be equal to that of the ellipse. At the same time that a planet departs from A the aphelion, a body begins to move with a uniform motion from C through the periphery CEGF, and performs a whole revolution in the same period that the planet describes the ellipse; the motion of this body will represent the equal or mean motion of the earth, and it will describe around S areas or sectors of circles which are proportional to the times, and equal to the elliptic areas de-

Fig. 71.



[redacted]

proportional to the difference of the areas ACE and *m*ER, or to the area GBR*m*, V being the situation of the body moving equably; for the sector CSV will be equal to the elliptic area ASR, and taking away the common spaces ACE, RE*m*=the sector VS*m*=the equation. At the points E and F, where the circle and ellipse intersect, the radius vector of the earth and the radius of the circle of equable motion are equal, and of course those radii then describe equal areas in equal times; hence, when the real motion of the earth is equal to the mean motion, the equation of the center is greatest.* The *mean motion* for any given time is easily found; for the periodic time : 360: :the given time : the number of degrees for that time. Observation shows when the *actual motion* of the planet is the same with this.

377. Now the equation of the center is greatest twice in the revolution, on opposite sides of the orbit, as at E and F, which points lie at equal distances from the apsides; and since the whole arc EAF or EBF is known from the time occupied in describing it, therefore, by bisecting this arc, we find the points A and B, the *aphelion* and *perihelion*, and consequently the position of *the line of the apsides.* The time of describing the area EBF being known, by bisecting this interval, we obtain the moment of passing the perihelion, which gives us *the place of the planet in its orbit at a particular epoch.*

The amount of the greatest equation obviously depends on the eccentricity of the orbit, since it arises wholly from the departure of the ellipse from the figure of a perfect circle; hence, the greatest equation affords the means of determining the eccentricity itself. In orbits of small eccentricity, as is the case with most of the planetary orbits, it is found that the arc which measures the greatest equation is very nearly equal to the distance between the foci,† which always equals twice the eccentricity, the eccentricity being the distance from the center to the focus. Consequently, 57° 17′ 44″.8‡ : rad.: :half the greatest equation : *the eccentricity.*

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* Gregory's Astronomy, p. 197. † Vince's Complete System, I, 113.

‡ The value of an arc equal to radius; for 3.14159 : 1 : : 180 : 57° 17′ 44″.8.

The foregoing explanations of the methods of finding the elements of the orbits, will serve in general to show the learner how these particulars are or may be ascertained, yet the methods actually employed are usually more refined and intricate than these. In astronomy scarcely an element is presented simple and unmixed with others. Its value when first disengaged, must partake of the uncertainty to which the other elements are subject; and can be supposed to be settled to a tolerable degree of correctness, only after multiplied observations and many revisions.*

So arduous has been the task of finding the elements of the planetary orbits.

QUANTITY OF MATTER IN THE SUN AND PLANETS.

378. It would seem at first view very improbable, that an inhabitant of this earth would be able to weigh the sun and planets, and estimate the exact quantity of matter which they severally contain. But the principles of Universal Gravitation conduct us to this result, by a process remarkable for its simplicity. By comparing the relations of a few elements that are known to us, we ascend to the knowledge of such as appeared beyond the pale of human investigation. *We learn the quantity of matter in a body by the force of gravity it exerts.* Let us see how this force is ascertained.

379. *The quantities of matter in two bodies, may be found in* [redacted]

sun, by comparing the distance and periodic time of the moon, revolving around the earth, with the distance and periodic time of the earth revolving around the sun. For the cube of the moon's distance from the earth divided by the square of her periodic time, is to the cube of the earth's distance from the sun divided by the square of her periodic time, as the quantity of matter in the earth is to that in the sun. That is, $\frac{238545^3}{27.32^2} : \frac{95,000,000^3}{365.256^2} :: 1 : 353,385$. The most exact determination of this ratio, gives for the mass of the sun 354,936 times that of the earth. Hence it appears that the sun contains more than three hundred and fifty four thousand times as much matter as the earth. Indeed the sun contains eight hundred times as much matter as all the planets.

Another view may be taken of this subject which leads to the same result. Knowing the velocity of the earth in its orbit, we may calculate its *centrifugal force.* Now this force is counterbalanced, and the earth retained in its orbit, by the attraction of the sun, which is proportional to the quantity of matter in the sun. Therefore we have only to see what amount of matter is required in order to balance the earth's centrifugal force. Is is found that the earth itself or a body as heavy as the earth acting at the distance of the sun, would be wholly incompetent to produce this effect, but that in fact it would take more than three hundred and fifty four thousand such bodies to do it.

380. The mass of each of the other planets *that have satellites* may be found, by comparing the periodic time of one of its satellites with its own periodic times around the sun. By this means we learn the ratio of its quantity of matter to that of the sun. The masses of those planets which have no satellites, as Venus or Mars, have been determined, by estimating the force of attraction which they exert in disturbing the motions of other bodies. Thus, the effect of the moon in raising the tides, leads to a knowledge of the quantity of matter in the moon; and the effect of Venus in disturbing the motions of the earth, indicates her quantity of matter.*

* These estimates are made by the most profound investigations in La Place's Mecanique Celeste, Vol. III.

381. The quantity of matter in bodies varies as their magnitudes and densities conjointly. Hence, their *densities* vary as their masses divided by their magnitudes; and since we know the magnitudes of the planets, and can compute as above their masses, we can thus learn their densities, which, when reduced to a common standard, give us their specific gravities, or show us how much heavier they are than water. Worlds therefore are weighed with almost as much ease as a pebble or an article of merchandize.

The *densities and specific gravities* of the sun, moon, and planets, are estimated as follows:*

	Density.	Specific Gravity.
Sun, . . .	0.2543	1.40†
Moon, . . .	0.6150	3.37
Mercury, . . .	2.7820	15.24
Venus, . . .	0.9434	5.17
Earth, . . .	1.0000	5.48
Mars, . . .	0.1293	0.71
Jupiter, . . .	0.2589	1.42
Saturn, . . .	0.1016	0.56
Uranus, . . .	0.2797	1.53

From this table it appears, that the sun consists of matter but little heavier than water; but that the moon is more than three times as heavy as water, though less dense than the earth. It also appears that the planets near the sun are, as a general fact,

[redacted]

formity with the law of Universal Gravitation. Venus and Mars, approaching as they do at times comparatively near to the earth, sensibly disturb its motions, and the satellites of the remoter planets greatly disturb each other's movements.

STABILITY OF THE SOLAR SYSTEM.

383. The derangement which the planets produce in the motion of one of their number will be very small in the course of one revolution; but this gives us no security that the derangement may not become very large in the course of many revolutions. The cause acts perpetually, and it has the whole extent of time to work in. Is it not easily conceivable then that in the lapse of ages, the derangements of the motions of the planets may accumulate, the orbits may change their form, and their mutual distances may be much increased or diminished? Is it not possible that these changes may go on without limit, and end in the complete subversion and ruin of the system? If, for instance, the result of this mutual gravitation should be to increase considerably the eccentricity of the earth's orbit, or to make the moon approach continually nearer and nearer to the earth at every revolution, it is easy to see that in the one case, our year would change its character, producing a far greater irregularity in the distribution of the solar heat: in the other, our satellite must fall to the earth, occasioning a dreadful catastrophe. If the positions of the planetary orbits with respect to that of the earth, were to change much, the planets might sometimes come very near us, and thus increase the effect of their attraction beyond calculable limits. Under such circumstances we might have years of unequal length, and seasons of capricious temperature; planets and moons of portentous size and aspect glaring and disappearing at uncertain intervals; tides like deluges sweeping over whole continents; and, perhaps, the collision of two of the planets, and the consequent destruction of all organization on both of them. The fact really is, that changes are taking place in the motions of the heavenly bodies, which have gone on progressively from the first dawn of science. The eccentricity of the earth's orbit has been diminishing from the earliest observations to our times. The moon has been moving quicker from the time

of the first recorded eclipses, and is now in advance by about four times her own breadth, of what her own place would have been if it had not been affected by this acceleration. The obliquity of the ecliptic also, is in a state of diminution, and is now about two fifths of a degree less than it was in the time of Aristotle.*

384. But amid so many seeming causes of irregularity, and ruin, it is worthy of grateful notice, that effectual provision is made for the *stability of the solar system.* The full confirmation of this fact, is among the grand results of Physical Astronomy. Newton did not undertake to demonstrate either the stability or instability of the system. The decision of this point required a great number of preparatory steps and simplifications, and such progress in the invention and improvement of mathematical methods as occupied the best mathematicians of Europe for the greater part of the last century. Towards the end of that time, it was shown by La Grange and La Place, that the arrangements of the solar system are stable; that, in the long run, the orbits and motions remain unchanged; and that the changes in the orbits, which take place in shorter periods, never trangress certain very moderate limits. Each orbit undergoes deviations on this side and on that side of its average state; but these deviations are never very great, and it finally recovers from them, so that the average is preserved. The planets produce perpetual [redacted]

in orbits of small eccentricity, and but slightly inclined to each other, their secular irregularities are periodical and included within narrow limits; so that the planetary system will only oscillate about a mean state, and will never deviate from it except by a very small quantity. The ellipses of the planets have been and always will be nearly circular. The ecliptic will never coincide with the equator; and the entire extent of the variation in its inclination, cannot exceed three degrees.

385. To these observations of La Place, Professor Whewell* adds the following on the importance, to the stability of the solar system, of the fact that those planets which have *great masses* have orbits of *small eccentricity*. The planets Mercury and Mars, which have much the largest eccentricity among the old planets, are those of which the masses are much the smallest. The mass of Jupiter is more than two thousand times that of either of these planets. If the orbit of Jupiter were as eccentric as that of Mercury, all the security for the stability of the system, which analysis has yet pointed out, would disappear. The earth and the smaller planets might in that case change their nearly circular orbits into very long ellipses, and thus might fall into the sun, or fly off into remote space. It is further remarkable that in the newly discovered planets, of which the orbits are still more eccentric than that of Mercury, the masses are still smaller, so that the same provision is established in this case also.

CHAPTER XIII.

OF COMETS.

386. A COMET,† when perfectly formed, consists of three parts, the Nucleus, the Envelope, and the Tail. The *Nucleus*, or body of the comet, is generally distinguished by its forming a bright point in the center of the head, conveying the idea of a solid, or

* Bridgewater Treatises, p. 131. See also Playfair's Outlines, 2, 290.
† κόμη *coma*, from the *bearded* appearance of comets.

at least of a very dense portion of matter. Though it is usually exceedingly small when compared with the other parts of the comet, yet it sometimes subtends an angle capable of being measured by the telescope. The *Envelope*, (sometimes called the *coma*) is a dense nebulous covering, which frequently renders the edge of the nucleus so indistinct, that it is extremely difficult to ascertain its diameter with any degree of precision. Many comets have no nucleus, but present only a nebulous mass extremely attenuated on the confines, but gradually increasing in density towards the center. Indeed there is a regular gradation of comets, from such as are composed merely of a gaseous or vapory medium, to those which have a well defined nucleus. In some instances on record, astronomers have detected with their telescopes small stars through the densest part of a comet.

The *Tail* is regarded as an expansion or prolongation of the coma; and, presenting as it sometimes does, a train of appalling magnitude, and of a pale, disastrous light, it confers on this class of bodies their peculiar celebrity.

387. The *number* of comets belonging to the solar system, is probably very great. Many, no doubt, escape observation by being above the horizon in the day time. Seneca mentions, that during a total eclipse of the sun, which happened 60 years before the Christian era, a large and splendid comet suddenly made its appearance, being very near the sun. The elements of at least

comet (the same which re-appeared in 1735) is distinguished as that whose return was first successfully predicted, and whose orbit is best determined; and Biela's and Encke's comets are well known, for their short periods of revolution, which subject them frequently to the view of astronomers.

388. In *magnitude* and *brightness* comets exhibit a great diversity. History informs us of comets so bright as to be distinctly visible in the day time, even at noon and in the brightest sunshine. Such was the comet seen at Rome a little before the assassination of Julius Cæsar. The comet of 1680 covered an arc of the heavens of 97°, and its length was estimated at 123,000,000 miles.* That of 1811, had a nucleus of only 428 miles in diameter, but a tail 132,000,000 miles long.† Had it been coiled around the earth like a serpent, it would have reached round more than 5,000 times. Other comets are of exceedingly small dimensions, the nucleus being estimated at only 25 miles; and some which are destitute of any perceptible nucleus, appear to the largest telescopes, even when nearest to us, only as a small speck of fog, or as a tuft of down. The majority of comets can be seen only by the aid of the telescope.

The same comet, indeed, has often very different aspects, at its different returns. Halley's comet in 1305 was described by the historians of that age, as *cometa horrendæ magnitudinis;* in 1456 its tail reached from the horizon to the zenith, and inspired such terror, that by a decree of the Pope of Rome, public prayers were offered up at noon-day in all the Catholic churches to deprecate the wrath of heaven, while in 1682, its tail was only 30° in length, and in 1759 it was visible only to the telescope, until after it had passed its perihelion. At its recent return in 1835, the greatest length of the tail was about 12°.‡ These changes in the appearances of the same comet are partly owing to the different positions of the earth with respect to them, being sometimes much nearer to them when they cross its track than at others; also one spectator so situated as to see the coma at a higher

* Arago. † Milne's Prize Essay on Comets.

‡ But might be seen much longer by *indirect* vision. (*Prof. Joslin*, Am. Jour. Science, 31, 328.)

angle of elevation or in a purer sky than another, will see the train longer than it appears to another less favorably situated; but the extent of the changes are such as indicate also a real change in their magnitude and brightness.

389. The *periods* of comets in their revolutions around the sun, are equally various. Encke's comet, which has the shortest known period, completes its revolution in 3⅓ years, or more accurately, in 1208 days; while that of 1811 is estimated to have a period of 3383 years.*

390. The *distances* to which different comets recede from the sun, are also very various. While Encke's comet performs its entire revolution within the orbit of Jupiter, Halley's comet recedes from the sun to twice the distance of Uranus, or nearly 3600,000,000 miles. Some comets, indeed, are thought to go to a much greater distance from the sun than this, while some even are supposed to pass into parabolic or hyperbolic orbits, and never to return.

391. Comets shine *by reflecting the light of the sun.* In one or two instances they have exhibited distinct *phases*,† although the nebulous matter with which the nucleus is surrounded, would commonly prevent such phases from being distinctly visible, even when they would otherwise be apparent. Moreover, certain qualities of polarized light enable the optician to decide whether the [redacted]

The tails of comets extend in a direct line from the sun, although they are usually more or less curved, like a long quill or feather, being convex on the side next to the direction in which they are moving; a figure which may result from the less velocity of the portions most remote from the sun. Expansions of the Envelope have also been at times observed on the side next the sun,* but these seldom attain any considerable length.

393. The *quantity of matter* in comets is exceedingly small. Their tails consist of matter of such tenuity that the smallest stars are visible through them. They can only be regarded as great masses of thin vapor, susceptible of being penetrated through their whole substance by the sunbeams, and reflecting them alike from their interior parts and from their surfaces. It appears, perhaps, incredible that so thin a substance should be visible by reflected light, and some astronomers have held that the matter of comets is self-luminous; but it requires but very little light to render an object visible in the night, and a light vapor may be visible when illuminated throughout an immense stratum, which could not be seen if spread over the face of the sky like a thin cloud. From the extremely small quantity of matter of these bodies, compared with the vast spaces they cover, Newton calculated that if all the matter constituting the largest tail of a comet, were to be compressed to the same density with atmospheric air, it would occupy no more than a cubic inch.† This is incredible, but still the highest clouds that float in our atmosphere, must be looked upon as dense and massive bodies, compared with the filmy and all but spiritual texture of a comet.‡

394. The small quantity of matter in comets is proved by the fact that *they have sometimes passed very near to some of the planets without disturbing their motions in any appreciable degree.* Thus the comet of 1770, in its way to the sun, got entangled among the satellites of Jupiter, and remained near them

* See Dr. Joslin's remarks on Halley's comet, Amer. Jour. Science, Vol. 31.
† Principia, iii, 41. ‡ Sir J. Herschel.

four months, yet it did not perceptibly change their motions. The same comet also came very near the earth; so near, that, had its mass been equal to that of the earth, it would have caused the earth to revolve in an orbit so much larger than at present, as to have increased the length of the year 2h, 47m.* Yet it produced no sensible effect on the length of the year, and therefore its mass, as is shown by La Place, could not have exceeded $\frac{1}{5000}$ of that of the earth, and might have been less than this to any extent. It may indeed be asked, what proof we have that comets have any matter, and are not mere reflexions of light. The answer is that, although they are not able by their own force of attraction to disturb the motions of the planets, yet they are themselves exceedingly disturbed by the action of the planets, and in exact conformity with the laws of universal gravitation. A delicate compass may be greatly agitated by the vicinity of a mass of iron, while the iron is not sensibly affected by the attraction of the needle.

By approaching very near to a large planet, a comet may have its orbit entirely changed. This fact is strikingly exemplified in the history of the comet of 1770. At its appearance in 1770, its orbit was found to be an ellipse, requiring for a complete revolution only 5½ years; and the wonder was, that it had not been seen before, since it was a very large and bright comet. Astronomers suspected that its path had been changed, and that it had been recently compelled to move in this short ellipse, by the disturbing force of Jupiter and his

result showed that it then moved in an ellipse of greater extent, having a period of 50 years, and having its *perihelion* instead of its aphelion near Jupiter. It was therefore evident why, as long as it continued to circulate in an orbit so far from the center of the system, it was never visible from the earth. In January 1767, Jupiter and the comet happened to be very near one another, and as both were moving in the same direction, and nearly in the same plane, they remained in the neighborhood of each other for several months, the planet being between the comet and the sun. The consequence was, that the comet's orbit was changed into a smaller ellipse, in which its revolution was accomplished in 5½ years. But as it was approaching the sun in 1779, it happened again to fall in with Jupiter. It was in the month of June, that the attraction of the planet began to have a sensible effect; and it was not until the month of October following that they were finally separated.

At the time of their nearest approach, in August, Jupiter was distant from the comet only $\frac{1}{491}$ of its distance from the sun, and exerted an attraction upon it 225 times greater than that of the sun. By reason of this powerful attraction, Jupiter being farther from the sun than the comet, the latter was drawn out into a new orbit, which even at its perihelion came no nearer to the sun than the planet Ceres. In this third orbit, the comet requires about 20 years to accomplish its revolution; and being at so great a distance from the earth, it is invisible, and will forever remain so unless, in the course of ages, it may undergo new perturbations, and move again in some smaller orbit as before.*

ORBITS AND MOTIONS OF COMETS.

395. The planets, as we have seen, (with the exception of the four new ones, which seem to be an intermediate class of bodies between planets and comets,) move in orbits which are nearly circular, and all very near to the plane of the ecliptic, and all move in the same direction from west to east. But the orbits of comets are far more excentric than those of the planets; they are

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* Milne.

inclined to the ecliptic at various angles, being sometimes even nearly perpendicular to it; and the motions of comets are sometimes retrograde.

396. The *Elements* of a comet are five, viz. (1) The *perihelion distance;* (2) *longitude of the perihelion;* (3) *longitude of the node;* (4) *inclination of the orbit;* (5) *time of the perihelion passage.*

The investigation of these elements is a problem extremely intricate, requiring for its solution, a skilful and laborious application of the most refined analysis. Newton himself, pronounced it *Problema longe difficilimum;* and with all the advantages of the most improved state of science, the determination of a comet's orbit is considered one of the most complicated problems in astronomy. This difficulty arises from several circumstances peculiar to comets. In the *first* place, from the elongated form of the orbits which these bodies describe, it is during only a very small portion of their course, that they are visible from the earth, and the observations made in that short period, cannot afterwards be verified on more convenient occasions; whereas in the case of the planets, whose orbits are nearly circular, and whose movements may be followed uninterruptedly throughout a complete revolution, no such impediments to the determination of their orbits occur. In the *second* place, there are many comets which move in a direction opposite to the order of the signs in the zodiac, and sometimes [redacted]

On account of these circumstances, it is found exceedingly difficult to lay down the path which a comet actually follows through the whole system, and least of all, possible to ascertain with accuracy, the length of the major axis of the ellipse, and consequently the periodical revolution.* An error of only a few seconds may cause a difference of many hundred years. In this manner, though Bessel determined the revolution of the comet of 1769 to be 2089 years, it was found that an error of no more than 5″ in observation, would alter the period either to 2678 years, or to 1692 years. Some astronomers, in calculating the orbit of the great comet of 1680, have found the length of its greater axis 426 times the earth's distance from the sun, and consequently its period 8792 years; whilst others estimate the greater axis 430 times the comet's distance, which alters the period to 8916 years. Newton and Halley, however, judged that this comet accomplished its revolution in only 570 years.

397. Disheartened by the difficulty of attaining to any precision in that circumstance, by which an elliptic orbit is characterized, and, moreover, taking into account the laborious calculations necessary for its investigation, astronomers usually satisfy themselves with ascertaining the elements of a comet on the supposition of its describing a parabola; and, as this is a curve whose axis is infinite, the procedure is greatly simplified by leaving entirely out of consideration, the periodical revolution. It is true that a parabola may not represent with mathematical strictness the course which a comet actually follows; but as a parabola is the intermediate curve between the hyperbola and ellipse, it is found that this method, which is so much more convenient for computation, also accords sufficiently with observations, except in cases when the ellipse is a comparatively short one, as that of Encke's comet, for example.

398. The elements of a comet, with the exception of its periodic time, are calculated in a manner similar to those of the plan-

* For when we know the length of the major axis, we can find the periodic time by Kepler's law, which applies as well to comets as to planets.

ets. Three good observations on the right ascension and declination of the comet (which are usually found by ascertaining its position with respect to certain stars, whose right ascensions and declinations are accurately known) afford the means of calculating these elements.

The appearance of the same comet at different periods of its return are so various, (Art. 388,) that we can never pronounce a given comet to be the same with one that has appeared before, from any peculiarities in its physical aspect. The identity of a comet with one already on record, is determined by the identity of the elements. It was by this means that Halley first established the identity of the comet which bears his name, with one that had appeared at several preceding ages of the world, of which so many particulars were left on record, as to enable him to calculate the elements at each period. These were as in the following table.

Time of appear.	Inclin. of the orbit.	Long. of the Node.	Long. of Per.	Per. Dist.	Course.
1456	17° 56′	48° 30′	301° 00	0.58	Retrograde.
1531	17 56	49 25	301 39	0,57	Retrograde.
1607	17 02	50 21	302 16	0.58	Retrograde.
1682	17 42	50 48	301 36	0.58	Retrograde.

On comparing these elements, no doubt could be entertained that they belonged to one and the same body; and since the interval between the successive returns was seen to be 75 or 76 years, Halley ventured to predict that it would again return in

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399. The return of Halley's comet in 1835, was looked for with no less interest than in 1759. Several of the most accurate mathematicians of the age had calculated its elements with inconceivable labor. Their zeal was rewarded by the appearance of the expected visitant at the time and place assigned; it traversed the northern sky presenting the very appearances, in most respects, that had been anticipated; and came to its perihelion on the 16th of November, within two days of the time prescribed by Pontecoulant, a French mathematician who had, it appeared, made the most successful calculation.* On its previous return, it was deemed an extraordinary achievement to have brought the prediction within a month of the actual time.

Many circumstances conspired to render this return of Halley's comet an astronomical event of transcendent interest. Of all the celestial bodies, its history was the most remarkable; it afforded most triumphant evidence of the truth of the doctrine of universal gravitation, and of course of the received laws of astronomy; and it inspired new confidence in the power of that instrument, (the Calculus,) by means of which its elements had been investigated.

400. Encke's comet, by its frequent returns, affords peculiar facilities for ascertaining the laws of its revolution; and it has kept the appointments made for it, with great exactness. On its late return (1839) it exhibited to the telescope a globular mass of nebulous matter, resembling fog, and moved towards its perihelion with great rapidity.

But what has made Encke's comet particularly famous, is its having first revealed to us the existence of a *Resisting Medium* in the planetary spaces. It has long been a question whether the earth and planets revolve in a perfect void, or whether a fluid of extreme rarity may not be diffused through space. A perfect vacuum was deemed most probable, because no such effects on the motions of the planets could be detected as indicated that they encountered a resisting medium. But a feather or a lock of cotton propelled with great velocity, might render obvious the

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* See Professor Loomis's Observations on Halley's Comet, *Amer. Jour. Science*, 30. 209.

resistance of a medium which would not be perceptible in the motions of a cannon ball. Accordingly, Encke's comet is thought to have plainly suffered a retardation from encountering a resisting medium in the planetary regions. The effect of this resistance, from the first discovery of the comet to the present time, has been to diminish the time of its revolution about two days. Such a resistance by destroying a part of the projectile force, would cause the comet to approach nearer to the sun, and thus to have its periodic time shortened. The ultimate effect of this cause will be to bring the comet nearer to the sun at every revolution, until it finally falls into that luminary, although many thousand years will be required to produce this catastrophe.* It is conceivable, indeed, that the effects of such a resistance may be counteracted by the attraction of one or more of the planets near which it may pass in its successive returns to the sun.

401. It is peculiarly interesting to see a portion of matter of a tenuity exceeding the thinnest fog, pursuing its path in space, in obedience to the same laws as those which regulate such large and heavy bodies as Jupiter or Saturn. In a perfect void, a speck of fog if propelled by a suitable projectile force would revolve around the sun, and hold on its way through the widest orbit, with as sure and steady a pace as the heaviest and largest bodies in the system.

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proached 166 times nearer the sun than the earth, being only 130,000 miles from the surface of the sun.* The heat which it must have received, was estimated to be equal to 28,000 times that which the earth receives in the same time and 2000 times hotter than red hot iron. This temperature would be sufficient to volatalize the most obdurate substances, and to expand the vapor to vast dimensions; and the opposite effects of the extreme cold to which it would be subject in the regions remote from the sun, would be adequate to condense it into its former volume.

This explanation however, does not account for the direction of the tail, extending as it usually does, only in a line opposite to the sun. Some writers therefore, as Delambre, suppose that the nebulous matter of the comet after being expanded to such a volume, that the particles are no longer attracted to the nucleus unless by the slightest conceivable force, are carried off in a direction from the sun, by the impulse of the solar rays themselves.† But to assign such a power of communicating motion to the sun's rays while they have never been proved to have any momentum, is unphilosophical; and we are compelled to place the phenomena of comets' tails among the points of astronomy yet to be explained.

403. Since those comets which have their perihelion very near the sun, like the comet of 1680, cross the orbits of all the planets, *the possibility that one of them may strike the earth*, has frequently been suggested. Still it may quiet our apprehensions on this subject, to reflect on the vast extent of the planetary spaces, in which these bodies are not crowded together as we see them erroneously represented in orreries and diagrams, but are sparsely scattered at immense distances from each other. They are like insects flying in the expanse of heaven. If a comet's tail lay with its axis in the plane of the ecliptic when it was near the sun, we can imagine that the tail might sweep over the earth; but the tail may be situated at any angle with the ecliptic as well as in the same plane with it, and the chances that it will not be in the same plane, are almost infinite. It is also

* See Principia, Lib. III, 41.

† Delambre's Astronomy, t. 3, p. 401.

extremely improbable that a comet will cross the plane of the ecliptic precisely at the earth's path in that plane, since it may as probably cross it at any other point, nearer or more remote from the sun. Still some comets have occasionally approached near to the earth. Thus Biela's comet in returning to the sun in 1832, crossed the ecliptic very near to the earth's track, and had the earth been then at that point of its orbit, it might have passed through a portion of the nebulous atmosphere of the comet. The earth was within a month of reaching that point. This might at first view seem to involve some hazard; yet we must consider that a month short implied a distance of nearly 50,000,000 miles. La Place has assigned the consequences that would ensue in case of a direct collision between the earth and a comet;* but terrible as he has represented them on the supposition that the nucleus of the comet is a solid body, yet considering a comet (as most of them doubtless are) as a mass of exceedingly light nebulous matter, it is not probable, even were the earth to make its way directly through a comet, that a particle of the comet would reach the earth. The portions encountered by the earth, would be arrested by the atmosphere, and probably inflamed; and they would perhaps exhibit on a more magnificent scale than was ever before observed, the phenomena of shooting stars, or meteoric showers.

* Syst. du Monde, l. iv, c. 4.

PART III.—OF THE FIXED STARS AND SYSTEM OF THE WORLD.

CHAPTER I.

OF THE FIXED STARS—CONSTELLATIONS.

404. THE FIXED STARS are so called, because, to common observation, they always maintain the same situations with respect to one another.

The stars are classed, by their apparent *magnitudes*. The whole number of magnitudes recorded are *sixteen*, of which the first six only are visible to the naked eye; the rest are *telescopic stars*. These magnitudes are not determined by any very definite scale, but are merely ranked according to their relative degrees of brightness, and this is left in a great measure to the decision of the eye alone, although it would appear easy to measure the comparative degree of light in a star by a photometer, and upon such measurement to ground a more scientific classification of the stars. The brightest stars to the number of 15 or 20 are considered as stars of the *first* magnitude; the 50 or 60 next brightest, of the *second* magnitude; the next 200 of the *third* magnitude; and thus the number of each class increases rapidly as we descend the scale, so that no less than fifteen or twenty thousand are included within the first seven magnitudes.

405. The stars have been grouped in *Constellations* from the most remote antiquity: a few, as Orion, Bootes, and Ursa Major, are mentioned in the most ancient writings under the same names as they bear at present. The names of the constellations are sometimes founded on a supposed resemblance to the objects to which the names belong; as the Swan and the Scorpion were evidently so denominated from their likeness to those animals;

but in most cases it is impossible for us to find any reason for designating a constellation by the figure of the animal or the hero which is employed to represent it. These representations were probably once blended with the fables of pagan mythology. The same figures, absurd as they appear, are still retained for the convenience of reference; since it is easy to find any particular star, by specifying the part of the figure to which it belongs, as when we say a star is in the neck of Taurus, in the knee of Hercules, or in the tail of the Great Bear. This method furnishes a general clue to its position; but the stars belonging to any constellation are distinguished according their apparent magnitudes as follows:—first, by the Greek letters, Alpha, Beta, Gamma, &c. Thus α *Orionis*, denotes the largest star in Orion, β *Andromedæ*, the second star in Andromeda, and γ *Leonis*, the third brightest star in the Lion. Where the number of the Greek letters is insufficient to include all the stars in a constellation, recourse is had to the letters of the Roman alphabet, a, b, c, &c.; and, in cases where these are exhausted, the final resort is to numbers. This is evidently necessary, since the largest constellations contain many hundreds or even thousands of stars. *Catalogues* of particular stars have also been published by different astronomers, each author numbering the individual stars embraced in his list, according to the places they respectively occupy in the catalogue. These references to particular catalogues are sometimes entered on large celestial globes. Thus we meet with a star [redacted]

much greater than this; but it is found that the catalogue of Hipparchus, embraces nearly all that can now be seen in the same latitude, and that on the equator, when the spectator has the northern and southern hemispheres both in view, the number of stars that can be counted does not exceed 3000. A careless view of the firmament in a clear night, gives us the impression of an infinite multitude of stars; but when we begin to count them, they appear much more sparsely distributed than we supposed, and large portions of the sky appear almost destitute of stars.

By the aid of the telescope, new fields of stars present themselves of boundless extent; the number continually augmenting as the powers of the telescope are increased. Lalande, in his Histoire Celesté, has registered the positions of no less than 50,000; and the whole number visible in the largest telescopes amount to many millions.

407. It is strongly recommended to the learner to acquaint himself with the leading constellations at least, and with a few of the most remarkable individual stars. The task of learning them is comparatively easy, and hardly any kind of knowledge, attained with so little labor, so amply rewards the possessor. It will generally be advisable, at the outset, to get some one already acquainted with the stars, to point out a few of the most conspicuous constellations, those of the Zodiac for example; the learner may then resort to a celestial globe,* and fill up the outline by tracing out the principal stars in each constellation as there laid down. By adding one new constellation to his list every night, and reviewing those already acquired, he will soon become familiar with the stars, and will greatly augment his interest and improve his intelligence in celestial observations and practical astronomy.

CONSTELLATIONS.

408. We will point out particular marks by which the leading constellations may be recognized, leaving it to the learner, af-

* For the method of rectifying the globe so as to represent the appearance of the heavens on any particular evening, see page 26, Prob. 76.

ter he has found a constellation, to trace out additional members of it by the aid of the celestial globe, or by maps of the stars. Let us begin with the *Constellations of the Zodiac*, which succeeding each other as they do in a known order, are most easily found.

Aries (The Ram) is a small constellation, known by two bright stars which form his head, α and β *Arietis*. These two stars are three degrees* apart, and directly south of β at the distance of one degree, is a smaller star, γ *Arietis*. It has been already intimated (Art. 193) that the vernal equinox probably was near the head of Aries, when the signs of the Zodiac received the present names.

Taurus (The Bull) will be readily found by the seven stars or *Pleiades*, which lie in his neck. The largest star in Taurus is *Aldebaran*, in the Bull's eye, a star of the first magnitude, of a reddish color somewhat resembling the planet Mars. Aldebaran and four other stars in the face of Taurus, compose the *Hyades*.

Gemini (The Twins) is known by two very bright stars, Castor and Pollux, four degrees asunder. Castor (the northern) is of the first, and Pollux of the second magnitude.

Cancer (The Crab). There are no large stars in this constellation, and it is regarded as less remarkable than any other in the Zodiac. It contains however an interesting group of small stars, called *Præsepe* or the Nebula of Cancer, which resembles a comet,

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VIRGO (The VIRGIN) extends a considerable way from west to east, but contains only a few bright stars. *Spica*, however is a star of the first magnitude, and lies very near the place of the autumnal equinox. Four degrees eastward of Spica, and six degrees south of Denebola, is *Vindemiatrix*, in the head of Virgo, a star of the third magnitude.

LIBRA (The BALANCE) is distinguished by three large stars, of which the two brightest constitute the beam of the balance, and the smallest forms the top or handle.

SCORPIO (The SCORPION) is one of the finest of the constellations. His head is formed of five bright stars arranged in the arc of a circle, which is crossed in the center by the ecliptic nearly at right angles, near the brightest of the five, β *Scorpionis*. Four degrees southeast of this, is a remarkable star of the first magnitude, of a reddish color, called *Cor Scorpionis*, or *Antares*. South of this a succession of bright stars sweep round towards the east, terminating in several small stars, forming the tail of the Scorpion.

SAGITTARIUS (The ARCHER). Northeast of the tail of the Scorpion, are three stars in the arc of a circle which constitute the *bow* of the Archer, the central star being the brightest, directly west of which is a bright star which forms the *arrow*.

CAPRICORNUS (The GOAT) lies northeast of Sagittarius, and is known by two bright stars, two degrees apart, which form the head.

AQUARIUS (The WATER BEARER) is recognized by two stars in a line with α *Capricorni*, forming the shoulders of the figure. These two stars are 10° apart, and 3° southeast is a third star, which together with the other two, makes an acute triangle, of which the westernmost is the vertex.

PISCES (The FISHES) lie between Aquarius and Aries. They are not distinguished by any large stars, but are connected by a series of small stars, that form a crooked line between them. *Piscis Australis*, the Southern Fish, lies directly below Aquarius, and is known by a single bright star far in the south, having a declination of 30°. The name of this star is *Fomalhaut*, and is much used in astronomical measurements.

409. The Constellations of the Zodiac, being first well learned, so as to be readily recognized, will facilitate the learning of others that lie north and south of them. Let us therefore next review the principal *Northern Constellations*, beginning north of Aries and proceeding from west to east.

ANDROMEDA, is characterised by three stars of the second magnitude, situated in a straight line, extending from west to east. The middle star is about 17° north of *β Arietis*. It is in the girdle of Andromeda, and is named *Mirach*. The other two lie at about equal distances, 14° west and east of Mirach. The western star, in the head of Andromeda, lies in the Equinoctial Colure. The eastern star, *Alamak*, is situated in the foot.

PERSEUS lies directly north of the Pleiades, and contains several bright stars. About 18° from the Pleiades is *Algol*, a star of the second magnitude, in the Head of Medusa, which forms a part of the figure; and 8° north of Algol is *Algenib*, of the same magnitude in the breast of Perseus. Between Algenib and the Pleiades are three bright stars, at nearly equal intervals, which compose the right leg of Perseus.

AURIGA (the WAGONER) lies directly east of Perseus, and extends nearly parallel to that constellation from north to south. *Capella* a very white and beautiful star of the first magnitude, distinguishes this constellation. The feet of Auriga are near the Bull's Horns.

The LYNX comes next, but presents nothing particularly inter- [redacted]

CORONA BOREALIS (The CROWN) which is situated to the N. E. of Bootes, is very easily recognized, composed as it is of a semicircle of bright stars. In the center of the bright crown, is a star of the second magnitude, called *gemma;* the remaining stars are all much smaller.

HERCULES, lying between the Crown on the west and the Lyre on the east, is very thick set with stars, most of which are quite small. The Constellation covers a great extent of the sky, especially from N. to S., the head terminating within 15° of the equator, and marked by a star of the third magnitude, called *Rasalgethi*, which is the largest in the Constellation.

OPHIUCUS is situated directly south of Hercules, extending some distance on both sides of the equator, the feet resting on the Scorpion. The head terminates near the head of Hercules, and like that, is marked by a bright star within 5° of *α Herculis.* Ophiucus is represented as holding in his hands the SERPENT, the head of which, consisting of three bright stars, is situated a little south of the Crown. The folds of the serpent will be easily followed by a succession of bright stars which extend a great way to the east.

AQUILA (The EAGLE) is conspicuous for three bright stars in its neck, of which the central one, *Altair*, is a very brilliant white star of the first magnitude. *Antinous* lies directly south of the Eagle, and north of the head of Capricornus.

DELPHINUS (The DOLPHIN) is a small but beautiful Constellation, a few degrees east of the Eagle, and is characterized by four bright stars near to one another, forming a small rhombic square. Another star of the same magnitude 5° south, makes the tail.

PEGASUS lies between Aquarius on the south and Andromeda on the north. It contains but few large stars. A very regular square of bright stars is composed of *α Andromedæ*, and the three largest stars in Pegasus, namely, *Scheat*, *Markab*, and *Algenib.* The sides composing this square are each about 15°. Algenib is situated in the equinoctial colure.

410. We may now review the *Constellations which surround the North Pole*, within the circle of perpetual apparition. (Art. 54.)

Ursa Minor (The Little Bear) lies nearest the pole. The Pole-star, *Polaris*, is in the extremity of the tail, and is of the third magnitude. Three stars in a straight line 4° or 5° apart, commencing with the Pole-star, lead to a trapezium of four stars, and the whole seven form together a *dipper*, the trapezium being the body, and the three stars the handle.

Ursa Major (The Great Bear) is situated between the pole and the Lesser Lion, and is usually recognized by the figure of a larger and more perfect dipper, which constitutes the hinder part of the animal. This has also seven stars, four in the body of the dipper, and three in the handle. All these are stars of much celebrity. The two in the western side of the dipper, α and β, are called *Pointers*, on account of their always being in a right line with the Pole-star, and therefore affording an easy mode of finding that. The first star in the tail, next the body, is named *Alioth*, and the second *Mizar*. The head of the Great Bear lies far to the westward of the Pointers, and is composed of numerous small stars; and the feet are severally composed of two small stars very near to each other.

Draco (The Dragon) winds round between the Great and Little Bear; and commencing with the tail, between the Pointers and the Pole-star, it is easily traced by a succession of bright stars extending from west to east, passing under Ursa Minor, it returns westward, and terminates in a triangle which forms the head of Draco, near the feet of Hercules, northwest of Lyra.

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LYRA (The LYRE) is directly west of the Swan, and is easily distinguished by a beautiful white star of the first magnitude, α *Lyræ*.

411. The *Southern Constellations* are comparatively few in number. We shall notice only the Whale, Orion, the Greater and Lesser Dog, Hydra, and the Crow.

CETUS (The WHALE) is distinguished rather for its extent than its brilliancy, reaching as it does through 40° of longitude, while none of its stars except one, are above the third magnitude. *Menkar* (α Ceti) in the mouth, is a star of the second magnitude, and several other bright stars directly south of Aries, mark the head and neck of the Whale. *Mira* (*o Ceti*) in the neck of the Whale, is a variable star.

ORION is one of the largest and most beautiful of the constellations, lying southeast of Taurus. A cluster of small stars form the head; two large stars, *Betalgeus* of the first and *Bellatrix* of the second magnitude, make the shoulders; three more bright stars compose the buckler, and three the sword; and *Rigel*, another star of the first magnitude, makes one of the feet. In this Constellation there are 70 stars plainly visible to the naked eye, including two of the first magnitude, four of the second, and three of the third.

CANIS MAJOR lies S. E. of Orion, and is distinguished chiefly by its containing the largest of the fixed stars, *Sirius*.

CANIS MINOR a little north of the equator, between Canis Major and Gemini, is a small Constellation, consisting chiefly of two stars, of which *Procyon* is of the first magnitude.

HYDRA has its head near Procyon, consisting of a number of stars of ordinary brightness. About 15° S. E. of the head, is a star of the second magnitude, forming the heart, (*Cor Hydræ*); and eastward of this, is a long succession of stars of the fourth and fifth magnitudes composing the body and the tail, and reaching as far as a few degrees south of Spica Virginis.

CORVUS (The CROW) is represented as standing on the tail of Hydra. It consists of small stars, only three of which are as large as the third magnitude.

412. The foregoing brief sketch is designed merely to aid the student in *finding* the principal constellations and the largest fixed stars. When we have once learned to recognize a constellation by some characteristic marks, we may afterwards fill up the outline by the aid of a celestial globe or a map of the stars. It will be of little avail however, merely to commit this sketch to memory; but it will be very useful for the student at once to render himself familiar with it, from the actual specimens which every clear evening presents to his view.

CHAPTER II.

CLUSTERS OF STARS—NEBULÆ—VARIABLE STARS—TEMPORARY STARS—DOUBLE STARS.

413. In various parts of the firmament are seen large groups or *clusters*, which, either by the naked eye, or by the aid of the smallest telescope, are perceived to consist of a great number of small stars. Such are the Pleiades, Coma Berenices, and Præsepe or the Bee-hive, in Cancer. The Pleiades, or *Seven Stars*, as they are called, in the neck of Taurus, is the most conspicuous cluster. When we look *directly* at this group, we cannot distinguish [redacted]

414. *Nebulæ* are those faint misty appearances which resemble comets, or a small speck of fog. The Galaxy or Milky Way presents a continued succession of large nebulæ. A very remarkable Nebula, visible to the naked eye, is seen in the girdle of Andromeda. No powers of the telescope have been able to resolve this into separate stars. Its dimensions are astonishingly great. In diameter it is about 15′. The telescope reveals to us innumerable objects of this kind. Sir William Herschel has given catalogues of 2000 Nebulæ, and has shown that the nebulous matter is distributed through the immensity of space in quantities inconceivably great, and in separate parcels of all shapes and sizes, and of all degrees of brightness between a mere milky appearance and the condensed light of a fixed star. Finding that the gradations between the two extremes were tolerably regular, he thought it probable that the nebulæ form the materials out of which nature elaborates suns and systems; and he conceived that, in virtue of a central gravitation, each parcel of nebulous matter becomes more and more condensed, and assumes a rounder form. He infers from the eccentricity of its shape, and the effects of the mutual gravitation of its particles, that it acquires gradually a rotary motion; that the condensation goes on increasing until the mass acquires consistency and solidity, and all the other characters of a comet or a planet; that by a still further process of condensation, the body becomes a real star, self-shining; and that thus the waste of the celestial bodies, by the perpetual diffusion of their light, is continually compensated and restored by new formations of such bodies, to replenish forever the universe with planets and stars.*

415. These opinions are recited here rather out of respect to their notoriety and celebrity, than because we suppose them to be founded on any better evidence than conjecture. The Philosophical Transactions for many years, both before and after the commencement of the present century, abound with both the observations and speculations of Sir William Herschel. The former are deserving of all praise; the latter of very little confidence.

* Phil. Trans. 1811.

Changes, however, are going on in some of the nebulæ, which plainly show that they are not, like planets and stars, fixed and permanent creations. Thus the great nebula in the girdle of Andromeda, has very much altered its structure since it first became an object of telescopic observation.* Many of the nebulæ are of a globular form, (Fig. 72, *a*) but frequently they present the appear-

(Fig. 72, *a*.) (Fig. 72, *b*.)

ance of a rapid increase of numbers towards the center, (Fig. 72, *b*) the exterior boundary being irregular, and the central parts more nearly spherical.

416. The Nebula in the sword of Orion is particularly celebrated, being very large and of a peculiarly interesting appearance. According to Sir John Herschel, its nebulous character is very different from what might be supposed to arise from the assemblage of an immense collection of small stars. It is formed of little flocculent masses like wisps of clouds; and such wisps seem to adhere to many small stars at its outskirts, and especially to one considerable star which it envelops with a nebulous atmos-

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oval figure; and in some instances, the nebula consists of a long, narrow spindle-shaped ray, tapering away at both ends to points.

Annular Nebulæ also exist, but are among the rarest objects in the heavens. The most conspicuous of this class, is to be found exactly half way between the stars β and γ Lyræ, and may be seen with a telescope of moderate power.*

Planetary Nebulæ constitute another variety, and are very remarkable objects. They have, as their name imports, exactly the appearance of planets. Whatever may be their nature, they must be of enormous magnitude. One of them is to be found in the parallel of γ Aquarii, and about 5m. preceding that star. Its apparent diameter is about 20″. Another in the Constellation Andromeda, presents a visible disk of 12″, perfectly defined and round. Granting these objects to be equally distant from us with the stars, their real dimensions must be such as, on the lowest computation, would fill the orbit of Uranus. It is no less evident that, if they be solid bodies, of a solar nature, the intrinsic splendor of their surfaces must be almost infinitely inferior to that of the sun. A circular portion of the sun's disk, subtending an angle of 20″, would give a light equal to 100 full moons; while the objects in question are hardly, if at all, discernible with the naked eye.†

418. The *Galaxy* or *Milky Way* is itself supposed by some to be a nebula of which the sun forms a component part; and hence it appears so much greater than other nebulæ only in consequence of our situation with respect to it, and its greater proximity to our system. So crowded are the stars in some parts of this zone, that Sir William Herschel, by counting the stars in a single field of his telescope, estimated that 50,000 had passed under his review in a zone two degrees in breadth during a single hour's observation. Notwithstanding the apparent contiguity of the stars which crowd the galaxy, it is certain that their mutual distances must be inconceivably great.

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* A list of 288 bright nebulæ, with references to well known stars, near which they are situated, is given in the Edinburg Encyclopœdia, Art. *Astronomy*, p. 781. It is convenient for finding any required nebula.

† Sir J. Herschel.

419. Variable Stars are those which undergo a periodical change of brightness. One of the most remarkable is the star *Mira* in the Whale, (*o Ceti*). It appears once in 11 months, remains at its greatest brightness about a fortnight, being then, on some occasions, equal to a star of the second magnitude. It then decreases about three months, until it becomes completely invisible, and remains so about five months, when it again becomes visible, and continues increasing during the remaining three months of its period.

Another very remarkable variable star is *Algol* (β Persei). It is usually visible as a star of the second magnitude, and continues such for 2d. 14h. when it suddenly begins to diminish in splendor, and in about 3½ hours is reduced to the fourth magnitude. It then begins again to increase, and in 3½ hours more, is restored to its usual brightness, going through all its changes in less than three days. This remarkable law of variation appears strongly to suggest the revolution round it of some opake body, which, when interposed between us and Algol, cuts off a large portion of its light. It is (says Sir J. Herschel) an indication of a high degree of activity in regions where, but for such evidences, we might conclude all lifeless. Our sun requires almost nine times this period to perform a revolution on its axis. On the other hand, the periodic time of an opake revolving body, sufficiently large, which would produce a similar temporary obscuration of the sun, seen from a fixed star, would be less than fourteen hours.

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sented themselves. Thus the appearance of a star in 1572, was so sudden, that Tycho Brahe returning home one day was surprized to find a collection of country people gazing at a star which he was sure did not exist half an hour before. It was then as bright as Sirius, and continued to increase until it surpassed Jupiter when brightest, and was visible at mid-day. In a month it began to diminish, and in three months afterwards it had entirely disappeared.

It has been supposed by some that in a few instances, the same star has returned, constituting one of the periodical or variable stars of a long period.

Moreover, on a careful re-examination of the heavens, and a comparison of catalogues, many stars are now found to be missing.*

421. Double Stars are those which appear single to the naked eye, but are resolved into two by the telescope; or, if not visible to the naked eye, are seen in the telescope so close together as to be recognized as objects of this class. Sometimes three or more stars are found in this near connexion, constituting *triple* or *multiple* stars. Castor, for example, when seen by the naked eye, appears as a single star, but in a telescope even of moderate powers, it is resolved into two stars of between the third and fourth magnitudes, within 5″ of each other. These two stars are nearly of equal size, but more commonly one is exceedingly small in comparison with the other, resembling a satellite near its primary, although in distance, in light, and in other characteristics, each has all the attributes of a star, and the combination therefore cannot be that of a planet with a satellite. In most instances, also, the distance detween these objects is much less than 5″, and in many cases it is less than 1″. The extreme closeness, together with the exceeding minuteness of most of the double stars, requires the best telescopes united with the most acute powers of observation. Indeed, certain of these objects are regarded as the severest *tests* both of the excellence of the instruments, and of the skill of the observer.

* Sir J. Herschel.

422. Many of the double stars exhibit the curious and beautiful phenomena of contrasted or *complementary colors.** In such instances, the larger star is usually of a ruddy or orange hue, while the smaller one appears blue or green, probably in virtue of that general law of optics, which provides that when the retina is excited by any bright colored light, feebler lights which seen alone would produce no sensation but of whiteness, appear colored, with the tint complementary to that of the brighter. Thus a yellow color predominating in the light of the brighter star, that of the less bright one in the same field of view will appear blue; while, if the tint of the brighter star verges to crimson, that of the other will exhibit a tendency to green, or even under favorable circumstances, will appear as a vivid green. The former contrast is beautifully exhibited by ι Cancri, the latter by γ Andromedæ, both fine double stars. If, however, the colored star is much the less bright of the two, it will not materially affect the other. Thus for instance, η Cassiopeiæ exhibits the beautiful combination of a large white star, and a small one of a rich ruddy purple. It is by no means, however, intended to say, that in all such cases, one of the colors is the mere effect of contrast, and it may be easier suggested in words, than conceived in imagination, what variety of illumination *two suns*, a red and green, or a yellow and a blue sun, must afford a planet circulating about either; and what charming contrasts and "grateful vicissitudes," a red and green day for instance, alternating with a white one and

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423. Our knowledge of the double stars almost commenced with Sir William Herschel, about the year 1780. At the time he began his search for them, he was acquainted with only *four*. Within five years, he discovered nearly 700 double stars.* In his memoirs published in the Philosophical Transactions,† he gave most accurate measurements of the distances between the two stars, and of the angle which a line joining the two, formed with the parallel of declination.‡ These data would enable him, or at least posterity, to judge whether these minute bodies ever change their position with respect to each other.

Since 1821, these researches have been prosecuted with great zeal and industry by Sir James South and Sir John Herschel in England, and by Professor Struve at Dorpat in Russia, and the whole number of double stars now known, amounts to several thousands.§ Two circumstances add a high degree of interest to the phenomena of the double stars,—the first is, that a few of them at least are found to have a revolution around each other, and the second, that they are supposed to afford the means of obtaining the parallax of the fixed stars. Of these topics we shall treat in the next chapter.

CHAPTER III.

MOTIONS OF THE FIXED STARS—DISTANCES—NATURE.

424. In 1803, Sir William Herschel first determined and announced to the world, that there exist among the stars, separate systems, composed of two stars revolving about each other in regular orbits. These he denominated *Binary Stars*, to distinguish them from other double stars where no such motion is detected, and whose proximity to each other may possibly arise from casual juxta-position, or from one being in the range of the

* During his life he observed in all, 2400 double stars.

† Phil. Trans. 1782—1785. ‡ Baily *Astron. Trans.* II, 542.

§ The Catalogue of Struve, contains 3063.

other. Between fifty and sixty instances of changes to a greater or less amount of the relative position of double stars, are mentioned by Sir William Herschel; and a few of them had changed their places so much within 25 years, and in such order, as to lead him to the conclusion that they performed revolutions, one around the other, in regular orbits.

425. These conclusions have been fully confirmed by later observers, so that it is now considered as fully established, that there exist among the fixed stars, binary systems, in which two stars perform to each other the office of sun and planet, and that the periods of revolution of more than one such pair have been ascertained with something approaching to exactness. Immersions and emersions of stars behind each other have been observed, and real motions among them detected rapid enough to become sensible and measurable in very short intervals of time.* The following table exhibits the present state of our knowledge on this subject.

Names.	Period in years.	Major axis of the orbit.	Eccentricity.
η Coronæ,	43.40	———	———
ζ Cancri,	55.00	———	———
ξ Ursæ Majoris,	58.26	7″.714	0.4164
70 Ophiuchi,	80.34	8.784	0.4667
Castor,	252.66	16.172	0.7582
σ Coronæ,	286.00	7.358	0.6112
61 Cygni,	452.00	30.860	———

[redacted]

of apparent distance, and rapid increase of angular motion about each other of the individuals composing it. It is a bright star of the fourth magnitude, and its component stars are almost exactly equal. It has been known to consist of two stars since the beginning of the eighteenth century, their distance being then between six and seven seconds; so that any tolerably good telescope would resolve it. Since that time, they have been constantly approaching, and are at present hardly more than a single second asunder; so that no telescope that is not of a very superior quality, is competent to show them otherwise than as a single star, somewhat lengthened in one direction. It fortunately happens that Bradley (Astronomer Royal) in 1718, noticed, and recorded in the margin of one of his observation books, the apparent direction of their line of junction, as being parallel to that of two remarkable stars α and δ of the same constellation, as seen by the naked eye,—a remark which has been of signal service in the investigation of their orbit. It is found that it passed its perihelion, August 18th, 1834, and that in the interval from 1839 to 1841, this star will have completed a full revolution from the epoch of the first measurement of its position in 1781; and the regularity with which it has maintained its motion, is said to have been exceedingly beautiful.*

426. The revolutions of the binary stars have assured us of that most interesting fact, that *the law of gravitation extends to the fixed stars.* Before these discoveries, we could not decide except by a feeble analogy that this law transcended the bounds of the solar system. Indeed, our belief of the fact rested more upon our idea of unity of design in all the works of the Creator, than upon any certain proof; but the revolution of one star around another in obedience to forces which must be similar to those that govern the solar system, establishes the grand conclusion, that the law of gravitation is truly the law of the material universe.

We have the same evidence (says Sir John Herschel) of the revolutions of the binary stars about each other, that we have of those of Saturn and Uranus about the sun; and the correspond-

* Astron. Trans. v, 35.

ence between their calculated and observed places in such elongated ellipses, must be admitted to carry with it a proof of the prevalence of the Newtonian law of gravity in their systems, of the very same nature and cogency as that of the calculated and observed places of comets round the center of our own system.

But (he adds) it is not with the revolutions of bodies of a planetary or cometary nature round a solar center that we are now concerned; it is with that of sun around sun, each, perhaps, accompanied with its train of planets and their satellites, closely shrouded from our view by the splendor of their respective suns, and crowded into a space, bearing hardly a greater proportion to the enormous interval which separates them, than the distances of the satellites of our planets from their primaries, bear to their distances from the sun itself.

427. *Some of the fixed stars appear to have a real motion in space.*

The *apparent* change of place in the stars arising from the precession of the equinoxes, the nutation of the earth's axis, the diminution of the obliquity of the ecliptic, and the aberration of light, have been already mentioned; but after all these corrections are made, changes of place still occur, which cannot result from any changes in the earth, but must arise from changes in the stars themselves. Such motions are called the *proper motions* of the stars. Nearly 2000 years ago, Hipparchus and Ptolemy made the most accurate determinations in their power of the relative

the apparent approach of the stars in the region which he is leaving, and the recession of those which lie in the part of the heavens towards which he is travelling. Were two groves of trees situated on a plain at some distance apart, and we should go from one to the other, the trees before us would gradually appear farther and farther asunder, while those we left behind would appear to approach each other. Some years since, Sir William Herschel supposed he had detected changes of this kind among two sets of stars in opposite points of the heavens, and announced that the solar system was in motion towards a point in the constellation Hercules;* but other astronomers have not found the changes in question such as would correspond to this motion, or to any motion of the sun; and while it is a matter of general belief that the sun has a motion in space, the fact is not considered as yet entirely proved.

429. In most cases where a proper motion in certain stars has been suspected, its annual amount has been so small, that many years are required to assure us, that the effect is not owing to some other cause than a real progressive motion in the stars themselves; but in a few instances the fact is too obvious to admit of any doubt. Thus the two stars 61 Cygni, which are nearly equal, have remained constantly at the same, or nearly at the same distance of 15″ for at least fifty years past. Meanwhile they have shifted their local situation in the heavens, 4′ 23″, the annual proper motion of each star being 5.″3, by which quantity this system is every year carried along in some unknown path, by a motion which for many centuries must be regarded as uniform and rectilinear. A greater proportion of the double stars than of any other indicate proper motions, especially the binary stars or those which have a revolution around each other. Among stars not double, and no way differing from the rest in any other obvious particular, μ Cassiopeiæ has the greatest proper motion of any yet ascertained, amounting to nearly 4″ annually.

DISTANCES OF THE FIXED STARS.

430. *We cannot ascertain the actual distance of any of the fixed stars, but can certainly determine that the nearest star is*

* Phil. Trans. 1783, 1805, and 1806.

more than (20,000,000,000,000,) *twenty billions of miles from the earth.*

For all measurements relating to the distances of the sun and planets, the *radius of the earth* furnishes the base line (Art. 87). The length of this line being known, and the horizontal parallax of the body, whose distance is sought, we readily obtain the distance by the solution of a right angled triangle. But any star viewed from the opposite sides of the earth, would appear from both stations, to occupy precisely the same situation in the celestial sphere, and of course it would exhibit no horizontal parallax.

But astronomers have endeavored to find a parallax in some of the fixed stars by taking the *diameter of the earth's orbit* as a base line. Yet even a change of position amounting to 190 millions of miles, proves insufficient to alter the place of a single star, from which it is concluded that the stars have not even any *annual parallax;* that is, the angle subtended by the semi-diameter of the earth's orbit, at the nearest fixed star is insensible. The errors to which instrumental measurements are subject, arising from the defects of the instruments themselves, from refraction, and from various other sources of inaccuracy, are such, that the angular determinations of arcs of the heavens cannot be relied on to less than 1″. But the change of place in any star when viewed at opposite extremities of the earth's orbit, is less than 1″, and therefore cannot be appreciated by direct measurement. It follows, that, when viewed from the nearest star, the diameter of the earth's orbit would be insensible: the spider line of the telescope would more than cover it.

431. Taking, however, the annual parallax of a fixed star at 1″, let *a b* (Fig. 73) represent the radius of the earth's orbit and *c* a fixed star, the angle at *c* being 1″ and the angle at *b* a right angle; then,

$$\text{Sin. } 1'' : \text{Rad.} :: 1 : 200{,}000, \text{ nearly.}$$

Hence the hypothenuse of a triangle whose vertical angle is 1″ is about 200,000 times the base; consequently the distance of the nearest fixed star *must exceed* $95000000 \times 200000 = 190000000 \times 100000$, or one hundred thousand times one hundred and ninety millions of miles. Of a distance so vast we can

Fig. 73.

form no adequate conceptions, and even seek to measure it only by the time that light, (which moves more than 192,000 miles per second and passes from the sun to the earth in 8m. 7sec.,) would take to traverse it, which is found to be more than three and a half years.

If these conclusions are drawn with respect to the largest of the fixed stars, which we suppose to be vastly nearer to us than those of the smallest magnitude, the idea of distance swells upon us when we attempt to estimate the remoteness of the latter. As it is uncertain, however, whether the difference in the apparent magnitudes of the stars is owing to a real difference or merely to their being at various distances from the eye, more or less uncertainty must attend all efforts to determine the relative distances of the stars; but astronomers generally believe that the lower orders of stars are vastly more distant from us than the higher. Of some stars it is said, that thousands of years would be required for their light to travel down to us.

432. We have said that the stars have no annual parallax; yet it may be observed that astronomers are not exactly agreed on this point. Dr. Brinkley, a late eminent Irish astronomer, supposed that he had detected an annual parallax in α Lyræ amounting to 1″.13 and in α Aquilæ of 1″.42.* These results were controverted by Mr. Pond of the Royal Observatory of Greenwich; and Mr. Struve of Dorpat has shown that in a number of cases, the parallax is *negative*, that is in a direction opposite to that which would arise from the motion of the earth. Hence it is considered doubtful whether in all cases of an apparent parallax, the effect is not wholly due to errors of observation.

433. Indirect methods have been proposed for ascertaining the parallax of the fixed stars by means of observations on the *double stars*. If the two stars composing a double star are at different distances from us, parallax would affect them unequally, and change their relative position with respect to each other; and

* Phil. Trans. 1821.

since the ordinary sources of error arising from the imperfection of instruments, from precession, nutation, aberration, and refraction, would be avoided, (since they would affect both objects alike, and therefore would not disturb their relative positions,) measurements taken with the micrometer of changes much less than $1''$ may be relied on. Sir John Herschel proposes a method* by which changes may be determined which amount to only $\frac{1}{40}$ of a second.†

434. The immense distance of the fixed stars is inferred also from the fact that the largest telescopes do not increase their apparent magnitude. They are still points, when viewed with the highest magnifiers, although they sometimes present a spurious disk, which is owing to irradiation.‡

NATURE OF THE STARS.

435. *The stars are bodies greater than our earth.* If this were not the case they could not be visible at such an immense distance. Dr. Wollaston, a distinguished English philosopher, attempted to estimate the magnitudes of certain of the fixed stars from the light which they afford. By means of an accurate photometer (an instrument for measuring the relative intensities of light) he compared the light of Sirius with that of the sun. He next inquired how far the sun must be removed from us in

give out twice as much light as the sun; or that, in point of splendor, Sirius must be at least equal to two suns. Indeed, he has rendered it probable that the light of Sirius is equal to fourteen suns.

436. *The fixed stars are suns.* We have already seen that they are large bodies; that they are immensely farther off than the farthest planet; that they shine by their own light; in short, that their appearance is, in all respects, the same as the sun would exhibit if removed to the region of the stars. Hence we infer that they are bodies of the same kind with the sun.

437. We are justified therefore by a sound analogy, in concluding that the stars were made for the same end as the sun, namely, as the centers of attraction to other planetary worlds, to which they severally dispense light and heat. Although the starry heavens present, in a clear night, a spectacle of ineffable grandeur and beauty, yet it must be admitted that the chief purpose of the stars, could not have been to adorn the night, since by far the greatest part of them are wholly invisible to the naked eye; nor as landmarks to the navigator, for only a very small proportion of them are adapted to this purpose; nor, finally, to influence the earth by their attractions, since their distance renders such an effect entirely insensible. If they are suns, and if they exert no important agencies upon our world, but are bodies evidently adapted to the same purpose as our sun, then it is as rational to suppose that they were made to give light and heat, as that the eye was made for seeing and the ear for hearing. It is obvious to inquire next, to what they dispense these gifts if not to planetary worlds; and why to planetary worlds, if not for the use of percipient beings? We are thus led, almost inevitably, to the idea of a *Plurality of Worlds*; and the conclusion is forced upon us, that the spot which the Creator has assigned to us is but a humble province of his boundless empire.*

* See this argument, in its full extent, in *Dick's Celestial Scenery*.

CHAPTER III.

OF THE SYSTEM OF THE WORLD.

438. *The arrangement of all the bodies that compose the material universe, and their relations to each other, constitute the System of the World.*

It is otherwise call the Mechanism of the Heavens; and indeed in the System of the world, we figure to ourselves a machine, all the parts of which have a mutual dependence, and conspire to one great end. " The machines that are first invented (says Adam Smith) to perform any particular movement, are always the most complex; and succeding artists generally discover that with fewer wheels and with fewer principles of motion than had originally been employed, the same effects may be more easily produced. The first systems, in the same manner, are always the most complex; and a particulår connecting chain or principle is generally thought necessary to unite every two seemingly disjointed appearances; but it often happens, that *one great connecting principle* is afterwards found to be sufficient, to bind together all the discordant phenomena that occur in a whole species of things." This remark is strikingly applicable to the origin and progress of systems of astronomy.

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Hesiod and Homer; and the "sweet influences of the Pleiades" and the "bands of Orion," are beautifully alluded to in the book of Job.

2. *Eclipses.* Pythagoras knew both the causes of eclipses and how to predict them;* not indeed in the accurate manner now employed, but by means of the Saros (Art. 233).

3. Pythagoras had divined the true system of the world, holding that the sun and not the earth, (as was generally held by the ancients, even for many ages after Pythagoras,) is the center around which all the planets revolve, and that the stars are so many suns, each the center of a system like our own.† Among lesser things, he knew that the earth is round; that its surface is naturally divided into five Zones; and that the ecliptic is inclined to the equator. He also held that the earth revolves daily on its axis, and yearly around the sun; that the galaxy is an assemblage of small stars; and that it is the same luminary, namely, Venus, that constitutes both the morning and the evening star, whereas all the ancients before him had supposed that each was a separate planet, and accordingly the morning star was called Lucifer, and the evening star Hesperus.‡ He held also that the planets were inhabited, and even went so far as to calculate the size of some of the animals in the moon.§ Pythagoras was so great an enthusiast in music, that he not only assigned to it a conspicuous place in his system of education, but even supposed the heavenly bodies themselves to be arranged at distances corresponding to the diatonic scale, and imagined them to pursue their sublime march to notes created by their own harmonious movements, called the "music of the spheres;" but he maintained that this celestial concert, though loud and grand, is not audible to the feeble organs of man, but only to the gods.

440. With few exceptions, however, the opinions of Pythagoras on the System of the World, were founded in truth. Yet they were rejected by Aristotle and by most succeeding astronomers down to the time of Copernicus, and in their place was

* Long's Astronomy, 2, 671.

† Library of Useful Knowledge, *History of Astronomy.*

‡ Long's Ast. 2. 673. § Ed. Encyclopædia.

substituted the doctrine of *Crystalline Spheres*, first taught by Eudoxus. According to this system, the heavenly bodies are set like gems in hollow solid orbs, composed of crystal so pellucid that no anterior orb obstructs in the least the view of any of the orbs that lie behind it. The sun and the planets have each its separate orb; but the fixed stars are all set in the same grand orb; and beyond this is another still, the *Primum Mobile*, which revolves daily from east to west, and carries along with it all the other orbs. Above the whole, spreads the *Grand Empyrean*, or third heavens, the abode of perpetual serenity.*

To account for the planetary motions, it was supposed that each of the planetary orbs as well as that of the sun, has a motion of its own eastward, while it partakes of the common diurnal motion of the starry sphere. Aristotle taught that these motions are effected by a tutelary genius of each planet, residing in it, and directing its motions, as the mind of man directs his motions.

441. On coming down to the time of Hipparchus, who flourished about 150 years before the Christian era, we meet with astronomers who acquired far more accurate knowledge of the celestial motions. Hipparchus was in possession of instruments for measuring angles, and knew how to resolve spherical triangles. He ascertained the length of the year within 6m. of the truth. He discovered the eccentricity of the solar orb, (although he supposed the sun actually to move uniformly in a circle, but the earth to be placed out of the center,) and the positions of the sun's apogee and perigee. He formed very accurate estimates of the obliquity of the ecliptic and of the precession of the equinoxes. He computed the exact period of the synodic revolution of the moon, and the inclination of the lunar orbit; discovered the motion of her node and of her line of apsides; and made the first attempts to ascertain the horizontal parallaxes of the sun and moon.

Such was the state of astronomical knowledge when Ptolemy wrote the Almagest, in which he has transmitted to us an encyclopædia of the astronomy of the ancients.

* Long's Ast. 2. 640—Robinson's Mech. Phil. 2. 83—Gregory's Ast. 132—Playfair's Dissertation, 118.

442. The systems of the world which have been most celebrated are three—the Ptolemaic, the Tychonic, and the Copernican. We shall conclude this part of our work with a concise statement and discussion of each of these systems of the Mechanism of the Heavens.

THE PTOLEMAIC SYSTEM.

443. The doctrines of the Ptolemaic System were not originated by Ptolemy, but being digested by him out of materials furnished by various hands, it has come down to us under the sanction of his name.

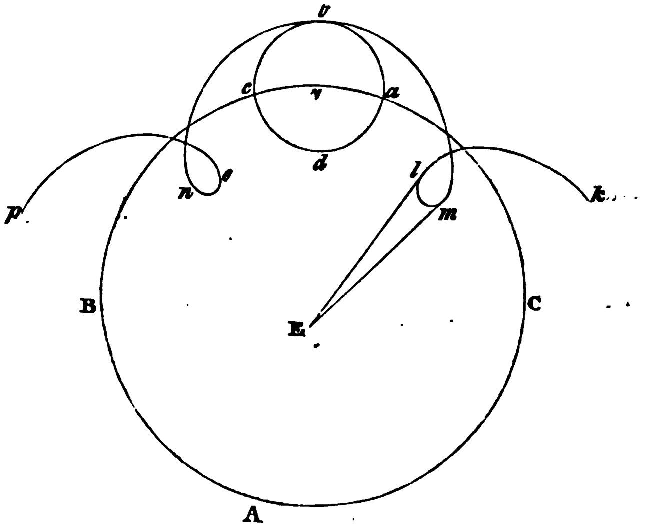
According to this system, the earth is the center of the universe, and all the heavenly bodies daily revolve around it from east to west. In order to explain the planetary motions, Ptolemy had recourse to *deferents* and *epicycles*,—an explanation devised by Apollonius one of the greatest geometers of antiquity.* He conceived that, in the circumference of a circle, having the earth for its center, there moves the center of another circle, in the circumference of which the planet actually revolves. The circle surrounding the earth was called the *deferent*, while the smaller circle whose center was always in the periphery of the deferent, was called the *epicycle*. The motion in each was supposed to be uniform. Lastly, it was conceived that the motion of the center of the epicycle in the circumference of the deferent, and of the planet in that of the epicycle, are in opposite directions, the first being towards the east, and the second towards the west.

444. But these views will be better understood from a diagram. Therefore, let ABC (Fig. 74,) represent the *deferent*, E being the earth a little out of the center. Let *abc* represent the *epicycle*, having its center at *v*, on the periphery of the deferent. Conceive the circumference of the deferent to be carried about the earth every twenty four hours in the order of the letters; and at the same time, let the center *v* of the epicycle *abcd*, have a slow motion in the opposite direction, and let a body revolve in this circle in

* Playfair, Dissertation Second, 119.

the direction *abcd*. Then it will be seen that the body would actually describe the looped curves *klmnop*; that it would appear

(Fig. 74.)



stationary at *l* and *m*, and at *n* and *o*; that its motion would be direct from *k* to *l*, and then retrograde from *l* to *m*; direct again from *m* to *n*, and retrograde from *n* to *o*.

445. Such a deferent and epicycle may be devised for each planet as will fully explain all its ordinary motions; but it is in-

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446. The *objections* to the Ptolemaic system, in general, are the following: First, it is a mere hypothesis, having no evidence in its favor, except that it explains the phenomena. This evidence is insufficient of itself, since it frequently happens that each of two hypotheses, directly opposite to each other, will explain all the known phenomena. But the Ptolemaic system does not even do this, as it is inconsistent with the phases of Mercury and Venus, as already observed. Secondly, now that we are acquainted with the distances of the remoter planets, and especially of the fixed stars, the *swiftness of motion* implied in a daily revolution of the starry firmament around the earth, renders such a motion wholly incredible. Thirdly, the *centrifugal force* that would be generated in these bodies, especially in the sun, renders it impossible that they can continue to revolve around the earth as a center.

These reasons are sufficient to show the absurdities of the Ptolemaic System of the World.

THE TYCHONIC SYSTEM.

447. Tycho Brahe, like Ptolemy, placed the earth in the center of the universe, and accounted for the diurnal motions in the same manner as Ptolemy had done, namely, by an actual revolution of the whole host of heaven around the earth every twenty four hours. But he rejected the scheme of deferents and epicycles, and held that the moon revolves about the earth as the center of her motions; that the sun, and not the earth, is the center of the planetary motions; and that the sun accompanied by the planets moves around the earth once a year, somewhat in the manner that we now conceive of Jupiter and his satellites as revolving around the sun.

448. The system of Tycho serves to explain all the common phenomena of the planetary motions, but it is encumbered with the same objections as those that have been mentioned as resting against the Ptolemaic system, namely, that it is a mere hypothesis; that it implies an incredible swiftness in the diurnal motions; and that it is inconsistent with the known laws of universal grav-

itation. But if the heavens do not revolve, the earth must, and this brings us to the system of Copernicus.

THE COPERNICAN SYSTEM.

449. Copernicus was born at Thorn in Prussia in 1473. The system that bears his name was the fruit of forty years of intense study and meditation upon the celestial motions. As already mentioned, (Art. 6,) it maintains (1) That the *apparent* diurnal motions of the heavenly bodies, from east to west is owing to the *real* revolution of the earth on its own axis from west to east; and (2) That the sun is the center around which the earth and planets all revolve from west to east. It rests on the following arguments:

450. First, *the earth revolves on its own axis.*

1. Because this supposition is vastly more *simple.*

2. It is agreeable to *analogy,* since all the other planets that afford any means of determining the question, are seen to revolve on their axes.

3. The *spheroidal figure* of the earth, is the figure of equilibrium, that results from a revolution on its axis.

4. The *diminished weight* of bodies at the equator, indicates a centrifugal force arising from such a revolution.

5. Bodies let fall from a high eminence, fall *eastward of their base,* indicating that when farther from the center of the earth they were subject to a greater velocity, which in consequence of their inertia, they do not entirely lose in descending to the lower level.*

451. Secondly, *the planets, including the earth, revolve about the sun.*

1. The *phases* of Mercury and Venus are precisely such, as would result from their circulating around the sun in orbits within that of the earth; but they are never seen in opposition, as they would be if they circulated around the earth.

2. The superior planets do indeed revolve around the earth; but they also revolve around the sun, as is evident from their

* Biot.

phases and from the known dimensions of their orbits; and that the sun and not the earth, is the *center* of their motions, is inferred from the greater symmetry of their motions as referred to the sun than as referred to the earth, and especially from the laws of gravitation which forbid our supposing that bodies so much larger than the earth, as some of these bodies are, can circulate permanently around the earth, the latter remaining all the while at rest.

3. The annual motion of *the earth* itself is indicated also by the most conclusive arguments. For, first, since all the planets with their satellites, and the comets, revolve about the sun, *analogy* leads us to infer the same respecting the earth and its satellite. Secondly, The motions of the satellites, as those of Jupiter and Saturn, indicate that it is a law of the solar system that the smaller bodies revolve about the larger. Thirdly, on the supposition that the earth performs an annual revolution around the sun, it is embraced along with the planets, in Kepler's law, that the squares of the times are as the cubes of the distances; otherwise, it forms an exception, and the only known exception to this law. Lastly, the aberration of light affords a sensible proof of the motion of the earth, since that phenomenon indicates both a progressive motion of light, and a motion of the earth from west to east. (Art. 195).

452. It only remains to inquire, whether there subsist higher orders of relations between the stars themselves.

The revolutions of the *binary stars* (Art. 424) afford conclusive evidence of at least subordinate systems of suns, governed by the same laws as those which regulate the motions of the solar system. The *nebulæ* also compose peculiar systems, in which the members are evidently bound together by some common relation.

In these marks of organization,—of stars associated together in clusters,—of sun revolving around sun,—and of nebulæ disposed in regular figures, we recognize different members of some grand system, links in one great chain that binds together all parts of the universe; as we see Jupiter and his satellites combined in one subordinate system, and Saturn and his satellites in another, —each a vast kingdom, and both uniting with a number of other individual parts to compose an empire still more vast.

453. This fact being now established, that the stars are immense bodies like the sun, and that they are subject to the laws of gravitation, we cannot conceive how they can be preserved from falling into final disorder and ruin, unless they move in harmonious concert like the members of the solar system. Otherwise, those that are situated on the confines of creation, being retained by no forces from without, while they are subject to the attraction of all the bodies within, must leave their stations, and move inward with accelerated velocity, and thus all the bodies in the universe would at length fall together in the common center of gravity. The immense distance at which the stars are placed from each other, would indeed delay such a catastrophe; but such must be the ultimate tendency of the material world, unless sustained in one harmonious system by nicely adjusted motions.* To leave entirely out of view our confidence in the wisdom and preserving goodness of the Creator, and reasoning merely from what we know of the stability of the solar system, we should be justified in inferring, that other worlds are not subject to forces which operate only to hasten their decay, and to involve them in final ruin.

We conclude, therefore, that the material universe is one great system; that the combination of planets with their satellites constitutes the first or lowest order of worlds; that next to these planets are linked to suns; that these are bound to other suns, composing a still higher order in the scale of being; and, finally, that all the different systems of worlds, move around their common center of gravity.

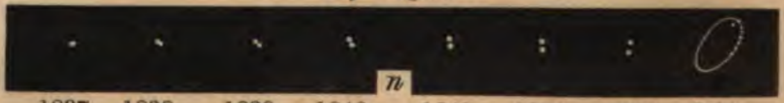
* Robison's Physical Astronomy.

PLATE I.

1. Castor. 2. γ Leonis. 3. 39 Drac. 4. λ Oph. 5. 11 Monoc. 6. ζ Cancri.

7. 4 & 5 ε Lyrae, et Debilissima. 8. σ Orionis.

9. γ Virginis.



1837. 1838. 1839. 1840. 1845. 1850. 1860. Orbit.

Fig. 10

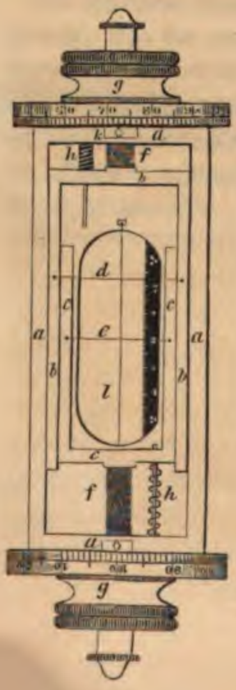
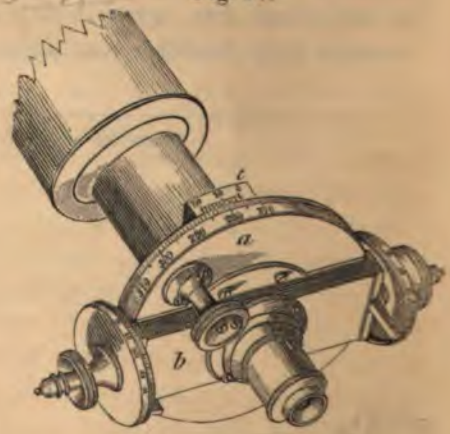


Fig. 11.



INTRODUCTION

TO

PRACTICAL ASTRONOMY,

DESIGNED AS A

SUPPLEMENT TO OLMSTED'S ASTRONOMY;

CONTAINING SPECIAL RULES FOR THE

ADJUSTMENT AND USE OF ASTRONOMICAL INSTRUMENTS,

TOGETHER WITH THE

CALCULATION OF ECLIPSES AND OCCULTATIONS,

AND THE METHODS OF FINDING THE

LATITUDE AND LONGITUDE.

BY EBENEZER PORTER MASON.

NEW YORK: COLLINS, KEESE, & Co.

PRINTED BY B. L. HAMLEN—NEW HAVEN, CONN.

1841.

EBENEZER PORTER MASON.

It imparts a melancholy interest to the following pages, that they constitute the last efforts of a youth of extraordinary promise, cut off just as he seemed entering upon a brilliant career of astronomical discovery. Before he had completed the present article, a fatal malady, the consumption, had marked him for its victim. When, only three weeks before his decease, he wrote at my desk the concluding paragraph, it was an interesting but moving spectacle, to witness the energies of a mind of the first order soaring above its crumbling tenement, and exhibiting 'the ruling passion strong in death.'

Immediately after completing this treatise, which he could not be persuaded to leave unfinished, he yielded to the solicitations of his relatives at Richmond, Virginia, who had for some time been urging him to hasten to that milder climate, with the hope of preserving, or at least of prolonging, his valuable life. In less than two weeks after he reached his friends, he experienced a sudden prostration, and quietly sunk into the arms of death. He died on the morning of Dec. 26th, in the twenty second year of his age. He was a native of Washington, Ct., where his father the Rev. Stephen Mason, was formerly settled in the ministry, but now resides at Marshall, Michigan.

The present treatise on Practical Astronomy was chiefly written in the Spring of 1840, before his health failed. Early the ensuing Summer, symptoms of consumption began to develop themselves; and hoping to receive benefit from the invigorating climate of Maine, and eager to embrace every opportunity for making astronomical observations, he obtained the post of assist-

ant in the Commission under Professor Renwick, which explored the northeastern boundary of the United States, during the last Autumn. Sustained by a temper remarkably cheerful and resolute, he was able to fulfil the duties of his appointment; but on his return, the latter part of October, it was manifest that his disease had made regular progress and was carrying him to the grave.

Young Mason was truly a man of genius; and short as was his career as an astronomer, he accomplished enough to inspire in his scientific friends the highest expectations of his future eminence, in the exalted study to which he had devoted himself. The peculiar assemblage of faculties requisite to form the great astronomer, is seldom found united in the same individual, comprising as it does so many of the higher attributes of genius,—a *hand* of exquisite delicacy to construct and adjust—an *eye* endued with extraordinary powers of vision to observe—an *intellect* the most profound to follow out all the consequences of astronomical discovery—and that unconquerable *enthusiasm* which is regardless of loss of rest, of exposures by night, and even of life itself. These qualities were *severally* possessed by Mr. Mason in an unusual degree; but it was their striking and harmonious *union*, which, from the time I first discovered it, led me to recognize in him the promise of one probably destined to enlarge the boundaries of astronomical science.

In one of my last interviews with my departed friend, I asked him "What first turned his attention to the study of astronomy?" He replied that, when a child, (I think about twelve years of age,) he for the first time looked through a pair of concave glasses. Being very near-sighted, he had never before seen the external world so distinctly, and he was charmed with the new and lovely aspect of Nature. Going abroad in the evening on an errand, he wore his glasses, and was so delighted with the appearance of the stars,

that he remained out to a late hour, gazing upon the brilliant spectacle. From this period he seems never to have lost his passion for the observation and study of the heavenly bodies; for before he came to college, which he entered at sixteen, he had, as he informed me, perused Sir John Herschel's Elements of Astronomy with the deepest interest. Obscure as this work has been thought by some, young Mason, nevertheless, acquired by the aid of it a remarkably clear conception of the leading phenomena and laws of astronomy, of which I found him possessed before he had had opportunity to study any other author. After entering college, he soon formed an intimacy with one of his classmates, (Mr. H. L. Smith,) who had also imbibed an early taste for astronomical observation, and had already made considerable proficiency in the construction of telescopes. Mason joined him, with great enthusiasm, and they sometimes spent the whole night in casting and polishing specula, until they each completed a telescope of sufficient power and accuracy to show the leading phenomena of the heavenly bodies.*

From this time our young astronomer began to employ almost every fair evening in telescopic observations, sometimes protracting his delightful labors to a late hour of the night, advancing gradually from the planets and more common objects to the double stars, and the most intricate of the Nebulæ, and finally applying the nicest micrometrical measurements. An elaborate paper of his containing observations on some of the most obscure Nebulæ, communicated to the American Philosophical Society, and recently published in their Transactions, will attest his skill and ingenuity, as well as his perseverance, in the most refined observations of the sidereal heavens; and the engravings accom-

* Mr. Smith afterwards constructed a Herschelian telescope, of great power, with which Mr. Mason made many of his best observations.

panying that article, which were copies from his drawings, will indicate the delicacy of his hand.

During the last two years of his college course, although, by an uncommon facility in learning his lessons, he was able to maintain a rank among the first of his class, yet his affections were chiefly engrossed by his master-passion; and the lighter works of fiction and romance, which had occupied, at an earlier period, his moments of leisure and amusement, were now all thrown aside for the ponderous quartos of his favorite science; and such books as the London Astronomical Transactions, and Pearson's Practical Astronomy, were constantly at his side, awaiting every moment he could redeem from his other engagements. As soon as his college course was completed, in the Summer of 1839, his thoughts were earnestly bent on some mode of providing for his support, (which would thenceforth devolve wholly on himself,) consistent with his ultimate purpose of devoting his life to astronomy. It was partly with the view of furnishing him with employment, that I proposed to him to assist me in preparing this treatise, which I designed as a fourth part to my "Introduction to Astronomy." When, however, I became better acquainted with his skill in the use and adjustment of instruments, and with his peculiar qualifications to interest and instruct young learners in this department of astronomy, I gladly committed to him the sole preparation of the article, confident that the public would lose nothing by the change. I think it will be found, on trial, more peculiarly adapted to the exigencies of young students of Practical Astronomy, than any similar treatise hitherto published; and I cannot but believe that all who peruse it, will unite with me in deploring the untimely fate of a youth, who has given such signal proofs of his capacity to attain to the highest walks of astronomy.

DENISON OLMSTED.

Yale College, Jan. 1841.

ANALYSIS.

SUPPLEMENT

TO

OLMSTED'S ASTRONOMY.

CHAPTER I.

OF THE TELESCOPE.

1. THE application of the telescope to the purposes of practical astronomy, is as varied as universal. When attached to a system of circles, and by connection with them compelled to trace out in its motion corresponding circles in the heavens, it appears under the various forms of a transit, equatorial, altitude and azimuth instrument, mural circle, and sextant. In such combinations it is usually employed in exactly fixing the places of the heavenly bodies, and determining their distances from each other and from fixed points of reference. When unconnected with any such appendages, it becomes a simple telescope, free to sweep the heavens at large, and its principal use consists in the observation of phenomena and events which occur beyond our earth, and of the natural history of celestial objects. Under this, its simple form, and applied to observations of the latter kind, it will be considered in the present article.

2. We shall presume in the student an acquaintance with the principles of telescopic vision, and with the *general* construction of the different forms, under which the instrument has appeared since its first invention. That we may most simply and easily initiate him into the *practice* of astronomy, we shall suppose him at once an amateur observer, about to choose an instrument, and wishing to apply it in a way interesting to himself and useful to science. We shall direct our reader simply and plainly how to

choose his telescope, and judge of its excellence;—how to remedy its defects, manage its adjustments, find objects which he wishes to examine, examine them to the best advantage,—and finally, what kind of observations to make, and in what way to proceed, if he would add utility to pleasure.

3. *General description of refracting and reflecting telescopes.* The *refracting* telescope is usually mounted in a brass tube, and supported on a stand of various construction,—but oftenest a pillar of brass, with three folding supports. The object-glass must be achromatic, consisting of two (sometimes three) lenses so combined as to destroy very nearly the ill effects of color and aberration. The available diameter of the object-glass is called the *aperture* of the telescope, and is usually a little less than that of the tube. It forms its image near the eye-end of the telescope; the distance between the object-glass and this image is called the *focal length* of the telescope, and is commonly rather greater than that of the main tube. A microscope, consisting of two or more small lenses, is applied at the eye-end to magnify this image as if it were a tangible object; this is called an *eye-piece*, and with every large telescope, several of them are always furnished by the maker, to allow of variety in its magnifying power. The eye-piece is set in a sliding tube, and thus pushed in and out from the main tube by a milled head, which controls a concealed rack and pinion. The object of this contrivance is to enable the observer to adjust his microscope or eye-piece accurately upon the image, and is especially necessary where several of different foci are to be successively screwed into the eye tube. Such a telescope is pointed to any celestial object by wheeling it around the perpendicular support or axis till it arrives in the vertical plane of the star, and then turning it on a second pivot which allows of motion in altitude. If the instrument is of considerable magnitude and power, contrivances are attached, so that by a screw or pinion the telescope may have slow motions in altitude and azimuth, and yet may admit of disengagement when it is to be turned through any considerable arc.

4. *Reflecting* telescopes are of four kinds; the Newtonian, Herschelian, Gregorian and Cassegranian. Their principles of

construction are usually explained in elementary works on natural philosophy.* The Cassegranian is nearly obsolete; it differs but slightly from the Gregorian, and both are mounted in much the same style as refracting telescopes. In the Newtonian form, however, the eye does not look in at the lower extremity, and a different kind of mounting is necessary. When pointed at a star, the pencil of rays reflected from the large mirror at the lower end of the tube is turned off at right angles by the inclined plane mirror, and an eye-piece on the side of the tube receives it. The small mirror and eye-piece are attached to a slide, which may be moved to and from the large mirror in adjusting the focus.

In this form of the reflecting telescope, the eye looks at right angles to the tube of the telescope. But in the Herschelian construction, the small mirror is removed, and the eye looks immediately down the tube towards the large mirror. In order, however, that the head of the observer in this position may intercept no portion of the broad cylindrical beam of light, which enters the telescope from the star, the large mirror is slightly inclined so as to cast its focal image at the extreme left hand margin of the mouth of the instrument, where it is magnified by an eye-piece sliding along that side of the tube, and viewed with the right eye of the observer. It may be here remarked that it is a very common misconception to suppose that the interposition of the head or of any irregular body over the aperture of a telescope will obscure or cut out a corresponding portion of the disc of a star, of a planet or the sun; but a little consideration of that part of optics which teaches, that each point of the image is formed by rays from every portion of the speculum, will show that the only effect of such interposition is to diminish proportionally the light of the image.

5. Every telescope of considerable magnifying power should be furnished with a *finder*, which is a small telescope of a very low power, attached firmly to the side of the larger, and exactly parallel with it. In the common focus of the object and eye glass is a pair of coarse cross-wires. The necessity of this appendage becomes evident when we reflect, that a high magnify-

* Olmsted's Nat. Phil. Art. 918–920.

ing power requires a very small *actual* field of view. Thus the field of a telescope magnifying between 100 and 200 times is a circle in the heavens not as large as the full moon, and will scarcely include the Pleiades or seven stars. The telescope is approximately pointed to a star by glancing the eye along the tube; the object then appears in the field of the finder, because its low magnifying power allows of a wide field of view of several degrees in diameter. It may be easily brought upon the intersection of the cross-wires in the finder, and will then be somewhere in the field of view of the telescope. If the smaller instrument is out of adjustment, however, it will not be the index of the larger, and perfect correspondence of direction between the two must be thus obtained;—direct the telescope to a star, or better, to a distant terrestrial point not in motion, by the uncertain and tedious process of ranging for it till it is found; then turn the screws which alter the position of the finder, or of its cross-wires, until the object is covered by the intersection of the latter. Ever after, the finder will be a sure guide to any other object.

6. *Description of particular refracting and reflecting telescopes.* The largest reflecting telescope ever constructed was the celebrated one of 40 ft. in focal length and 4 ft. aperture, by Sir Wm. Herschel. We shall occasionally allude to it, especially in speaking of the *light* of a telescope. It is now out of repair, in consequence of exposure to the weather; and 20-ft. telescopes, of easier management, and more regular performance under [redacted]

7. Such telescopes as these are, and will be for a time, to say the least, very rare in America. In confining our remarks more exclusively to those in use in this country, we shall mention particularly the Herschelian telescopes of Mr. Holcombe, which are in very general use among observers, and from their excellence, will probably become much more so. For the benefit of the American observer, we will briefly describe its construction, and point out its advantages and disadvantages. It is of the Herschelian form, with a tube of sheet iron painted, and the end containing the large mirror rests on the ground. The other end is supported by two folding legs, which with the tube of the telescope form a kind of tripod. It is quickly directed to any point by spreading out more or less the two legs, and as each of these are double, and may be gradually lengthened or shortened at pleasure, a slow motion is obtained for following a star. The chief recommendation of this telescope is, that with an excellence of figure in the speculum which enables it to compete with the telescopes of the best European artists in its performance on the closest test-objects, it is afforded at a comparatively very moderate price, the cost of foreign instruments of equal power and light being two or three times greater. The telescope is perfectly supported at precisely the two points most important, the place of the large mirror, and of the eye-piece; its steadiness therefore gives it the advantage over most European instruments even actually superior, except in very calm nights. In elegance of external appearance, it will not compare with foreign instruments of equal intrinsic excellence; the style of mounting, however, is as neat and convenient as it is simple and inexpensive.* Other telescopes, American and foreign, but chiefly of the refracting kind, are frequently to be met with, whose astronomical excellence will recommend them to the notice of the observer.

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* The establishment of Mr. Holcombe is at Southwick, Mass., and his telescopes are from 5 to 14 feet in focal length, with apertures from 4 to 10 inches, at prices ranging from 100 to 600 dollars. The performance of some of these will be spoken of in our description of the test-objects of telescopes.

The largest telescope in this country is a Herschelian of 14 feet focal length and 12 inches aperture, principally constructed by Mr. Smith, lately a student of Yale College. It was first erected in New-Haven, Conn., but has recently been removed, and remounted at Ohio City, Ohio.

8. *In the choice of an instrument*, many particulars are to be attended to, among which we rank the following as the principal. The telescope should show a star free from burs or tails of light, or haziness consequent on aberration. If an achromatic, it should be noticed, whether the correction of color is perfect or not. In a good telescope, a slight alteration of no more than the $\frac{1}{16}$th of an inch in the focal adjustment, in either direction from the point of distinct vision, will considerably enlarge the image of a star, and render it indistinct. Day objects to an inexperienced observer, appear nearly as well in an ordinary instrument, as in an excellent one; the moon and planets are much better objects of comparison, and a bright fixed star affords the severest trial of excellence, especially if it be double, like Castor or γ Virginis.

Such questions as these, "How far can I see with this telescope?" or "How many times does it magnify?" are not likely to elicit satisfactory information with regard to the optical capacity of an instrument. For with an inferior telescope objects may be seen immeasurably remote, by directing it towards the stars; and may be magnified almost any number of times, more or less distinctly, by applying eye-glasses of sufficient power. The highest power supplied with a telescope is commonly much greater than can be employed to the utmost advantage, and therefore is no very sure indication of its distinctness or defining power. The true way of arriving at a knowledge of the excellence of an instrument is to find out *what test-objects it has resolved*, and is competent to exhibit clearly, either by inquiry, or actual observation. By referring to our table of test-objects in this chapter, and the remarks concerning them, the reader will have the best means of deciding on the merits of any telescope.

The qualities of the stand should not be neglected, the essential requisites of which are,—steadiness, ease of direction to any point of the heavens, and ease in following any celestial body in its diurnal motion. A star in the field of an unsteady telescope becomes like an irregularly whirled point of flame, upon which it is as impossible to exercise close observation, as to read from a printed leaf in rapid motion. Every tremor and vibration of the instrument is magnified by its whole optical power. A telescope supported at two points at a distance from each other, is

far more steady than one balanced on its centre. The other two requisites are obtained when the telescope is capable of good quick and slow motions in directions at right angles to one another; the quick motion affords the means of turning immediately to the star, and the slower motion of following it with regularity.

9. *The power and light of telescopes.*

A telescope magnifies celestial objects in two ways. First, it expands their *linear dimensions* by what is called its magnifying power; this power is expressed in numbers, and is the ratio of the visual angle under which the telescopic image is seen, to that under which the object appears to the naked eye. Secondly, it magnifies their *light* by what is termed its illuminating power; which also may be expressed in numbers, and is nearly the ratio of a cylindrical beam of light, of the diameter of the object-glass or speculum, to the much narrower pencil of light, which is collected by the naked eye, and which is of the same diameter as the pupil.

10. The *magnifying power* of a telescope may always be ascertained by dividing the solar focal length of the object-glass (S) by that of the eye-glass (φ). If the eye-piece is compound, and consists of two glasses, let F be the solar focal length of the inner, f that of the outer glass, and d the distance between them; then $\frac{Ff}{F+f-d}=\varphi$, or the focal length of the equivalent lens; and $\frac{S}{\varphi}$ will be the magnifying power, as before. The focus of any lens or mirror is found by casting the sun's image through or from it, on paper, moving the latter, till the image is most neatly defined and smallest.

Every telescope is accompanied with a set of eye-pieces, furnishing an ascending series of powers. For day observation, a *terrestrial* eye-piece of low power, and consisting of four glasses set in a tube of considerable length, is useful to represent objects in an erect position. With celestial objects the inversion of the image is of little consequence, and short combinations of two glasses each are employed. These are called astronomical eye-pieces, and are of two kinds. In that invented by Huygens, the

focal length of the glass next the eye, the distance between the two, and the focal length of the other, are nearly as the numbers 1, 2, and 3, and the plane surfaces of each lens are towards the eye. This eye-piece receives the pencil of rays from the object-glass before they reach the focus, and no aerial image is formed except between its two lenses; it is therefore called a negative eye-piece, because, like a convex mirror, it has only a negative or imaginary focus. The positive eye-piece was invented by Ramsden, and consists of two lenses of nearly equal focus, and distant by $\frac{2}{3}$ or $\frac{3}{4}$ the focal length of either, with their convex surfaces towards each other. The image of the object-glass in this case is formed in the air just before the pencil of rays meets the inner glass of the eye-piece. Few other combinations of two glasses will afford good vision. Ramsden's eye-piece and the Huygenian are nearly equal in their merits. The latter affords perhaps a larger and more uniformly distinct field; but the former admits of the insertion of micrometrical and other cross-lines at the place of the aerial image, and moreover, has what is called a flat field,—that is, does not distort the outline of an object.

Very low powers, as from 30 to 80, are useful to view ill defined objects, as comets and many of the nebulæ,—to command a large field of view, when desirable,—and to exhibit celestial objects to those who are unaccustomed to the management of a telescope. As the magnifying power increases, the intrinsic brightness of the object is diminished. The planet Jupiter, for instance, sends an absolute quantity of light to the eye, which with the same aperture is constant under all powers. By magnifying his surface more and more, the light which belongs to it must necessarily be spread over that surface more and more thinly; so that very soon a point is reached, where indistinctness begins, because the light at any one point of the retina is too faint to stimulate perfect vision. With a good 7-ft. achromatic or reflector, this cause alone renders 250 as high a power as can be employed to much advantage on the planets. An ordinary observer will be far better pleased by the view with 100 or 120, because much brighter and apparently more distinct; minute features of the object may, however, be often better brought out by pushing the magnifying power even to indistinctness.

Saturn will scarcely bear more than 200 well, because of its greater distance from the sun, and consequent feebleness of light; the moon, on the contrary, will occasionally bear 300 in the inspection of minute features, although 80 or 100 is always better to show it in whole. The fixed stars shine by their own intrinsic light, and will bear much higher powers. In very favorable circumstances, and with an excellent telescope, some advantage may be gained with 600 or even 1000, at the expense of much aberration and troublesome indistinctness. Sir Wm. Herschel, with his exquisite 7-ft. Newtonian, occasionally on rare nights, pushed his powers even to 6000, but the common observer will do well if he ever gains any advantage by going beyond 600. For the planets, powers between 100 and 200 are *most commonly* available; for double stars, those between 200 and 300, and if very close, 400 or 500 may be employed. Single lenses are of advantage in very high powers, because there is less room for loss of light, for imperfections of density in the interior, and of figure in the surfaces, of one glass than of two. They are more difficult to use, since the object under inspection must be kept scrupulously in the middle of the field, both in adjustment of focus and in subsequent examination; for in no other part of the field is there good vision.

11. The *light* of a telescope depends chiefly on its aperture. Through a 7-ft. telescope with 150 Jupiter is seen with ease and comfort; the same power on a 20-ft. telescope renders it so bright as to be painful to the eye, and too dazzling for distinctness. For this reason, the 7-ft. is preferable, unless the aperture of the 20-ft. be sufficiently contracted. Again, the field of the 20-ft. is strown with stars, which are utterly invisible in the 7-ft; here the 20-ft. has the advantage. Large telescopes and a great aperture are chiefly useful to view faint objects, as nebulæ, and to bring out faint points of light. The 40-ft. of Sir Wm. Herschel would not bear as high a power as his 20-ft., and neither could at all admit of such high powers, as we have already said he sometimes applied to his 7-ft.; but yet the former, from its power of light alone, afforded him immediately several of his most celebrated discoveries.

When we consider that the pupil of the eye, and consequently

the pencil of rays which enters it from any star, is but 0.2 of an inch in diameter, we may conceive how greatly we should increase our perception of its brightness by aid of an instrument, which receives a beam four feet in diameter, and concentrates it upon a single point of the retina. Since the areas or sections of the two beams are as the squares of their diameters, the brightness of the star *would be* increased by the telescope in the proportion of $48^2 : 0.2^2$ or 57,600 times, if the speculum were perfectly reflective. But only about $\frac{2}{3}$ of the light is reflected from the speculum, and of this, about $\frac{1}{10}$ more is lost in transmission through two lenses; the illuminating power is therefore $0.6 \times 57{,}600 = 34{,}560$. A single quotation from one of Sir Wm. Herschel's papers will illustrate the effect of a concentration of light so enormous. "I remember that after a considerable sweep with the 40-ft. instrument, the appearance of Sirius announced itself at a great distance, like the dawn of morning, and came on by degrees, till this brilliant star at last entered the field of the telescope with all the splendor of the rising sun, and forced me to take the eye from that beautiful sight."—Phil. Trans. vol. xc. p. 54.

12. *Atmospheric obstacles, and other causes of limitation to magnifying power.* The four surfaces of an achromatic object-glass, or the single one of a speculum, cannot be wrought to absolute perfection, or if they could, they could not remain so, but would oscillate on either side of it by change of temperature. As the effects of such imperfection are magnified by the whole optical power of the instrument, this cause alone prevents the application of glasses of extremely short foci.

Again, as the power is increased, the field is very much diminished, and the diurnal motion of the earth is magnified in the same ratio. The field with 600 or 1000 will little more than comprise one of the larger planets, and a star vanishes at the edge of the field in 2 or 3 seconds. It requires much skill in the observer to keep the object in the field at all, and more to follow the star, manage the focal adjustment, and exercise severe scrutiny, all at once.

But the chief bar to magnifying power is in the atmosphere. The air is seldom so well balanced, as to be without contrary

currents and motions, which produce slight transitory undulations and variations of density. In a section of the broad beam of light which is to enter the telescope, there is room for considerable momentary differences of density, and consequently of refraction. Where the rays before incidence are thus irregularly deflected from perfect parallelism with each other, and quivering or vibrating about a mean state, the point of light which they form after reflection must be troubled and confused. The star in the field of view is in constant and rapid agitation, like an object seen through smoke or heated air, or the image of the sun on gently rippling water. According to different circumstances of atmospheric disturbance, the appearances of the star to an attentive observer, are very different. Sometimes the individuals of a close double star twirl round each other, altering much their angle of position; sometimes it appears like a drop of agitated mercury,—very frequently writhing and convulsed, breaking in pieces and reuniting,—occasionally heaving gently like the sun reflected on a calm swell;—and the rarest of all states is that of perfect rest. These motions are minute, but are excessively troublesome, and in the majority of evenings prevent the application of very high powers, or the separation of very close and difficult stars. An excellent telescope might very easily be condemned for supposed want of optical capacity on an unfavorable evening. Its merits should not therefore be judged of too hastily, nor until several evenings' patient trial. And unless the star is free from any rapid quivering or agitation, it cannot be judged whether the atmosphere or the telescope is most in fault.

13. There are many causes of disturbed equilibrium which it is in the power of the observer to remove. A telescope pointed out of a window, especially in winter, or over houses and walls heated by the sun during the day, will exhibit objects in a very inferior manner; and, when brought out of doors, it needs a quarter or half an hour to acquire perfectly the temperature of the surrounding air. It is difficult to decide upon the atmospheric circumstances most favorable to perfect action in telescopes. As a general rule, it may be observed, that stars seldom appear well in clear frosty nights, or when they twinkle much to the naked eye. Sir Wm. Herschel remarks that "when the outsides of our

telescopes are dropping with moisture discharged from the atmosphere, there are now and then favorable hours in which it is hardly possible to put a limit to magnifying power;" and it is not uncommon to find that a slight haze improves very much the definition of close double stars.

As the aperture of a telescope is increased, the breadth of the beam of light which traverses the atmosphere becomes larger, and affords room for greater inequalities of density. The atmosphere thus puts a speedy bound to all human efforts made within a medium so fickle, and must frustrate attempts to employ telescopes of enormous magnitude. The 40-ft. telescope of Sir Wm. Herschel was nearly useless from this cause alone; for there were scarcely 100 hours during a year, in which it could be used to any advantage.

14. The observer will be a little surprised to find that stars have neat, well defined discs, especially since it is customary in philosophical treatises, to say that they are absolute points, without apparent size. This very common remark is true in this, that no magnifying can give the stars a *real* diameter. But α Lyræ, or any other bright star, appears in the telescope with a well defined and sharply terminated disc, of definite and sensible diameter, as certainly as Jupiter or Saturn. The difference between them is not in appearance, but in fact. The discs of the stars are spurious, and are due to irradiation, which always increases the diameters of intensely bright objects,—a phenomenon probably due to retinal sympathy. They are very small, and often confused amidst the aberration caused by imperfections of figure, tremors, and their own dazzling brightness. In the best telescopes, on the finest nights, and with high powers, they are most neatly defined, and beautifully seen;—and are then steadily surrounded with a number of slender circular rings of light, concentric with each other and with the star. Stars of the first magnitude have the largest diameter, and below a certain brightness this phenomenon ceases on account of growing indistinctness and want of stimulating power.

15. *Comparison of Refractors and Reflectors.*

We are now prepared to form some comparison between the

respective advantages of achromatics and reflectors, of which latter we will take the Herschelian as the simplest form. The achromatic is less subject to atmospheric disturbances, and performs more uniformly at all times and under all circumstances; while the speculum of the reflecting telescope is often liable to tremors, and requires very perfect equilibration of temperature. On the other hand, the discs of stars appear smaller in reflectors, and are therefore more easily separated, when close double. Reflectors too are in their nature perfectly achromatic, and color can only arise from the use of deep eye-lenses. They admit of being mounted more steadily than refractors; for the object-metal of a reflector is below the eye of the observer, resting nearly or quite on the ground,—while the object-glass of the other is high above the observer's eye in the air, and very frequently set in vibration by every breath of wind. Again, vision is more easy in reflectors; for the eye in the Herschelian looks downward, as in reading a book, and in the Newtonian, it looks directly forward horizontally; but with achromatics, especially at high altitudes, the head must be bent backward and held in a posture very tiresome and inconvenient, unless supported.

The apertures of achromatics are usually less in proportion to their focal lengths, than those of reflectors. We will adduce a few instances by referring to several telescopes:

Achromatics.

Sir J. Herschel's	7-ft. equatorial,	5 inches aperture.
Several	10-ft., by G. Dollond,	5 " "

Reflectors.

The favorite	7-ft. of Sir Wm. Herschel,	6¼ inches aperture.
"	40-ft. "	48 " "

These, and many other instances, show that the proportion found best in practice is for reflectors about 1 foot of focal length for every inch of aperture, and for achromatics between 1 and 2 feet of focal length for the same aperture. But since glass transmits much more light than polished metal reflects, achromatics, though of smaller aperture than reflectors of the same focal length, are nearly equal in light and power, and we shall regard them as such in speaking of the test-objects.

16. *On the test-objects of telescopic power and light.*

The usual objects of telescopic examination, especially the test-objects, will afford to the observer occupation, rich in interest and novelty. The solar spots, irregular dark forms, surrounded with their umbræ,—the bright ridges or faculæ, interwoven like the vessels of a leaf,—and the mottled appearance of the whole disc of the sun, are easily seen in a 5-ft. telescope, with a power of 100, provided the dark glass be good. Mercury and Venus require magnifying power, diminution of aperture, or slight haze, to take off their dazzling brilliance, and are usually very tremulous. The phases of the former may be seen indistinctly, and those of the latter with beautiful precision and clearness under favorable circumstances. The variegated appearance of the moon, her diversity of light and shade, wide level plains, annular mountains, and lofty ridges, render her in any telescope the most striking and magnificent of astronomical objects. No employment can be more interesting to the amateur astronomer than to watch from night to night the boundary of light and shade, as it passes over the lunar disc,—to see the mountains first appearing as separate fragments of light, soon joining and advancing into the bright portion, but still casting their long shadows back into the darkness,—to see these shadows separating at last from the dark portion, and gradually shortening as the long lunar day advances. The strong contrasts of light and shade, and dazzling brightness of the lunar disc, are apt to surprise and confuse the observer; but a little familiarity with its appearance and changes will very soon accustom him to judge as correctly of the inequalities of her surface, as of the irregularities of the ground beneath him, when lighted by strong sunlight. A power of 100 or more will show that the circular edge of the moon's bright limb is slightly irregular, and broken by mountains and valleys, seen in profile. This beautiful object is best exhibited in her first and last quarters, and is least interesting when full.

The moons of Jupiter are within the reach of any telescope above 1 ft. in focal length. His two principal belts are rather more difficult objects, and to inspect them minutely requires a power of more than 100, and an aperture of several inches. With a good 7-ft. reflector or achromatic, the transits of satellites and their shadows over his disc may be observed with ease.

The ring of Saturn is easily seen with a 2 or 3-ft. achromatic. The belts upon his body require light as well as distinctness; but a power of 100 on a 5-ft. reflector should show them to advantage. The black division which separates the ring into two is a severe test for good telescopes. It can be seen in excellent telescopes of 4 or 5 feet focal length, or even less, but the powers of a 7-ft. are requisite to give a satisfactory view of it. Even with an instrument of this magnitude, it is very rare to see the ring divided any where but at the two extremities of the oval, as in the annexed cut. To trace the division where narrowed by the effect of projection, and nearly throughout the visible portion of the ring, requires the most favorable night, perfect steadiness in the instrument and great excellence of figure, a practised eye, and the planet near the meridian. The telescope that has exhibited the division well at the extremities of the ring, has passed a very severe test, and is competent to deal with the greater part of Herschel's first class, or the closest of double stars.

17. The double and triple stars present an endless variety of the most delicate objects to the attention of the observer. They are—first, of all degrees of proximity;—secondly, of all grades of inequality. Many are farther apart than the diameter of Jupiter, (from 30″ to 40″,) and some are closer than the breadth of the division in Saturn's ring, which is less than 1″. The black interval between the two stars of Castor (fig. 1.) is about once the diameter of the larger, or 1½ times that of the smaller star; those of γ Leonis (fig. 2.) very nearly touch one another; and the two nearest of ζ Cancri (fig. 6.) are seen nearly or quite in contact. A closer approximation makes the discs overlap one another, appearing like a double-headed shot, as γ Virginis in 1838 (fig. 9.), or if very unequal, wedge-shaped, like λ Ophiuchi (fig. 4.) If still closer, two equal stars appear merely a slight elongation of one, as γ Virginis in 1837 (fig. 9.), and a faint companion only distorts the disc of the larger star. A star, however, may present difficulties, not from the closeness, but from the faintness of its companion. And the observer, by visiting in succession the small stars attendant on Polaris, α Lyræ, and η

Coronæ,* will gradually descend to a degree of faintness, which requires long attention to be visible at all.

Double stars vary in still another particular—viz. color. A few offer splendid contrasts, among which are γ Andromedæ, red and green,—β Cygni and ε Bootis, in both of which the smaller is a fine blue star. The larger star is never blue or green; the smaller in these combinations may be either, and occasionally we find it of a fine purple.

18. The following list of test-objects will enable the observer to choose such as will try the powers of his instrument to the utmost.

A LIST OF TEST-OBJECTS.

For an Achromatic, Newtonian, or Gregorian of 3-ft. or less.

DEFINING POWER.				ILLUMINATING POWER.			
Object.	Dist.	Mags	Remarks.	Object.	Dist.	Mags.	Remarks.
β Cygni	34″	4,6	L. yellow; S. fine blue.	β Cephei	13″	3,8	
ζ Urs. Maj.	14	3,4	Coarse.	η Cass.	10	4,8	Sm. star purple. Binary.
γ Androm.	10	3,5	Easy. S. beautiful green.	ξ Boötis	8	5,7	Binary.
γ Arietis	9	4,4	Easy.	ξ Libræ	7	5,8	Binary.
Castor	5	3,3½	Binary.	Pleiades	....	.....	In Taurus. Very coarse cluster.
α Piscium	5	4,4	Close.	Præsepe	In Cancer.		Coarse cluster.
ζ Aquarii	4	5,5		1st Satellite of Saturn.			
Jupiter's moons and belts,—Saturn's ring.							

For a 5-ft. Achromatic, or Herschelian.

DEFINING POWER.				ILLUMINATING POWER.			
Object.	Dist.	Mags.	Remarks.	Object.	Dist.	Mags.	Remarks.
γ Aquarii	4″	5,5	Very easy.	ξ Libræ	7″	5,9	Triple and revolving.
ε¹ Lyræ	3¼	6,7	Beautiful double-double. Both binary.	Rigel	9	1,9	
ε² Lyræ	3	6,6		ε Boötis	3	3,8	
11 Monoc.	3,7	7,8,8½	6h. 20m.—6° 55′.† Easy triple.	Polaris	15	2,10	
				39 Drac.	3½	5,10	
ζ Leonis	3	2.3,4	Difficult. Binary.	σ Coronæ	40	7,13	Quadruple. The most distant.
γ Virginis	Var.	3,3	Rapid binary.	χ Persei	Two very rich clusters, crowded with stars.		
ξ Libræ	1¼	5,5	Very difficult. Binary.	Saturn's belts and 3 satellites.			
Division of Saturn's ring at the extremities.							

* This star is also close double, and has been a standard test-object for telescopes, but of late has closed so as to be inseparable by almost every telescope in the world.

† The places of objects not in the small "Maps of the Society for the Diffusion of Useful Knowledge," are designated by their right ascensions and declinations for the year 1830.

For a 7-ft. Achromatic, or Herschelian.

DEFINING POWER.

Object.	Dist.	Mags.	Remarks.
ζ Boötis	1″	5,5	
ζ Cancri	5,1	6,7,7	8*h*. 2.1*m*. +18° 10′. Triple. Rapid revolution.
σ Coronæ	Var.	7,9	16*h*. 7.5*m*.+34° 20′. Rapid binary.
λ Ophiuchi	1	4,6	Binary. Wedge-shaped.
π Aquilæ	1½	7,7	19*h*.40.7*m*.+11° 24′.
Can. Min. 31	1	8,9	7*h*. 31.1*m*. +5° 37′. In the field with and following Procyon.

ILLUMINATING POWER.

Object.	Dist.	Mags.	Remarks.
Polaris	15″	2,10	Easy.
α Lyræ	45	1,12	Difficult.
η Coronæ	40	6,13	15*h*. 16.1*m*.+30° 56′. Very difficult.
Debilissima inter ε¹ and ε² Lyræ	50	13,15	Very difficult.
Messier 5	....	.....	Resolvable; glimpses of innumerable stars.
" 13	....	.....	
5 satellites of Saturn.			

For a 10-ft. Achromatic, or Herschelian.

DEFINING POWER.

Object.	Dist.	Mags.	Remarks.
36 Androm.	½″	7,7	0*h*. 45.6*m*.+22° 41′.
P. XIII. 127	1¼	9,10	13*h*. 25.4*m*.+0° 35′. In the field with and north of ζ Virginis. Binary.
ε Arietis	¾	7,7½	
4 Aquarii	½	7,7½	20*h*. 42.1*m*.—6° 17′. Power 800. Requires light.
20 *h* Drac.	¾	8.8	16*h*. 55.5*m*.+65° 19′.

ILLUMINATING POWER.

Object.	Dist.	Mags.	Remarks.
γ Crateris	3″	4,13	
τ Orionis	18	4,14	
φ Virginis	4	5,14	
δ Cygni	2	3,10	Very difficult. Needs defining power.
Messier 5	....	.....	Partially resolved into stars.
" 13	....	.....	

For a 14-ft. Achromatic, or Herschelian.

DEFINING POWER.

The same as for a 10-ft. telescope.

ILLUMINATING POWER.

Object.	Dist.	Mags.	Remarks.
β Aquarii	20	3,15	
τ Boötis	20	4,16	
σ Coronæ	20	7,16	Nearest of the two faint stars.
Messier 5			Crowded with countless stars, which run up to a confused blaze of light in the centre.
" 13			

For a 20-ft. Herschelian.

DEFINING POWER.

The same as for a 10-ft. telescope.

ILLUMINATING POWER.

Object.	Dist.	Mags.	Remarks.
ξ Pegasi	11	5,16	The nearest of two.
α² Cancri	10	4.5,16	
α² Capric.	8	3,16	

These tests of distinctness and light are such as ought to be within the reach of very good telescopes of the focal lengths assigned, and of the apertures usually corresponding to them. Occasionally an object-glass or speculum is fortunately wrought to such perfection, that it may resolve nearly all of the tests which have been recommended for a larger telescope. The observer, however, with an instrument of but common excellence, can scarcely hope to do more than separate the tests appropriate to its size; and indeed, only the easiest of these, while unpractised in observation, and unfamiliar with atmospheric obstacles to perfect vision.*

19. A few of the more remarkable combinations of double and multiple stars are represented in figs. 1—9.

Figs. 1 and 2.—Castor and γ Leonis with a power of 300. Standard and beautiful objects, on account of the brightness of the individuals. Both slow binary.

Figs. 3 and 4.—39 Draconis and λ Ophiuchi with 300. The latter is binary, and so rapid in its revolution, that a few years will probably render it too close to be visible; it can now be seen only wedge-shaped. 39 Draconis consists of 2 stars, the larger of which is the double star.

Figs. 5 and 6.—11 Monocerotis and ζ Cancri with 300. The former is stationary. The distant star of ζ Cancri is in very slow motion; and the two nearest are in rapid rotation. ξ Libræ, and probably 12 Lyncis ($6^{h.}$ 30^{m}. $+59°$ $37'$) are also triple stars in revolution around a common centre of gravity.

* The actual performance of an excellent 7½-ft. Herschelian of Mr. Holcombe's manufacture will afford the student an illustration of the advantage of reference to test-objects. With the instrument referred to, ξ Libræ and ζ Bootis have been often and easily separated, and ζ Cancri well elongated. A rare night early in 1838, showed γ Virginis and λ Ophiuchi notched on either side, π Aquilæ was pronounced a very easy star, and Saturn's ring was seen double nearly throughout its visible portion. 36 Andromedæ is also within the reach of the instrument. "Debilissima" was first looked for through a very thin haze, but was distinctly seen with 5 inches aperture by glimpses, and correctly figured. And since on referring to European observations, we find that few or none of their telescopes of equal focal length are competent to resolve closer test-objects than these, we are thus enabled to decide on the great excellence of Mr. Holcombe's instrument. It may be of use to the inexperienced observer to add, that at first it was thought no small achievement to separate γ Leonis with this telescope, and ξ Libræ could not be seen at all.

Fig. 7. An accurate representation of the fine double-double star ε^1 and ε^2 Lyræ with 150. The unequal set is ε^1, the equal is ε^2, and both are slow binary stars; a star of the 10th mag. follows on the right hand. The faint "Debilissima" is between, the uppermost of the two being the brightest, and both necessarily much exaggerated.

Fig. 8. σ Orionis with a power of 80. Composed of 7 stars easily visible in a 3 or 5-ft. telescope.

Fig. 9. γ Virginis with 300. In June 1836, it passed its perihelion; then, and in 1837, scarce a telescope in the world could show it otherwise than *round*, and none more than a trifle *elongated;* in Jan. 1838, the elongation might be seen notched on each side; in Jan. 1839, there was a hair-breadth division; and now in 1840, it is fairly and easily separated with a large telescope. Its appearance for 1845, 1850, and 1860, are also given, and the several phases when combined, plainly indicate that the elliptical orbit annexed to the figure, is described by one star relatively around the other. These phases are taken from the distances and positions of the table in Art. 29.

These figures represent the stars as they are seen in an achromatic telescope when on the meridian, and by holding them before a looking-glass, they will appear as in a Herschelian. The letters *n*, *f*, *s*, *p*, signify respectively "north," "following," "south," "preceding," and mark out those directions in the heavens. The revolving stars naturally excite the most interest, and these representations therefore, (with the exception of 39 Draconis, 11 Monocerotis, and σ Orionis,) are chosen from that class, and exhibit them in their true positions for the year 1840.

20. The nebulæ and clusters of stars are in a large telescope objects of uncommon interest, because different from any thing we see with the naked eye. Of these the Nebula in Orion, of an irregular shape, and containing a minute trapezium of stars, is the brightest. It has a nebulous star in the same field. The Nebula in Andromeda (M. 31) is a very long ellipse, fading imperceptibly away from a bright centre; there is a nebulous star (M. 32) almost in the same field with it, and a faint nebula at a little greater distance. These two bright nebulæ are well seen in achromatics of 3 and 5-ft., and are splendid objects in large tele-

scopes. There is a pretty bright nebula in Sagitta, ($19^{h.}$ 52.2^{m}. +22° 16',) which in a 7 or 10-ft. may be seen as a double-headed shot, or double nebula not quite separated. Between β and γ Lyræ is a wonderful annular nebula (M. 57;) it requires a good eye to distinguish the "hole in it" in a 7-ft. reflector. Through the double star 52 *k* Cygni ($20^{h.}$ 38.9^{m}. +30° 7',) passes an exceedingly faint, forked ray of nebulosity, which can but just be seen in a 10 or 12-ft. instrument.

In Cancer, the naked eye may easily see a nebulous appearance, like a comet, called Præsepe, ($8^{h.}$ 30.4^{m}. +20° 34',) which a 3-ft. telescope shows to be a brilliant cluster of large scattered stars. Midway between Cassiopeia and Perseus, in the sword-handle of the latter, a hazy star, χ Persei ($2^{h.}$ 7—10^{m}. +56° 22') is faintly visible to the eye, which a 5 or 7-ft. telescope exhibits as two clusters of crowded stars, each filling the field—a glorious object. These, however, are but coarsely scattered collections when compared to Messier's 13th ($16^{h.}$ 35.7^{m}. +36° 48') and 5th, which in a 7-ft. are nebulous, with the addition of a few faint glimpses of stars; a 10-ft. shows many faint stars, but it requires at least a 14-ft. to resolve all the nebulosity into stars, and exhibit them as dense swarms, running up to a blaze of light in the centre. M. 13 is between η and ζ Herculis, and M. 5 is almost or quite in the same field with 5 Serpentis.

21. *How to find objects for examination.*

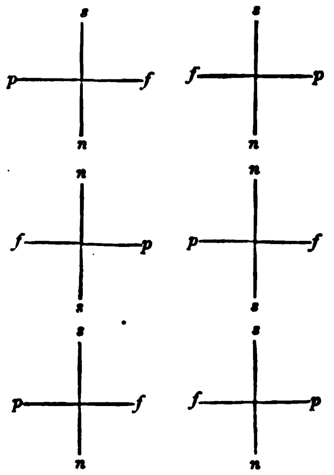
A familiarity with the geography of the heavens is so necessary for the observation of such objects as we have named, that in no way can it be acquired more rapidly than by looking for faint stars with the telescope. A good *map of the heavens* is here an indispensable auxiliary. The best of the kind easily obtained are those of "The Society for the Diffusion of Useful Knowledge, London." The larger set are expensive, but the price of the smaller does not exceed $2.

22. Stars are numbered on maps and in catalogues in several different ways. Thus—α Orionis—58 Orionis—P. V. 268 are synonyms for the bright star Betelgeux. 58 Orionis means the 58th star in order of right ascension, of those which Flamsteed numbered in Orion, and P. V. 268 refers to the 268th star in or-

der of right ascension between 4^h and 5^h R. A. in Piazzi's catalogue. Double stars and nebulæ are referred to in catalogues by abbreviations like the following: H. II. 79, is the 79th double star or nebula of Sir Wm. Herschel's second class; *h.* 1239, denotes a double star or nebula of that number in the younger Herschel's catalogues; M. 13, is the 13th nebula of Messier's catalogue, &c.

23. The points of the compass in the heavens are—north,—following,—south,—preceding,—denoted by their initial letters. A star that precedes another arrives at the meridian first, and its right ascension is therefore less than that of the other. The preceding point of the field of view is always that towards which a star moves by diurnal motion; and the observer, by noticing this circumstance, is at no loss to conceive of the other cardinal points in his field. For in an achromatic, the points *n*, *f*, *s*, *p*, follow the circumference of the field in a direction *contrary* to that in which the hands of a watch revolve; in a Herschelian, they follow in the opposite direction, or *with* the hand of a watch. The following cut exhibits the different positions of these points when the star moves horizontally, or is on the meridian.

In an
ACHROMATIC. HERSCHELIAN.



South of the zenith.

North of the zenith, above the pole.

North of the zenith, below the pole.

It is to be noticed that an achromatic inverts entirely, both up and down, and right and left; while in the Herschelian objects

are only inverted up and down, as when seen in a still lake. This is because the mirror, at the same time that it inverts like an achromatic, reverses right and left as in a common looking-glass, thus destroying the effect in a horizontal direction. In a Newtonian, because of the second reflection, objects appear as in an achromatic.

24. Suppose the object to be looked for is put down on the map, and *visible* to the naked eye, for instance, 39 Draconis. A kind of triangulation may be carried on by means of the map from the large and well-known stars in the neighborhood down to the point required. The star, when found, should be so carefully compared with the surrounding objects by means of the map, (especially by ranging it with stars nearly or quite in the same line,) that no doubt may remain of its identity. For without much care and precision, fancied resemblances of bearing and distance may be mistaken for real ones, and the observer may be mortified with the discovery, that he has been exercising much needless scrutiny and attention upon the wrong star. If the object is *invisible* to the naked eye, the exact point among the stars towards which the telescope is to be directed, may be determined in the same way. If not recorded on the map, a pencil dot may be made to mark its place, when its right ascension and declination are given.

When the star, or the point where it should be seen, if visible, is thus determined by the eye,—direct the tube of the telescope or finder as nearly as possible to the spot; and make the image of the cross-wires, viewed with one eye, approach the desired star or point, as seen with the other. It is usually best in practice, to bring the nearest bright star on the cross-wires, and then move in the direction of the object, still employing binocular vision. By remembering that the finder inverts, and noticing the scale of its enlargement, it soon becomes easy to trace the inverted and magnified but still similar configurations of the finder, proceed from star to star, and thus arrive at the object in the finder, though invisible to the naked eye.

25. Other modes of finding an object, when invisible to the naked eye, will be but hinted at. An object invisible even in

the finder, may often be detected by ranging in its vicinity, with one observer at the finder, and another watching the field of view; the annular nebula in Lyra is thus found without much difficulty. If an invisible object follows a visible star in the same parallel, and is less than a diameter of the field of vision north or south of that star, it may be found with the aid of a common watch, by allowing the diurnal motion of the heavens to sweep past the field of view the partial zone that intervenes between the star of departure and the object to be found. If the telescope can be so placed in the meridian as to rise and fall through a small zone with tolerable correctness, (for which purpose such simple methods as suspension of the upper end by a cord and pulley, or a bearing of the same against a wooden bar truly upright, will avail,) the faintest objects within the reach of the telescope may be found without the aid of a map by their right ascensions and declinations, taking care to note the passages and declinations of a sufficient number of known stars to furnish standards of comparison. A common watch will answer to measure right ascensions, and any good scale of equal parts will serve for the small zone of declination.

26. *How to examine a difficult object to advantage.*

Suppose the object is a close double star, which has been found with a low power. Put in as high a power as the object will easily bear, and adjust for distinct vision. It is easy to hit a point of tolerably distinct vision by turning one way and the other, for growing indistinctness on either side shows the necessity of turning back. But on a delicate object the most perfect vision of the object can only be obtained by adjusting this point with the utmost nicety and repeatedly, and the focal point that is best suited for one eye will not answer for another. A near-sighted eye requires that the eye-piece be pushed in a little farther than usual. The object should be kept in the middle of the field, or suffered to cross the field diametrically several times. If the telescope is really excellent, and the object looked at not a very difficult one, few evenings occur in which, however disturbed the star may be, there are not momentary pauses and quiescences, during which a quick eye may catch a glimpse of neat and perfect separation between two well defined discs. A few such

glimpses will settle the point. If the night is a rare one, the most difficult objects may be attempted, and these should be reserved for such occasions, for it is useless to attempt to deal with them at ordinary times.

If the object is the very faint companion of a star, or the most diffuse parts of a nebula, a surprising advantage is usually gained, by directing the eye to a part of the field at some little distance. Besides indirect vision, a careful seclusion and rest of the eye from all light, even from that of the sky, for a short time, increases its susceptibility to the impressions of light. A moderately high power is sometimes needed upon faint stars to render them visible, although usually low powers are considered brighter. In all cases, whether the object be close or faint, different eye-pieces and different single glasses should be tried successively, to ascertain with what combination the effect is the best.

27. *To remedy the defects of a telescope.*

The outer border of an object-glass or speculum is frequently imperfect, on account of the difficulty of figuring and polishing surfaces true to the very edges. A great advantage in defining power is frequently gained by a diaphragm, or a succession of diaphragms, that shall contract the aperture by little and little.

All optic glasses, when dirty or dusty, should be wiped carefully with chamois leather, or clean soft cotton, free from dust,—never with silk, which will in time wear many scratches. But as a general rule, it is best to wipe the glasses as seldom as possible, and then lightly and quickly. Specks of dust interfere very much with perfect vision, especially on eye-glasses, and it is therefore almost needless to caution the observer to keep them constantly and closely covered, when not in use. The speculum of a reflector is more liable to become tarnished and dirty than an object-glass; but since it is more easily scratched, it is best to wait until the evil becomes very troublesome. A closely adhering film of dirt, and even an incipient oxidation, may be removed by spreading over the surface a drop or two of oil of turpentine, and, when it is nearly dry and hard, applying a piece of soft leather, with a firm hard pressure. The strong adhesion of the oil will bring off with itself all that can be safely removed from the surface of the speculum.

The action of a telescope that is unsteady in windy weather may usually be improved by bearing the farther end of the tube against some steadily yielding support, as a bar of wood leaning against the side of the telescope, or a double-folding window-shutter.

28. We will conclude the subject by pointing out to the observer the different ways, in which he may employ himself agreeably and usefully with the aid of the telescope.

There is enough of strong interest and deep instruction in surveying the more secret of the wonders of the sidereal heavens, to render the use of the telescope, with no other object than personal information, an occupation elevating to the mind, no less than it is novel and delightful. And one who thus seeks for himself and his friends, a more intimate acquaintance with the grandest works of nature, than is commonly obtained in the progress of education, cannot be said to spend the fraction of time which he devotes to these studies otherwise than most usefully and profitably. At the same time, there are few observers, who, if in addition to these sources of gratification, they could be conscious of adding a tribute, however humble, to the sum of human knowledge, and of advancing a little a science, whose long and splendid history, and present greatness, is written in the successive contributions of individuals, would not feel a more keen interest and animation in the pursuit. A few of the more eligible modes of employing the telescope to advantage, will therefore be suggested.

29. There have been multitudes of those curious systems, called double and triple stars, observed, carefully registered in distance and position, and inserted in catalogues. A considerable part of them have been reviewed, and material, sometimes very considerable changes of angle and distance in a very few have at once detected a rapid revolution. Great numbers have never been examined but once; and among these some revolving or *binary* stars doubtless lie concealed for want of a second observation. The *angle of position* with the meridian is recorded in catalogues in two different ways; the old method reckons it from 0° to 90° in each quadrant from the parallel towards the

meridian; while that more recently adopted reckons from the north point, around the circle in the direction *n, f, s, p*. Thus 10° *s p* corresponds to 260° of the new nomenclature. The observer may sweep a small zone of the meridian in the way recommended in Art. 25, and examining each faint and very close double star that has not been repeatedly observed before at very distant intervals of time, record its approximate distance and angle of position. The diagrams in Art. 23, will readily suggest the mode of estimating the latter, and the insertion of corresponding cross-lines in the field of view faintly illuminated will much aid the judgment. Any star that has varied its angle of position by 10° or more, would in this way be instantly detected by a careful observer. And the demonstration of the comparative fixity of the remainder would be of no inconsiderable importance.

Thus the observer could not mistake the changes either in distance or position, which have occurred and which will occur in γ Virginis, as represented in fig. 9. That he may avail himself of its rapid motion to try the accuracy of his estimations from year to year, we subjoin a table of the successive meridional appearances of this interesting system until 1860:

γ Virginis.

Date.	Distance.	Position.	Date.	Distance.	Position	Date.	Distance.	Position.
1837.0	0.″5	109°	1840.0	1″.4	31°	1845.0	2″.3	10°
1838.0	0. 8	60	1841.0	1. 6	25	1850.0	3. 1	1
1839.0	1. 1	41	1842.0	1. 8	20	1860.0	4. 3	350

σ Coronæ, another of our test-objects, is rapidly increasing its distance in much the same way, and from nearly the same epoch of perihelion passage.

A good micrometer attached to a telescope will furnish employment enough to the most active astronomer. It will enable him to detect far slighter changes in distance and position than mere estimation can do,—to confirm old determinations,—and to settle the elements of new objects. (See Art. 33, on "the Micrometer.")

30. Again, there are probably many telescopic comets roaming abroad in different parts of the heavens, especially in that

part of it which is nearly lost in twilight. A telescope of large aperture and great light, with very low power and a wide field of a degree or two, or even more, (aptly termed a *sweeper*,) is admirably adapted to explore carefully the twilight sky, and keep watch for comets. Any nebulous appearance is quickly detected, and its motion and situation will show very soon whether it is a nebula or comet. The announcement of the discovery of such a body is instantly followed up by careful observations upon it at the principal European observatories.

31. *Eclipses* of the sun and moon, occultations of stars by the moon, the eclipses of Jupiter's satellites, &c., are events of much interest, and good observations of the exact moments of their occurrence are always esteemed important additions to practical science. If the observer has access to a transit instrument, or sextant, his telescope will afford very useful occupation in such kind of observations. An immersion or emersion of a bright star at the moon's dark limb is a beautiful sight, often absolutely startling, for the star flashes into view or vanishes from sight in dark space without a moment's warning, and as far as sense can discern, instantly. The time can usually be noted to the accuracy of the fraction of a second at the dark limb, but is more uncertain at the bright limb, especially when the star occulted is below the 3rd magnitude. But to be well prepared for phenomena of so sudden occurrence, it is necessary to know beforehand the approximate time of immersion and emersion, to the nearest minute or two, and the angles from the moon's vertex reckoned around her circumference, at which the star will disappear and reappear. Directions for calculating these roughly will be given in the chapter on "Eclipses and Occultations." The observer, with such preparation, has only to count from the clock for two or three minutes before the time appointed, with his eye on that part of the moon's edge indicated by the calculated angle, and very nearly at the expected instant and place, the star will appear or disappear, as the case may be. The chapter on "the Transit Instrument" will direct him how to conclude the true time of the observations from the recorded instants, and he will then have the means of determining his longitude, if he chooses, for which purpose he may consult the chapter on that

subject. But, if he does not wish to carry his observations to their conclusions, their publication will place them in the hands of those, who will make the requisite use of them.

But there are a number of highly interesting phenomena, whose laws are as yet little more than conjectural, connected with eclipses and occultations; these need the light that can be thrown upon them only by observation, and will render it worth the observer's while to watch these phenomena closely, even if he has no knowledge of the true time. Thus, the first indentation of the moon in a solar eclipse is probably in many cases a deep, acute notch in the sun's edge, soon blending with the moon's limb; and in an annular eclipse, the phenomena attendant on the formation and rupture of the ring are beginning to excite much interest and careful attention. In occultations, some bright stars, and Aldebaran especially, have been seen in a number of cases to advance apparently upon the moon's bright or dark limb, and that to a very perceptible distance, before they vanished,—a point strongly demanding the notice of future observers. And it is desirable to know whether there is ever any perceptible distortion or diminution of brightness in a star or planet on approaching the moon's dark limb, or any appreciable time occupied in the act of disappearance. These, and a number of other particulars, which are not yet free from mystery and uncertainty, need very much the aid, which active and careful observers, with no other instruments than excellent telescopes, can bestow.

They may be classed as follows;—first, those which serve to measure small celestial arcs, either directly in the field of view, or indirectly on the limb of an instrument. These are *micrometers* under their varied forms, the *vernier*, the *reading microscope*, &c.

Secondly, those which aid in determining fixed points of the sphere, or of the limb of an instrument, especially the zenith and horizontal points. These are the *level*, the *plumb-line*, the *artificial horizon*, the *floating collimator*, &c.

33. *The micrometer* is the most important of all those instruments which are to be used only in connection with others. Its object is to measure distances in the field of view. Suppose we insert a pair of movable wires exactly in the plane of the aerial image or focus of a telescope, and are enabled to measure their separation by a delicate scale outside of the tube; we obviously have the means of accurately measuring the dimensions and relative distances of the images of planets and stars. For the images of celestial objects and the movable spider-lines are equally in the focus of the eye-glass, and the eye refers both to the sphere of the heavens. The scale of the micrometer is, however, only a comparative one, and the *value* of any portion of it, as an inch, or fraction of an inch, must be found in seconds of space by measuring bodies of known diameter, or stars of known distance, or by noting the time in which a star of known rate of motion passes over a given interval.*

The numerous varieties of micrometers are all calculated to measure the focal images of the heavenly bodies by reference in some way or other to a delicate scale, and may be classed under three principal species. The *first* comprises those in which some simple form of scale is inserted in the field of view at the focal image; the principal of which is the wire or spider's line micrometer. Cavallo's transparent scale of mother of pearl, reticulated diaphragms and networks of lines, and annular micrometers belong to this class; and it may be said to include also the cross-lines in the field of view of all fixed instruments. The *second* are *double image* micrometers, in which, by the division

* See the method of finding "the equatorial interval" in the chapter on "the Transit Instrument."

of some lens, mirror, or prism, two images are actually made of the same star in the plane of the focal image, and their separation is measured by the separation of the halves of the lens or mirror, and an attached scale. The chief of these is Dollond's object-glass micrometer. The *third* are *binocular* micrometers, in which distances and diameters seen in the field of view with one eye, are compared with some species of scale seen outside of the telescope with the other.

34. We shall take for an example the most common form, the wire or spider-line micrometer, and refer to fig. 10. in our brief description of its parts and their use. It consists of a closed shallow box $\frac{1}{4}$ or $\frac{1}{2}$ inch thick in the direction of the axis of the telescope, 1 or 2 inches broad, and 3 or 4 long, with a divided screw head at each extremity. Its general appearance, when attached to a telescope, may be seen in fig. 11. If we should take out the eye-piece, and remove the cover, the appearance in fig. 10. would be presented: *aaaa* are the sides of the containing box, seen edgewise; the two forks of brass, *bbb*, *ccc* slide one within the other and in opposite directions, and across them are respectively stretched the spider-lines, *d* and *e*. To them are firmly fastened the screws *ff*, which, passing through the ends of the box, enter the nuts or divided heads *gg*. It is obvious, that whenever these latter are turned in the direction indicated by the figures on their circumferences, the forks *b*, *c* will be drawn outwards; and on turning in the contrary direction, the springs *hh* tend to push the forks inwards, and thus prevent any shake or loss of motion in the screw. The screws have about 100 threads to the inch, and one revolution of the divided head *g* therefore carries the line *d* over the $\frac{1}{100}$th of an inch; but to such exquisite perfection has the cutting of these screws been carried, that by dividing the circumference of the nut *g* into 100 parts, $\frac{1}{100}$ of each thread, or $\frac{1}{10000}$ of an inch may be perfectly reckoned. The field of view is oblong, and within it are seen the lines *d*, *e*, and on one side a notched scale of teeth corresponding in size to the threads of the screw; every fifth one of these are cut deeper than the rest, and they are numbered from zero at the centre by tens in each direction. The spider-lines may be brought to coincide at zero, and even to glide by each

other a little way, e passing very close under d. Suppose them both at coincidence at zero of the scale, and that it is required to measure the diameter of the sun; turn either nut, as g, until d is drawn so far out, as to touch one limb, while e touches the other; read off in the field of view how many notches have been passed over by the wire, and on the divided head against a fixed mark the fractional part. Thus, if d is between the 22nd and 23rd notch in the field, and the diamond mark k stands at 72, the measure will be 22.72 revolutions of the head. This can be reduced to seconds of space by knowing the *value* of each thread of the screw, (see Art. 33.) If e also has been moved in the opposite direction, the space passed over by that thread must be added to the other.

When this apparatus is attached to, and carried round by, a circle, divided into degrees and parts, it becomes a position micrometer. By placing the fixed wire l so as to bisect both the stars to be measured, their angle of position with the meridian may be ascertained. Fig. 11. represents such an arrangement; the divided circle a, and with it the box b and attached eye-piece, are carried around to any position desired by turning the milled head s, and the angle is read off by aid of the fixed vernier c.

35. *The vernier.*—It is an undertaking as difficult as tedious to divide the limb of an instrument with refined accuracy. This renders it desirable to make as few divisions as possible, and descend to any further subdivision by some contrivance, which can be applied successively to different parts of the limb. Such are the vernier and reading microscope.

The principle of the vernier is as follows: If a small arc of equal radius with the limb of an instrument, and sliding in coincidence with it, be divided so that any number n of its divisions shall correspond to $n-1$ or $n+1$ on the limb, it will enable the observer to subdivide each division of the limb into n parts. Without going into details, it is easy to see in fig. 12., that if the 20 divisions of the vernier aa are equal to 19 on the limb bb, and the 4th of the twenty exactly coincides with any one on the limb, the diamond mark must be $\frac{4}{20}$ of the distance from c to d. The space $c\ d$, between the divisions 32° and 32° 20′, is thus subdivided into 20 parts or minutes, and the reading of the limb is 32° 4′.

The reading microscope is the application of a spider's line micrometer to a microscope, to measure minute distances on the limb. The image of the limb is received in the same plane with the spider-lines like that of a celestial object, and the fractional part of *cd* pointed out by the zero of the micrometer may thus be measured. The instrument is represented at the end of the arc in fig. 12.

36. *The level* is the most important of those instruments, which serve to point out the zenith and horizontal points, and enable the observer to reckon distances from them. One of its most valuable applications is to the transit instrument, and a description of its use in this connexion will readily suggest the modifications in its adaptation to circles, and fixed instruments generally. There are two very common forms of the level,—termed the *hanging* and *riding* levels; these terms relate to the position of the instrument with respect to the axis of the transit, and sufficiently explain themselves. The tube of the level is nearly filled with alcohol and hermetically sealed, so as to include a bubble of air. It is ground within so that the upper side shall be very slightly convex upwards, and when nearly levelled, the bubble will of course rest at the highest point of the curve, or where a tangent to it is exactly horizontal. Fig. 13. represents a hanging level, suspended from the axis of the transit, and embracing each pivot in the way exhibited in fig. 14. Its use is to render the axis of the transit perfectly horizontal. How then are we enabled to effect this object?

[redacted]

before. But if the pivot P′ be higher than P, and the arms are equal, the bubble will still run towards P′ after reversion, and of course will no longer retain the same place that it did in the first position of the transit, but one as far on the other side of the central zero; although apparently the same as before, because the reversion of the level reverses also its scale.

Remembering, therefore, that a difference in the lengths of the arms of the level has *no* tendency to change the *real* place of the bubble in the two positions, but that all *change* or *difference* of place is due to inclination of the axis, we have an easy means of rectifying such inclination. For if after reversion, the bubble should be found not at the same real point of the scale as before, but should be removed towards either pivot, that pivot is of course too high. It must be lowered until the bubble has gone back half way to the point it occupied in the first position, for it is plain that the first point is as much too far on one side as the second is on the other. If now in the two positions, it remains at the same division of the scale, the axis is truly horizontal; otherwise, the error yet uncorrected must be diminished by a second trial.

After the transit axis is carefully levelled, if the bubble does not rest at the centre, it may be made to do so by lengthening the arm, toward which it tends. Where there is no screw for this purpose, the adjustment can always be effected by scraping or filing a little the internal angle of the shorter arm.

The place of the centre of the bubble on the scale may easily be inferred from the readings at its two extremities, and is half way between them.

37. *The plumb-line* is always perpendicular to the surface of still water, and therefore marks the observer's zenith. A single example will illustrate its astronomical utility. In the altitude and azimuth instrument, a plumb-line is attached to the vertical axis at its upper extremity, and hangs by its side, so as exactly to cover the image of a fine dot near the lower extremity of the axis, when viewed by a reading microscope. The test (and a very severe one) of the exact verticality of the axis, is the exact bisection of this dot by the plumb-line, while the axis is turned completely round in azimuth.

38. *The artificial horizon* is any level reflecting surface. Although the surface of still water, of pure fluid pitch, &c. will answer the purpose, mercury is found to be the best. It is usually contained in a shallow wooden vessel, and a roof, the two sides of which are of glass, protects it from the wind. The incident ray passes through one glass nearly perpendicularly, and the reflected ray passes out at the other. This instrument generally accompanies the sextant in astronomical observation.

39. *The floating collimator* is a late invention, and consists of a small telescope supported on a float in a vessel of mercury. A visible point of light may be placed in its focus, the rays diverging from which, after passing through the object-glass emerge parallel, and it may therefore be viewed as an infinitely distant star, by a telescope attached to any mural or other vertical circle. Since the axis of the floating telescope always preserves the same inclination to the horizon, a reversed observation on opposite sides of the fixed circle fixes the zenith point of that circle. There are two forms of this instrument, the *horizontal* and *vertical*, terms which designate the position of the floating telescope.

CHAPTER III.

OF THE SEXTANT—ALTITUDE AND AZIMUTH INSTRUMENT—EQUATORIAL.

40. *The sextant.*—By adapting the principle of reflection to the measurement of angles, a number of instruments have been invented, which are independent of any fixed support, and may therefore be used in any situation, however unsteady. The *sextant* is the most common of these instruments, and its peculiar importance to the navigator, no less than its general utility in the observatory, is sufficient reason for dwelling at some length on its several adjustments, and on the manner of using it in the measurement of angles. For a description of the principal parts of

the instrument the student may refer to Art. 129 of Olmsted's Astronomy.

41. The principal adjustments of the sextant are as follows:

(1.) *To make the index-glass perpendicular to the plane of the sextant.*—Move the arm ID (fig. 15.) from zero near F towards the middle of the arc; and turning the limb or arc FE from the eye, look at its reflection in the index-glass I. If the reflected portion of the limb is a perfect continuation of the part seen by direct vision, the index-glass is truly perpendicular; but if not, it must be made so, by alternately loosening and tightening the screws behind it. But this adjustment in a good sextant will seldom be found deranged, except by violence.

(2.) *To set the horizon-glass perpendicular to the plane of the sextant.*—Screw in the telescope T, and point it towards a star. Move the index-arm backwards and forwards past the zero 0 of the limb, and if the two images of the star do not exactly coincide in passing one another, turn a screw at the top or bottom of the horizon-glass H until such coincidence can be made.

(3.) *To make the horizon-glass parallel to the index-glass when the index is at zero.*—When 0 of the vernier is at 0 of the limb, the horizon and index glasses should be parallel, and the two images of a star should perfectly coincide. If they do not, adjust by the screw at the side of the horizon-glass. This adjustment should be repeated alternately with the last till both are as perfect as may be.

(4.) *To set the axis of the telescope parallel to the plane of the sextant.*—There are two parallel wires on opposite sides and equidistant from the centre of the field of the telescope, and usually crossed by two others. Turn either pair around until they are parallel to the plane of the instrument. Adjust by the screws in the ring which holds the telescope, until the images of two stars more than 90° distant from each other, having been brought to perfect coincidence on one wire, shall remain so on the other.

These adjustments may be performed, although less accurately, in the day time by means of the sun, and the dark glasses of the instrument are then to be employed. Of these there are two sets; one between I and H, and perpendicular to the reflected ray IH; the other just beyond H, in the line TS. As many as

are needed of the first set may be turned up when the image of the sun is received by reflection, or of the other when it is viewed directly. The two surfaces of each glass in a good sextant should always be perfectly plane and parallel, and are proved to be so, if the two images of the sun, when brought into exact contact, remain so after the glass under trial has been taken out and reversed.

42. On the completion of these adjustments, the sextant becomes an accurate instrument, and may be employed in angular measurement. But the observer must first know—how to move the index-arm in measuring any distance,—how to read off the angle when measured,—and how to determine the index error.

(1.) *To move the index-arm in measuring angles.*—There are two screws attached to the moving extremity of the index-arm ID; one may be seen in fig. 15, and is beneath the limb; the other (not represented in the figure) is called the *tangent screw*, and lies in the direction that its name implies. The first fastens the arm after it has been shifted nearly to the point desired, and the tangent screw, acting only when the other is fixed, serves by a slow motion to bring the images into perfect contact. This screw moves the arm but for a short arc of the limb, and the observer should be particularly careful not to force it at its limit of action.

(2.) *To read off the value of a measured arc.*—Observe what minute and part of a minute on the vernier coincides exactly with a division on the limb; and add this arc to the degree and minute on the limb immediately preceding the first division on the vernier. (See Chapter II. on "the Vernier.")

(3.) *To find the index error.*—The adjustment, No. (3,) will render the two images of the same star very nearly coincident when the index is at 0. But since they cannot be precisely so, it becomes important to know at what point of the arc exact coincidence takes place, since it is from this point that all angular measurement is reckoned as from the true zero. Therefore by means of the tangent screw, bring the two images of a star to perfect coincidence, (or to that point in passing by each other where they should coincide,) and read off the measure, calling it + when forward from zero, and — when backward, or towards

F. This is the index error, and is always to be subtracted from every other angle read off on the limb, paying attention to its sign.

Another method is to bring the two images of the sun into exact contact on one side, and then to make the reflected image pass the other and touch on the opposite side. The readings in the two cases must be marked + or —, according to their positions with regard to zero, and half their algebraic sum will be the index error.

43. *To measure the diameter of the sun.*—Proceed as in obtaining the zero error by means of the sun, and half the algebraic *difference* of the two readings will be the sun's diameter. In this case, two measures are taken in opposite directions from zero, and it is therefore needless to apply the zero error.

To take an altitude of a star or of the sun by reflection from mercury.—Set the index near zero, put in the telescope, and make the wires parallel to the plane of the instrument. By means of the handle, hold the instrument with the right hand, with its face to the left, and in the vertical plane of the star, towards which let the telescope be pointed. Two images will be seen in the field of view, one of which, viz. that formed by reflection, will apparently move downward when the index is pushed forward. Follow the reflected image as it travels downward, until it appears to be as far beneath the horizon as it was at first above, and the reflection of the star from the mercury also appears in the field of view; then fasten the index, and make the contact perfect by means of the tangent-screw, taking particular care that the images shall be midway between the parallel wires. The reading of the limb, diminished of course by the index error, will be twice the apparent altitude of the star at that moment.

To manage the sextant dexterously, the observer must acquire the art of giving it a swinging motion, as if turning on the axis of the telescope, which may be gained most easily by leaning the body over gently to the right and left alternately. The image reflected from the index-glass may thus be made to sweep the arc of a circle, convex downwards, and in slowly passing and repassing the image seen in the mercury, the contact may be very accurately judged of.

In taking an altitude of the sun, proceed as with a star, turning up dark glasses between H and I, and also beyond H, for the protection of the eye.* If the altitude of the *lower* limb is to be observed, and an inverting telescope is used, make the image reflected from the index-glass sweep *under* that seen in the mercury, turning the tangent-screw until it just touches in gliding by, and *vice versâ*. The image in the mercury can always be distinguished from the other by its remaining fixed, and being often affected with tremors.

The beginner, in attempting to bring down the star to its image in the mercury, will probably lose it, and be obliged to commence anew. If therefore he has the means of knowing the approximate altitude, he may set the index at twice that angle, and point the telescope at once to the star in the mercury. The swinging motion will then make the reflected image pass horizontally through the field.

To take an altitude by means of the natural horizon.—If the observer is at sea, the natural horizon must be employed, and the arc measured, (after subtracting the index error, dip, and refraction,) will be the altitude. The star or sun's limb must be made to graze the horizon by the swinging motion. This method of taking altitudes is sufficiently accurate for the navigator, but not for the astronomer.

To find the distance between the moon and sun, or between the moon and a star.—The same management is necessary here as in taking an altitude, except that instead of holding the sextant in a vertical plane, it must be held in the plane passing through the two objects and the eye of the observer. When the index has been set to the approximate distance, or brought thereto by following the reflected image, the swinging motion will bring the sextant exactly into this plane, and the two bodies will appear together. Let the reflected limb of the sun rise and fall by that of the moon until perfectly tangent to it, as in observing altitudes. So with a star, make the moon's reflected limb just touch it in swinging by. A little practice, or a rule similar to that prescribed in taking altitudes, will enable the observer at once to decide, whe-

* A combination of a light red and green glass commonly gives an image less unpleasant and heating to the eye, than the darkest image that can be made by red glasses alone.

ther it is the distance between the *nearer* or *farther* limbs of the moon and sun that has been taken.

The use of the sextant in connexion with the artificial horizon, will be again alluded to in the chapter on "the methods of determining the latitudes and longitudes of places."

Of the Altitude and Azimuth Instrument.

44. Instruments of this kind, however varied in their form, consist (1.) of a graduated circle confined to a horizontal plane, and turning freely in that plane along with its vertical axis; (2.) of another graduated circle secondary to the former, attached to its vertical axis, and capable of being brought into any vertical plane by its motion; and (3.) of a telescope firmly fastened to the secondary circle in its own plane, and turning with it in altitude. Each circle is read off by verniers or reading microscopes, usually three in number, and the mean result of such readings is recorded after each observation.

By turning the instrument so that the intersection of its cross-wires shall exactly coincide with the image of a star, and noting the instant of such bisection, the altitude and azimuth of the star at that moment will be obtained from the readings of the circles. But it is first necessary to determine the points on the circles from which the reckoning commences. The meridional point on the azimuth circle is its reading when the telescope is pointed north or south, and may be determined by observing a star at equal altitudes east and west of the meridian, and finding a point half way between the azimuthal angles recorded in the two cases. The horizontal point of the altitude circle is its reading when the axis of the telescope is horizontal, and may be found by aid of the level, plumb-line or collimator, or by alternate observation of a star directly and reflected from mercury, taking a mean between the two recorded angles.

45. This instrument, in its capability of determining the place of any star above the horizon at any hour of the night, has an advantage over such as are confined to the meridian, like the transit instrument and mural circle. But stars are not located in the heavens by altitudes and azimuths, because these are constantly changing from moment to moment; and measures with

this instrument are therefore not complete without a record of the instant of observation, and must be reduced to right ascensions and declinations by processes of spherical trigonometry, laborious when accumulated. This circumstance tends to confine the use of the instrument in practice to meridional observations; and here its construction gives it a great advantage in taking repeated measures of right ascension and declination a little on each side of the meridian, which admit of reduction to that circle by easy formulæ, and have all the weight of accumulated observations.

Of the Equatorial Instrument.

46. If an altitude and azimuth instrument be turned from its upright position so that its axis, instead of pointing to the zenith, shall be directed to the pole of the heavens, it becomes an equatorial. What was before the azimuth circle, now lies permanently in the plane of the equator, and the altitude circle in the new arrangement can be turned into no position, in which it will not coincide with a horary circle, or one of declination. The circles are called respectively the horary and declination circles; and the former is usually graduated into hours, minutes and seconds of time, instead of degrees, minutes and seconds of space, like the azimuth circle.

To determine if an equatorial is in approximate adjustment,—follow a star in its diurnal course by means of the horary circle, the declination circle remaining clamped. The star ought always to pass the field of view at the intersection of the crosswires at all hour angles, or rather, to pass apparently below them by the amount of vertical refraction due to its altitude.

To find a star,—set the telescope to the declination of the star in the meridian position, and then turn the hour circle to the star's horary angle at the moment, which is always the time elapsed since it last passed the meridian, and is equal to the sidereal time then shown by an adjusted clock, *minus* the right ascension of the star. It is best in practice to set the hour circle at a horary angle corresponding to an instant two or three minutes in advance, to allow of time for preparation.

To determine the approximate right ascension and declination of an unknown object,—bring the object on the intersection of

the cross-wires, and note the corresponding instant of time shown by the clock; the reading of the declination circle will be its declination, and its right ascension, (if the hour circle is graduated from west to east up to XXIV^h uninterruptedly,) will be the corrected sidereal time of observation, *minus* the reading of the hour circle. The result will not be very accurate unless the instrument is in good adjustment, the errors of the zero points of both circles allowed for, and the corrections for refraction, especially at low altitudes, applied.

47. The oblique position of the circles of this instrument and of their axes, renders it nearly impossible so to combine materials, as to prevent unequal strain and bending of its parts. This defect, though very minute in amount, is sufficient to exclude the equatorial from the highest accuracy of observation, and refer the formation of stellar catalogues to instruments more symmetrical in their positions relative to the horizon,—a circumstance much to be regretted, since no instrument, except the transit-circle, could furnish the requisite data more easily and rapidly. The equatorial, however, is nevertheless exceedingly useful in determining *small differences* of right ascension and declination, such as between objects in the same field of view;—in enabling the observer to follow with ease and measure with certainty double stars and other objects, and to settle their places at the same time so as to be recognisable thereafter;—and in fixing with sufficient accuracy the places of comets and other bodies, which cross the meridian at a time when it is either impossible or inconvenient to observe them. For its peculiar adaptation to these important uses, the equatorial is in very frequent demand, and holds a high place among astronomical instruments.

CHAPTER IV.

OF THE TRANSIT INSTRUMENT.

48. This most important of astronomical instruments, in connexion with a clock or chronometer, is applicable to such a variety of purposes, and especially gives the observer so complete a command over that primary element, *the time*, that we may be permitted to consider at some length its several adjustments, the means of rectifying them, the conduct of actual observation, and the best mode of eliminating instrumental errors. In the body of the chapter, such minuteness is not aimed at, as would be uninteresting to the more numerous class of students, who are desirous of understanding only the general method of proceeding in an observatory, or such refinement, as would discourage any one, unaccustomed to mathematical details; while a few pages are appended, by which the observer will be enabled to choose his mode of proceeding, and reduce his observations, so as to obtain the most accurate results within the reach of his instrument.

49. *Location of the instrument.*

Let us suppose that the reader is in possession of a good portable transit and clock, and is desirous of using them to advantage. The situation most suitable for the instrument would engage his earliest attention. And first, it should have a firm basis; the pier on which it stands should be of stone, brick, or other solid material,—should descend into the ground several feet, so as not to be affected by frost, or by tremors arising from the vicinity of public roads,—and should rise only so high above the ground, as to afford a clear view of the north and south points. The next desirable requisite is, that it should command a view of the whole, or a considerable portion of the meridian. The room, therefore, in which it stands should have openings in the roof in the line of the meridian; these should be about 12 or 15 inches wide, and should be continued down the north and south walls so as to afford a clear sweep from the north to the south

point of the horizon. They should be closed by doors easily opened from below, and should be secure against the admission of rain or snow. If such a situation as this cannot be procured, the transit may be placed on a solid pier out of doors, and guarded by a tight cover from the weather; or if it can be firmly placed in a north or south window, so as to command a range of 60° or more in altitude, it may be used so as to afford very accurate results. A south window is preferable to a north, especially as it commands the passages of the sun, moon, and planets.

50. *Description of the mechanical contrivances for adjustment.* The general construction of the frame of the instrument may easily be understood by an inspection of fig. 16, or by reference to Olmsted's Astronomy, Art. 121. The two ends of the axis are called pivots, and great care is taken to make them exactly equal and cylindrical; they rest in angles of polished steel, or other very hard substance, shaped as at Y, (fig. 14.) because contact at only two points in the circumference of the pivot ensures greater accuracy of meridional motion. These angles are called Y's, from their resemblance in shape to that letter. At one end of the axis there is usually a screw, by which the Y of that extremity may be raised or lowered a little, and thus the axis be made perfectly horizontal. This adjustment is made by means of the level, (Chap. II., Art. 36,) and is called the adjustment for *horizontality of the axis.* Another screw moves the Y at the other extremity backwards or forwards, and by its azimuthal motion is of use in bringing the telescope into the plane of the meridian, when a little east or west of it. This adjustment is termed that of *position in the meridian.* When the frame of the instrument rests upon three feet screws, one of them may supply the place of the screw of elevation. The perpendicular wires of the telescope are either three, five, or seven in number, but most commonly five; in small instruments only one horizontal wire is inserted. To make these wires exactly perpendicular, there is a contrivance for moving them around with a circular motion in their own plane; and by another arrangement they admit of lateral motion for the *adjustment of collimation.* These contrivances are different in different instruments. The field of view is illuminated at night by a lamp placed so as to shine into

one of the pivots, the axis being perforated for this purpose. An oval ring of painted metal, or card paper, placed at an angle of 45° within the junction of the axis and telescope, and encircling without obstructing the pencil of light from the object-glass, turns the light down the tube, and makes the wires appear black on a bright ground, yet does not obliterate the brighter stars.

The axis is furnished at one of its extremities with a circle and vernier, by which the telescope can be directed to any required altitude or declination. The vernier usually admits of being unloosed and refastened so as to point to different portions of the circle while the telescope remains stationary.

All the screws of adjustment, and indeed, all parts of the transit, should be fitted without shake or improper motion; and when the adjustments are once made, they should be as permanent as possible. For this purpose, clamps or tightening screws are attached to such of them as require it, and these should always be attended to after adjustment.

With this brief description of the more important parts of the instrument, we will now proceed to the adjustments, which should succeed each other in the following order:

51. *Distinctness of vision and parallax.*

The optical adjustments of the telescope are the first to be examined. The system of wires or spider-lines is in a plane perpendicular to the axis of the tube, and set in a circular aperture very near the eye-end. It should be in the common focus of the eye-glass and object-glass. First, to place the lines in the focus of the eye-glass, push in or draw out the eye-tube until they are seen with perfect distinctness. Next, for the purpose of throwing the focal image of the object-glass in the exact plane of the wires, either the object-glass or the wires are set in a tube which admits of motion within the main tube, and which is fastened securely by a fixing screw after the proper adjustment has been obtained. This is only the case when (the wires appearing perfectly well defined by the first adjustment) the images of objects at the distance of one mile or several are also as distinct as possible. A still surer test of the perfection of this adjustment arises from the circumstance, that if the image be cast either before or be-

hind the lines, a parallax will be detected by moving the eye laterally, and distant objects will move on the lines. The images of near objects, however, ought neither to be entirely distinct or motionless, when the telescope is in perfect order. These adjustments are called the adjustments for *distinctness of vision* and *parallax.*

The optical excellence of the telescope may now be tested as directed in the chapter on "the Telescope," Art. 8. Any imperfections in the object-glass or eye-piece, which may render either the star or wires distorted or ill defined, especially if they exhibit a star of the first magnitude otherwise than round, will tend to diminish the accuracy of observation. After the transit has been placed in the meridian, and the wires adjusted as described hereafter, let a star run occasionally upon the horizontal wire, and if it does not remain perfectly bisected while the eye is moved up and down, the adjustment for parallax is not quite perfect.

52. *Horizontality of the axis.*

In our illustration of the use of the level, (Chap. II., Art. 36,) we have given ample directions for levelling the axis of the transit.

We have remarked that in a perfect instrument, the pivots are exactly equal and round. The level is competent to detect any imperfection in these respects. For suppose we take the transit axis from its Y's, and reverse it end for end; if the bubble does not give the same indications in the two positions of the axis, but shows a tendency towards either pivot, that pivot is of larger diameter than the other. And if the level be made to bear against a fixed support while the telescope is turned in altitude, any motion in the bubble will prove that one or both of the pivots are not exactly round. These are not accidental errors, but inherent faults of the instrument, and recourse to the original maker or to an excellent workman is the only means of remedy.

The level is liable to give erroneous indications, unless it hangs always with the same face upwards. If, therefore, by swinging it gently back and forth, the position of the bubble is materially altered, a cross level should be firmly attached to it, by means of which the same curve of the tube may always be made uppermost.

53. *Perpendicularity of wires.*

Some well defined terrestrial point may now be brought upon the intersection of the wires in the centre of the field by the screw of motion in azimuth. If, on turning the telescope in altitude, this point is perfectly bisected by the central wire from top to bottom, the wire is perpendicular to the horizontal axis. If not, the ring or tube containing the wires must be turned in a circular direction until it is so bisected, and there fastened.

54. *Collimation in azimuth.*

That point of an object-glass, through which a ray in passing suffers no refraction, is called its optical centre. A line drawn from this point to the central vertical spider's line is called the *line of collimation* in azimuth, and ought to revolve in the plane of the meridian. But if it is inclined to the transit axis, it will not trace out any great circle, but some small circle of the heavens. Suppose that the line of collimation produced to meet the same terrestrial point as before, leans towards the eastern pivot. By reversing the transit, it is evidently made to lean as far to the west, as it did before to the east, and the central wire will be thrown entirely off of the point which it before bisected. The true direction of a perpendicular to the axis is obviously halfway between its present and its former position. Therefore, by the contrivance for the lateral motion of the wires, make the central wire traverse half the distance by estimation from its present position to the point it first covered. To correct remaining error, bring the central wire again upon the distant point of reference by means of the screw of motion in azimuth, and repeat the process, until it is bisected from top to bottom of the central wire equally before and after reversion.

55. *Collimation in altitude.*

If the circle attached to the instrument is intended to indicate meridian altitudes, take the declination of any bright star that crosses south of the zenith during the evening, from the Nautical or American Almanac, calling it + if north, and — if south of the equator. Add to it the elevation of the equator to obtain its true altitude, and then the refraction due to its altitude. The sum will be the apparent meridian altitude. While the star is running

along the horizontal wire of the transit instrument, (supposed to be in the meridian, or very nearly so,) unloose the vernier, set it to the apparent altitude, fasten it firmly, and then see if the star is still on the horizontal wire. If declinations are indicated by the circle, the vernier must be set to the true declination, increased by the same refraction as before. By this process, the vernier will probably be correct within 2′ or 3′, and its zero error may be determined and allowed for. If the vernier be now set to the apparent altitude or declination of any expected star, the star will enter the field on or near the horizontal wire.

56. *Position in the meridian.*

This is the most difficult of the adjustments of the instrument, and requires that all the others should be first completed. In explaining the methods of adjustment, the clock will be considered as indicating sidereal time, in which case the right ascension of each star, as it arrives at the meridian, will be the same as the clock time at that moment. If, therefore, the pendulum of the clock has not been adjusted to the proper length, it should be shortened or lengthened until a star comes to the intersection at the centre of the field within a few seconds of the same time on two successive evenings. This may be done before the transit is brought very near a meridional position.

By the pole star.—When the telescope commands the northern portion of the meridian, this is the easiest and best mode of adjustment. First point it to the pole star, and then turn it to some other star about to cross the meridian at a distance from the pole. At the moment of its central passage, set the clock to its right ascension, and it will thenceforth indicate nearly sidereal time. The approximate times of the upper and lower culmination of the pole star are then known, being the clock times answering to its right ascension, and 12 hours thereafter. For a few minutes before and after either of these moments, on account of its extremely slow motion, it is almost exactly in the meridian. Follow the pole star therefore by turning the transit, till it arrives within half an hour or less of the meridian. The base of the frame may then be fixed by wedging or pouring hard cement underneath, if not sufficiently steady, and the horizontality of the axis should be tested. Still follow the star by means of the

screw of motion in azimuth until the clock shows its R.A., or its R.A.+12^h. The central wire then, if the previous adjustments have been well attended to, will almost precisely coincide with the meridian of the place, even if the clock be 2^m or 3^m in error.

Now, the axis being perfectly horizontal, and the line of collimation perpendicular to it, if the central wire by its motion bisect the small circle described by the pole star, the adjustment is complete. This will be the case, when the interval between the upper and lower is equal to that between the lower and upper transits, each being $=\frac{24^h}{2}$. Few portable instruments, however, are competent to render this star visible in the day time. Hence it is usually best in practice to rectify the clock or ascertain its correction the next evening, by stars at a distance from the pole, and repeat the process with a more perfect knowledge of the sidereal time.

By a pair of circumpolar stars.—Choose two stars which cross the meridian within a few minutes of each other, one above, and the other below the pole. Let A and T be the R. A. and observed time of passage of the upper star, and α and τ those of the lower. Then when the central wire of the transit coincides with the meridian, $A-\alpha=T-\tau+12^h$; when the wire deviates to the west at the north of the zenith, $(A-\alpha)<(T-\tau+12^h)$, and *vice versâ*. This will enable the observer to determine in which direction he must move the screw for azimuthal motion. Suppose he finds $(A-\alpha)>(T-\tau+12^h)$ by 5^m, and in consequence [redacted]

the east, but still by the application of the level is made a vertical circle, and of course, cuts the meridian at the zenith, and departs more and more from it towards the south. The southern or lower star obviously must be longer in crossing from the transit plane to the true meridian, than the upper star, which crosses where the interval is narrower: the time of its passage is therefore earlier than it should be in comparison with the other; that is, $T-\tau$ is greater than it should be, or $A-\alpha < T-\tau$, when the deviation is eastward, and *vice versâ*. The deviation should be corrected in the same way as by the last method. It may be proper to observe, that in this, as well as in the two preceding methods, the best way of correcting the error is by ascertaining its exact amount, which is easily obtained from the observed passages of the stars by the formulæ near the end of the chapter.*

57. *Location of the meridian mark.*

When the instrument has been once fairly brought to the meridian, a mark may be placed either to the north or south, or both, for the advantage of constant and speedy reference. It should be placed at such a distance, as not to be affected by parallax, (Art. 51,) and yet not so far as to be imperfectly seen. An

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* For, by form. 1, Art. 72, we have for the two stars,

$$\alpha = t + x + a \sin(\phi-\delta) \sec\delta \quad \text{for one star,}$$

$$\text{and } \alpha' = t' + x + a \sin(\phi-\delta') \sec\delta' \quad \text{for the other,}$$

where the two other errors b and c are supposed to be nothing.

Subtracting, we have $\alpha-\alpha' = t-t' + a \left\{\sin(\phi-\delta)\sec\delta - \sin(\phi-\delta')\sec\delta'\right\}$

$$\text{and } a = \frac{(\alpha-\alpha')-(t-t')}{\sin(\phi-\delta)\sec\delta - \sin(\phi-\delta')\sec\delta'} \text{ or the azimuthal deviation.}$$

For instance, take the stars α Piscis Australis and α Pegasi, an excellent pair of high and low stars, differing in right ascension by 8^m. By our Table V. for New Haven, $\sin(\phi-\delta)\sec\delta$ equals

for α Pisc. Aust.	1.102
for α Pegasi,	0.468
	0.634

which is therefore a constant divisor for these two stars at that place. Thus if the observed times of their passage, Jan. 1, 1840, differ by $+8^m\ 30^s.5$, then $(\alpha-\alpha')-(t-t') = +7^m\ 59^s.8 - 8^m\ 30^s.5 = -30^s.7 = 0.634\,a$, and a, or the azimuthal deviation, equals $\frac{-30^s.7}{0.634} = -48^s.4$. Therefore if the equatorial interval between each two of the wires is 64^s, the telescope must be pointed to a terrestrial object on the southern horizon, and screwed eastward about ¾ of one of the intervals.

excellent form of the meridian mark is a piece of sheet iron or copper 6 or 8 inches in diameter, cut into the shape of a hollow equilateral triangle; this, when placed on an eminence, so as to have the sky in the background, may be very perfectly bisected at the vertical angle by the central wire of the instrument. It should admit of being moved laterally, if not at first exactly placed in the meridian.

58. *The clock,—its rate and error.*

The clock is the indispensable companion of the transit instrument. The office of the transit is in fact, to point out the agreement or disagreement between the great and perfectly regular time-piece furnished by the apparent revolution of the sphere, and the irregular and imperfect clock of the observatory. It is available only as a plane of reference, an indicator or pointer to the grand motion of the sidereal heavens, thus rendering it comparable with the measurement of time by mechanism of human construction.

No clock is fit for the nicer purposes of astronomy, unless it is rendered as invariable as possible. For this purpose, much attention is always devoted to the construction of the escapement, which should not be such, as to allow of any inequality in the transmission of the moving power to the pendulum. Still more should the pendulum be of that class, called compensated; and so be free from the variable effects of heat and moisture. The two forms usually adopted for astronomical uses are what are commonly termed the gridiron, and the mercurial, (Art. 365, [redacted]

the interest of the observer carefully and *closely* to compare his own whenever he is forced to depend upon the latter.

59. If the clock is regulated to sidereal time, it is supposed in theory to keep exact pace with the stars, and to indicate at any moment the right ascension of the star then crossing the meridian. But in practice, it has both an *error* and *rate*. An example will best explain the meaning of these terms. If α Arietis,* in R.A. $2^h\ 0^m$, cross the meridian at $2^h\ 0^m\ 30^s$ by the clock, the clock is fast of the heavens 30^s, or its *error* $=+30^s$. The *correction* of the clock, or the quantity to be applied to its indications to obtain the right ascension of the star, is the same in amount, but with an opposite sign,—viz. in this case $=-30^s$. Now if another star α Aurigæ in R.A. $5^h\ 0^m$, cross 3^h after at $5^h\ 0^m\ 33^s$, and if the time of passage of 15 Hydræ, in R.A. $8^h\ 0^m$, on the same evening be $8^h\ 0^m\ 36^s$, it is very plain that the clock's error is increasing, or in other words, its *rate* $=+3^s$ in 3^h, and $+6^s$ in 6^h, or $+24^s$ per day. The *error* of the clock then, is its difference from true sidereal time at any given moment, + if faster, — if slower; its *rate* is its daily gain or loss on sidereal time, + if the former, and — if the latter. If the error at a given time be called e, and the daily rate r, the error at any time thereafter will of course be $e+tr$, where t denotes the number of days and parts of a day elapsed since e was determined. All that is necessary to find the true time at which any event happened, is to apply the correction of the clock, or $-e-tr$ to the observed time.

60. The rate and error are not by any means due to imperfection of the clock, and may be as large or larger in the most perfect as in the rudest time-piece. Yet it is convenient in practice that they should both be of small amount. The error may be nearly annihilated by setting the clock to the right ascension of a star, and starting the pendulum at the moment when that star is on the middle transit wire; and the rate may be readily brought within small limits, as 1^s or 2^s per day, by lowering the bob of the pendulum when positive, and raising it when negative. The number of turns by which the screw of the pendu-

* These stars have nearly, but not exactly, the right ascensions which are here assigned to them in round numbers for the sake of illustration.

lum is altered, may be compared with the consequent alteration of rate, and thus the ultimate reduction very certainly effected. After the rate has been reduced to a small quantity, say less than 1ˢ, it is better to let the error accumulate than to stop the clock. Indeed, it ought never to be stopped or to be suffered to run down, when in frequent use.

Whatever may be the error and rate, the clock is perfect if the rate is uniform, or equal in equal times. But in common clocks, the rate may be nothing for one day, and several seconds + or —, the next; and in ordinary uncompensated watches, this difference on different days may amount to 2 or 3 minutes. The latter cannot therefore be trusted for an accurate knowledge of the time. But with a compensated clock of the first order, an alteration of a second in the rate during a single day is scarcely to be apprehended.

61. *Method of observing and registering transits.*

We have hitherto supposed our observer to be so far acquainted with some method of observation, as to be able to note the times at which stars in the field cross the middle wire. A more particular explanation of the best modes of conducting observation becomes now necessary.

The field of view should be illuminated by the lamp mentioned in Art. 50, until the wires are perfectly and sharply visible. The vernier should be set so as to indicate the place of an expected star on the circle, allowing for refraction, (Art. 55,) and it will then enter the field at one side nearly upon the horizontal wire. In northern latitudes, the star enters upon the right hand side of the field if between the zenith and southern, or between the pole and northern horizon,—but if between the pole and zenith, it first appears on the left hand edge, and departs at the right; its apparent line of motion being inverted by the telescope. The stars should always be made to cross the same points on the perpendicular wires, that minute errors arising from their want of parallelism to each other and to the meridian may be avoided.

62. The star on entering the field will move slowly across it in a horizontal direction, and it is the business of the observer to

note the times of its passage across the transit wires with the utmost accuracy. The clock should be so placed, and its face so well illuminated, that the observer, stationed at the transit, can at any moment read the second indicated by the pointer. The second last counted *before* the star crosses the wire, is to be registered as the even second of passage; and the fractional part of the next second which elapses before the instantaneous bisection of the star may be most easily judged of by the eye, in comparing the small intervals of space by which the star is short of and beyond the wire, at the instants of the preceding and following beats. Thus, suppose the observer takes up the second 2 from the clock, and goes on silently counting 3,—4,—5,—6, &c., in exact coincidence with the beats as he turns to the field of view; if the star appears at the points 6, 7, 8, (fig. 17,) as he hears the corresponding seconds from the clock, 7 will plainly be the even second to be recorded. And if at the wire A the distance 7...A appears about $\frac{7}{10}$ of 7...8, as nearly as can be estimated when the object is in motion, $7^s.7$ is to be recorded on the journal. The minute during which the passage occurred should then be prefixed, taking care to make such allowance for the time occupied in estimating and writing down the seconds and tenths, that there shall be no danger in making the record one minute in advance.

63. The observer will not be long in perceiving that the spaces of time occupied by the stars in traversing the intervals between the wires, are very different on different points of the meridian; being shortest at the equator, and longer and longer as the star is more distant from that circle. The time occupied by a star at the equator in passing between any two of the wires is called their *equatorial interval;* and this time, converted into minutes and seconds of space, is the constant arc of a great circle included between such two wires. But a star whose declination is $+30°$ or $-30°$ moves more slowly than one at the equator; and the time in which it passes from one wire to another is equal to the equatorial interval, multiplied by the secant of the declination.* Consequently, to find the equatorial interval of a tran-

* Let PP′ (fig. 18.) be the axis of the heavens, EQ the equator, and DF a parallel of declination; let PEP′ be the meridian of the place, and PIP′ one a little inclined to it. A star at D moves over the arc DC in the same time that one at E moves

sit;—multiply the interval found from the passage of any known star by the cosine of its declination.

For the pole star, and others within 10° or 12° of the pole, a modification of this rule becomes necessary; for these do not pass perpendicularly from wire to wire, but describe considerable arcs of their respective small circles, as in fig. 19. Suppose the pole star describes the arc AS in 8^m; the arc AS of course $=2°$ of a circle whose radius is $\cos\delta$, and $AC = e = \sin 2° \cos\delta$, since AC is the sine, or very nearly the sine, of the arc AS.

64. In registering the times of observed transits, no rule is necessary other than to have a proper regard for convenience of reduction. The form of registry generally adopted by astronomers, is exhibited in our example, (Art. 76.) The best mode of obtaining the mean of the wires when 5 are used, is by the following rule:

Add together the seconds of the five transits, and multiply by the decimal .2, adding or subtracting from the product as many times $12^s \left(=\frac{60^s}{5}\right)$, *as will render it nearly the same as the number of seconds at the middle wire.*

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over EI; but since angle EOI = angle DXC $\therefore$ arc EI : arc DC :: circ EIQ : circ DCF :: EO : DX :: rad : sin PD or $\cos\delta$; δ being the declination of the star. Therefore $DC = EI \cos\delta$.

Now the time in which the star D would traverse the constant space DR or EI between the wires must be to the time in which E moves over the same, in the inverse ratio of their motions, or as EI : DC :: 1 : $\cos\delta$ :: $\sec\delta$: 1. If then e represent the

[redacted]

The *rationale* of the above rule is very apparent; multiplying by 0.2 is equivalent to dividing by 5, and the simple device of adding or subtracting multiples of 12 renders unnecessary the more tedious operation of adding the minutes as well as seconds, and dividing their sum as in compound division.

When 3 wires are used, divide the sum of the seconds by 3, and add or subtract $\frac{60^s}{3}=20^s$, as often as required.

In transits of the sun, the passages of each limb are to be taken and cast up by the above rule as those of two separate objects, and the mean of the results will give the passage of his centre. But if only one limb is observed, the passage of the centre may be inferred by adding or subtracting "the Sid. Time of Semidiameter passing the Meridian," as given on p. I. of each month in the Naut. Alm.

In transits of Jupiter and Saturn, when both limbs are taken, the appulses of the first limb may be noted at wires I, III, and V, and of the second limb at wires II and IV. If one limb only of a planet is observed, the ephemeris must be consulted for the time of passage of its semidiameter.

65. After obtaining from the mean of a great number of transits (from those of Polaris especially, if practicable,) the equatorial interval between each two wires respectively, (Art. 63,) if these are found to be unequal, a correction is necessary. Thus in the instrument which furnished the observations in Art. 76, the equatorial intervals deduced from a great number of stars, and agreeing also very well with the mean result for that evening, are as in the first of the following columns:

	Eq. Intervals.
Wire I— II	$63^s.90$
" II—III	64 .35
" III—IV	64 .19
" IV— V	64 .32

Hence we have distance of

	From mid. wire.	From mean of wires.
I	$-128^s.25$	$-128^s.27$
II	$-$ 64 .35	$-$ 64 .37
III	0 .00	$-$ 0 .02
IV	$+$ 64 .19	$+$ 64 .17
V	$+128$ .51	$+128$ .49

The mean of the transits over the 5 wires is the transit over an imaginary line nearly coinciding with the middle wire. In the instance before us, this line differs $0^s.02$ from the middle wire, and is towards wire V. Although the difference is accidentally

very small in the present instance, and might be safely neglected, we shall proceed as we would if it were of large amount. By addition of the equatorial intervals we obtain the numbers in the 2nd column, the sign — being applied to distances measured in a direction opposite to that of the star's motion. One fifth of the numbers in the 2nd column $=+0^s.02$, is the distance of the imaginary mean line from the middle wire, and applying it with changed sign to the distances in the 2nd column, we obtain those in the 3rd.

Now if through inadvertence, or unfavorable weather, the transits over only a portion of the wires are observed, the reduction to the imaginary centre may be performed by the following rule:

To the sum of the times of transit over the observed wires, add the sum of the distances of the unobserved wires from the mean or imaginary line, multiplied by sec δ; the amount divided by the number of wires observed will be the time of transit over the mean line.*

Thus, if a star is observed on wires I, II, and IV only, to the sum of the observed times add $(-0^s.02+128^s.49)\sec\delta = +128^s.47\sec\delta$, and divide by 3.

If II, III, and IV are observed, to the sum of the times add $(-128^s.27+128^s.49)\sec\delta = +0^s.22\sec\delta$, dividing by 3.

If the transit over the central wire only be noted, subtract (see Note) $-0^s.02\sec\delta$ from the time of transit. The result will be the same as if $(-128.27-64.37+64.17+128.49)\sec\delta = +0^s.02\sec\delta$ had been added, dividing the result by one.

This rule will apply to all cases that can be expected to occur. Under the pole, it must be recollected, the stars move across the wires in an opposite direction from the usual one, and all corrections to the middle wire must be with changed signs. The same must be done, if the transit is reversed, since then the wires are also reversed, and are crossed by the star in a different direction from before. For example, the distance of wire I from the mean is in the present case,

* These are taken in the present instance from the 3rd column given above. Instead of adding the distances of the unobserved wires, those of the observed wires may be subtracted, since the sum of the five must always $=0$. This course will be preferable where but one or two wires are observed.

Axis as usual	above the pole	$-128^s.27$
	below the pole	$+128\ .27$
Axis reversed	above the pole	$+128\ .27$
	below the pole	$-128\ .27$

and so for the others.

In filling up omitted wires of the planets, their motions in the intervals must be allowed for; in the case of the moon, this allowance increases the interval more than $\frac{1}{30}$th part, and her parallax at the side wires is also to be taken into account.

66. A few illustrations taken, with the exception of Polaris, from the general example at the end of the chapter, will render the application of these rules easy to the learner.

f Pegasi.

	$33^s.9$*
	$43\ .5$
$21^h\ 21^m$	$53\ .1$
	$2\ .8$
	$12\ .5$
	$145\ .8$
	$.2$
	$29\ .16$
Add	24
$21\ \ 21$	$53\ .16$

γ Pegasi.

	$56^s.8$	V.
	$3\ .4$	IV.
	$0^h\ 4^m\ 9\ .5$	III.
	$69\ .7$	
	$+198\ .80 = +192.64$ sec $14°\ 18'$.	
	$3)\ 268\ .50$	
	$89\ .50$	
Subtract	$80\quad = 4 \times 20$	
	$0\ \ 4\ \ 9\ .50$	

Polaris.

Wire V. . . . $+128^s.49 = +32'\ 7''.4$.

Decl. of Polaris, Oct. 17, 1840, . .	$+88°\ 27'\ 25''$.	
log. sin $32'\ 7''.4$	$+7.9705$	
log. sec $88°\ 27'\ 25''$	$+11.5698$	
	$+9.5403$	
	$+20°\ 18'\ 20''$	=

	23^h	40^m	$36^s.5$	I.
	0	21	36 .0	II.
	1	1	41 .5	III.
	1	41	35 .5	IV.
	98	45	29 .5	
	+	81	13 .33	
4)	100	6	42 .83	
	1	1	40 .71	

* It will probably be found most convenient in practice to set down the transits in this columnar form, at the time of observation, in any small note-book the observer has at hand. They can then be transferred to the regular journal as exemplified in Art. 76, the mean of the wires being previously cast up from the rough columns of the note-book. A regular system of book-keeping is no less important in the observatory than in the counting-house.

67. We thus obtain, in the most perfect way, the clock times of meridian passage for any number of bodies whose right ascension is known. And hence the error of the clock being known for a number of instants preceding and following any event, the error is of course known at the instant of its occurrence. This applied with a contrary sign at once transforms the clock time at which the phenomenon was observed to true sidereal time; which is usually the end and object of all the uses of a transit instrument out of the observatory.

All this, however, supposes that our instrumental adjustment of the transit instrument is perfect, which never is the case. So far from it, indeed, that without corrections to be applied for the errors of the instrument, the observer with an ordinary transit can seldom reckon with certainty on an accuracy greater than within 2^s or 3^s, and often much less. And even if no greater is desirable, yet some apprehension of the mode of correcting its errors is almost indispensable, that he may proceed with that knowledge of his means, and that acquaintance with the management of his instrument, so essential to inspire confidence in his results. The remainder of the chapter will therefore be devoted to a short consideration of the errors of the transit instrument, and their corrections.

68. *The errors of the transit instrument and their corrections.*

Let φ be the latitude of the place. If then δ represents the declination of any body upon the meridian above the pole, negative when south, and positive when north, and if when below [redacted]

transit. Or the same may be done imaginarily with fig. 18, calling PIP′ the meridian. The angle of deviation of the transit plane will then be measurable on the southern horizon, which call a, or

Let a = the azimuthal *deviation;* + when, measured on the southern horizon, it deviates towards the east, and − when towards the west; and let it be expressed in time,* and not in arc.

Now the deviation being equal to a at the horizon, will decrease as the two circles converge towards one another, till at the zenith it becomes 0; or at any known zenith distance, the distance between the two circles (Cor. 2, Note, Art. 63.) $= a \times$ sin zen. dist. $= a \sin (\varphi - \delta)$. Now a star south of the zenith in coming to the meridian, will cross this small space (Cor. 1, Note, Art. 63.) in a time $= a \sin (\varphi - \delta) \sec \delta$. If therefore the star arrives at the transit plane at the time t, it will pass from one circle to the other, and come to the meridian at the time

$$t + a \sin (\varphi - \delta) \sec \delta.$$

As $\varphi - \delta$, and of course $\sin (\varphi - \delta)$, is negative to the north of the zenith, the correction must always become subtractive between the zenith and the pole; and manifestly it ought, for the transit plane in that quarter passes over to the west of the meridian.

70. Suppose again the eastern pivot to be a little lower than the western, or which is the same thing, that the telescope revolves in a plane cutting the meridian in the north and south points, and turned a little over eastward from the meridian at the zenith. This position again may be imitated by bringing the poles on a globe or in fig. 18, to coincide with the horizon, and turning a colure from the zenith a very little over to the east. The error of inclination will then be expressed, of course, by the small arc of the prime vertical intercepted, which represent by

b = error of *inclination;* + when, measured on the prime vertical, it inclines towards the east, − when towards the west; and expressed in seconds of time.

The inclination being $= b$ at the zenith, decreases towards the

* Any great circle, as the horizon, may be supposed to be divided into hours, minutes, and seconds, instead of degrees, minutes, and seconds.

horizon either way, and the distance between the circles at any point is (Cor. 2, Note, Art. 63.) $b \times \sin$ altitude $= b \times \cos$ zen. dist. $= b \cos(\varphi - \delta)$; and a star arriving at the transit plane at the time t, will cross it, and (allowing for its rate of motion as before) arrive on the meridian at the time

$$t + b \cos(\varphi - \delta) \sec \delta.$$

Now $\cos(\varphi - \delta)$ does not change its sign when $\varphi - \delta$ becomes negative at the north of the meridian; and the reason is easily seen,—for the transit plane is in this case wholly to the east of the meridian from the north to the south point, or when b is negative wholly to the west.

71. Lastly, the telescope may not move in the meridian, but in a small circle parallel to the meridian, and every where a certain number of seconds (c) east of it. This will be the case if the optical axis, instead of being perpendicular to the horizontal axis, is inclined towards the eastern pivot a little. Let then

$c =$ the error of *collimation;* $+$ when eastward, $-$ when westward of the meridian, and reckoned in seconds of time.

The distance from this circle to the meridian being constant at all points, a star arriving at the transit plane at the time t, will pass from one to the other in $c \sec \delta$ seconds, and reach the meridian at the time

$$t + c \sec \delta.$$

If, during the observations, the transit should be reversed, the telescope will point as far to the westward of the meridian as [redacted]

ridian, the combined effect of the three at any point is the algebraic sum of the separate effects; consequently, a star, whose recorded time of passing the mean wire is t, is on the meridian at the time

$$t+a \sin(\varphi-\delta) \sec\delta+b \cos(\varphi-\delta) \sec\delta+c \sec\delta.$$

If a, b, c, are all $+$ or towards the east, this correction of t is entirely additive, as it is plain it should be, since the star moves from east to west; and *vice versâ.*

72. Once more, let x = the *correction* of the clock; $+$ when the clock is slow of sidereal time, $-$ when fast of the same.

Then t being the clock time of passing the mean of the transit wires, $t+x$ is the true sidereal time of passing the same, and as above . . . $t+x+a \sin(\varphi-\delta) \sec\delta +$ &c. is the true sidereal time of the star's crossing the meridian. But this = its own right ascension, which call α. We have therefore

$$\alpha = t+x+a \sin(\varphi-\delta) \sec\delta+b \cos(\varphi-\delta) \sec\delta+c \sec\delta. \quad (1.)$$

$$\text{or } x = \alpha-t-a \sin(\varphi-\delta) \sec\delta-b \cos(\varphi-\delta) \sec\delta-c \sec\delta. \quad (2.)$$

Now, our object being to find the error of the clock at any moment, we may do so by this equation, provided we know the values of a, b, and c. For all the other quantities are known, α, the R.A. of the star; t, the recorded time of its passage, and δ and φ, its known declination and the latitude of the place.

73. *To find a, b, and c.*—First, the level furnishes us with a direct means of ascertaining the quantity b. The horizontal axis, when the inclination is positive, is of course depressed below the east point by an angle equal to b; and the level indicates this depression, if we know to what arc in space each division of its scale corresponds. An accurate method of finding the value of a division, if unknown, is illustrated in our example, (Art. 76.)

Let the axis of the transit be nearly horizontal, and the bubble of the level extending on each side of the central zero of the scale. In this position, let e designate the reading of the east end, and w that of the west end; then $\frac{w-e}{2}$ represents the number of divisions by which the centre of the bubble is *west* of zero.* Let the east and west readings of the bubble in the re-

* Of course, if $\frac{w-e}{2}$ is negative, the centre of the bubble must be east of zero.

versed position be e' and w', then $\frac{e'-w'}{2}$ is the distance of the centre of the bubble *east* of zero. A consideration of Art. 36. shows that when the axis is horizontal, these two should be equal, 10^{div} W. in the usual position coinciding with 10^{div} E. in the reversed. And if the eastern pivot is depressed by b, expressed in divisions of the scale, $\frac{e'-w'}{2}$ must be less than $\frac{w-e}{2}$ by twice that amount, or $2b$. Therefore $2b=\frac{(w-e)-(e'-w')}{2}$, or more conveniently $b=\frac{(w+w')-(e+e')}{4}$. To express b, however, in seconds of time, let k be the value of 1^{div} of the scale; then

$$b=k\,\frac{(w+w')-(e+e')}{4}.$$

If the indications of the level are not constantly the same, or very nearly so, the level should be *read off* in both positions several times during observation, and the value of b at those times will thence be found with ease by the above formula.

74. The error of collimation (c) may be found in several ways. First, rudely, by making the central wire bisect the meridian mark, and after reversion, estimating its displacement in seconds of time by a comparison with the known distance between any two wires; one half of this displacement $=c$, and is positive, if, in an inverting telescope, the wire after reversion appears on the left hand of the meridian mark.

[redacted]

cess in the course of an evening, especially on slow-moving polar stars, will give a very correct value of c.

Frequently, however, a transit instrument is not situated so as to command circumpolar stars, and the following method is in all cases available, and generally more accurate with ordinary instruments. Having observed a number of stars on widely different points of the meridian, reverse, and observe about as many more. Then from each star in the first position we shall have an equation like eq. (2,) Art. 72, in which $c \sec \delta$ will have the sign $-$ before it; and from each observed after reversion, a similar equation, except that $c \sec \delta$ will be affected with the opposite sign. The quantity c may then be eliminated, as in our example, Art. 76.

75. The quantity a must be found by a combination of the observations themselves, in the form of equation (2,) Art. 72.

To each star observed, apply eq. (2,) giving to $\alpha - t$, $\sin(\varphi - \delta) \sec \delta$, $\cos(\varphi - \delta) \sec \delta$, $\sec \delta$, their respective values. The three latter are called the *coefficients* of a, b, and c. For any given latitude or place, they may be calculated for every degree, or every second degree of declination, both $+$ and $-$, as far as the range of the instrument extends. The calculation of such a table will occupy but a small space of time, and saves very much labor in reducing observations. Such a table for the latitude of New Haven is given in Table V, and any observer may in an hour or two fill out a similar one for his own station. By its aid, eq. (2) may be formed for each star almost instantly. For example:

July 17, 1838.—Observed transit of β Aquilæ... $19^h\ 47^m\ 19^s.12$.
By Naut. Alm.—R.A. of β Aquilæ... $19^h\ 47^m\ 24^s.22$; Decl. $+6°\ 0'$.

Therefore $\alpha - t = +5^s.10$, and looking in Table V, against Decl. $+6°\ 0'$, we have by mere inspection the coefficients of a, b, and c, as follows:

$$\beta \text{ Aquilæ} \ldots x = +5^s.10 - .58\,a - .82\,b - 1.01\,c.$$

A similar equation may be as easily formed for each star. That x may be the same in all the equations, allow for the known rate of the clock in each. Next, the value of b being ascertained by reducing the observations on the level, multiply it into its coefficient in each equation, and incorporate the same with the cor-

responding value of $\alpha-t$. The only unknown quantities are now x, a, and c.

If c has been obtained by reversing on several stars, the fraction of its value may be also annexed to $\alpha-t$ in like manner with that of b, and the equations combined so as to obtain x alone. But if c likewise is to be obtained from comparison of all the observations, we may proceed as follows:

(1.) Add a number of equations, which contain a large coefficient of a, to form one equation, and an equal number of those containing small coefficients of a for another; taking care that there shall be the same number of *plus* as of *minus* coefficients of c in each of the two newly formed equations. Subtracting one of these from the other, we eliminate x at once, and, if the equations are well selected, obtain a with so large, and c with so small a coefficient, that the latter may usually be neglected, or allowed for afterwards. Dividing by its coefficient, we obtain the value of a.

(2.) Again, form two other equations, one embracing *plus* coefficients of c, the other as many with the *minus* sign, at the same time choosing those in which the coefficients of a nearly balance one another. By subtracting as before, we have a very large coefficient of c, and a very small one of a, and a being already known, c becomes so of course.

The student will remark, that although the unknown quantities x, a, c, might be eliminated from any three of the equations, yet the results would not perfectly agree with those deduced [redacted]

76. ***Example; an evening's observations, and their reduction.***

We conclude with an example of the observations of a single evening, made with an uncompensated clock, and a transit instrument of but ordinary excellence, which commanded the southern meridian up to 70° of altitude. The actual reduction of such a series will illustrate and enforce the previous descriptions of processes, and furnish a model for the observer, which he may follow in whole, or in part, according to the degree of accuracy he is desirous of obtaining.

New Haven, Lat. 41° 18′. Oct. 17, 1839.—*The occultation of δ Capricorni having been observed,—it is required to deduce from the transit observations of that evening the true sidereal times of immersion and emersion.*

The transit journal furnishes the following observations:

Objects.	I.	II.	III.	IV.	V.	Reduction.
	s.	s.	h. m. s.	s.	s.	h. m. s.
η Capricorni..................	17.6	25.9	20 54 34.8	43.1	51.9	20 54 34.66
ε "	56.9	3.0	21 6 9.5	16.3	23.1	" 6 9.76
f Pegasi..................	33.9	43.5	" 21 53.1	2.8	12.5	" 21 53.16
☽'s 1 Limb..................	50.9	59.8	" 35 8.8	17.5	26.4	" 35 8.68
δ Capricorni..................	14.9	21.7	" 37 28.7	35.8	43.1	" 37 28.84
ι Aquarii..................	.....	57.4	" 57 3.8	9.9	16.3	" 57 3.71
Immersion	of	δ	Capricorni			22 22 43.80
ξ Pegasi..................	30.9	35.6	22 32 40.8*	.....	.....	" 32 40.98
	V.	IV.	III.	II.	I.	Lamp West.†
" "	.....	.....	" "	44.6*	49.4	" 32 39.19
α Piscis Australis..................	55.3	0.0	" 48 4.5	9.0	13.0	" 48 4.36
α Pegasi..................	44.6	51.1	" 55 57.3	3.7	9.9	" 55 57.32
Emersion	of	δ	Capricorni			23 14 42.5
γ Pegasi..................	56.8	3.4	0 4 9.5	.....	.....	0 4 9.50
β Ceti..................	31.8	39.8	" 34 47.5	55.3	3.1	" 34 47.50
	I.	II.	III.	IV.	V.	Lamp East.†
η Andromedæ..................	31.3	40.3	" 47 50.2	59.5	9.2	" 47 50.10

At 21h 45m.	East.	West.
Cross level E.	47.6	61.3
W.	55.5	53.2
E.	48.1	59.7
W.	55.0	53.2
E.	47.0	60.6
W.	54.8	52.1
Subtract sum of E.	308.0	340.1
from sum of W. readings..............		308.0
Divide by the number of readings‡	12	+ 32.1
		$b = +2.67\,k$

At 0h 10m.		
Cross level E.	48.6	62.2
W.	55.8	53.7
	104.4	115.9
		104.4
	4	+ 11.5
		$b = +2.88\,k$

* The axis being reversed during the passage of ζ Pegasi, (see Art. 74.)

† The illuminating lamp in the present case was at the west end of the axis after reversion, and serves to distinguish between the two positions.

‡ The process indicated in the formula (Art. 73,) is here repeated three times for the sake of diminishing small errors, and 12 is therefore the divisor instead of 4.

The reductions in the last column are made by the rules in Arts. 64 and 65. The apparent right ascensions of the stars observed are next to be obtained from the Nautical Almanac, or from some catalogue by Problem IV, Art. 81; also from the same catalogue their declinations to the nearest minute of space, allowing for precession. The work will then be as follows:

Stars.	α	t	$\alpha - t$	δ	Equations I.
η Capr.	20 55 17.68	54 34.66	+43.02	−20° 29′	$x_1 = +43.02 - .94\,a - .50\,b - 1.07\,c$
ε "	21 6 53.40	6 9.76	+43.64	−15 50	$x_2 = +43.64 - .87\,a - .56\,b - 1.04\,c$
ƒ Peg.	" 22 42.27	21 53.16	+49.11	+22 56	$x_3 = +49.11 - .34\,a - 1.03\,b - 1.00\,c$
☽'s 1 L.	" 25 1.66	35 8.68	−10 7.02	−17 4	$-10^m\,7.02 - .89\,a - .55\,b - 1.05\,c$
δ Capr.	" 38 12.39	37 28.84	+43.55	−16 51	$x_4 = +43.55 - .89\,a - .55\,b - 1.05\,c$
ε Aq.	" 57 47.84	57 3.71	+44.13	−14 39	$x_5 = +44.13 - .86\,a - .58\,b - 1.03\,c$
ζ Peg.	22 33 29.38	32 40.98	+48.40	+10 0	$x_6 = +48.40 - .53\,a - .87\,b - 1.01\,c$
	Axis	reversed.			
ζ "	" 33 29.38	32 39.19	+50.19	+10 0	$x_7 = +50.19 - .53\,a - .87\,b + 1.01\,c$
α P. A.	" 48 48.64	48 4.36	+44.28	−30 28	$x_8 = +44.28 - 1.10\,a - .36\,b + 1.16\,c$
α Peg.	" 56 48.19	55 57.32	+50.87	+14 21	$x_9 = +50.87 - .47\,a - .92\,b + 1.03\,c$
γ Peg.	0 5 0.94	4 9.50	+51.44	+14 18	$x_{10} = +51.44 - .47\,a - .92\,b + 1.03\,c$
β Ceti	" 35 34.43	34 47.50	+46.93	−18 52	$x_{11} = +46.93 - .92\,a - .53\,b + 1.06\,c$
	Axis	returned.			
η And.	" 48 40.47	47 50.10	+50.37	+22 33	$x_{12} = +50.37 - .35\,a - 1.03\,b - 1.08\,c$

Art. 75 sufficiently explains the manner of obtaining "Equations I." The R.A. of the moon is such as it was when crossing

[redacted]

and ι Aq.; therefore for $21^h.8\left(=\frac{21^h\ 38^m+21^h\ 58^m}{2}\right)-21^h.1$, or $0^h.7$, the rate $=\frac{43.55+44.13}{2}-43.64=+0^s.20$, or $+0^s.29$ pr. hour. Giving a double weight to the former of these two values, on account of its deduction from a longer interval, we have $r=+0^s.34$ pr. hour.

For the value of b, the observations give us $b=+2.67\ k$, and $b=+2.88\ k$, at $21^h.8$ and $0^h.2$ respectively. Being nearly the same, we may combine them, giving the first double weight, and $b=+2.74\ k$. To determine k, it had been found by a scale and vernier, that 28 turns of the screw of elevation raised its Y 0.677 inches; also 25 turns were equal to 0.605 inches; $\therefore$ 1 turn $=.02418$ by the 1st, and $=.02420$ by the last measure; therefore 1 turn $=.02419$ inches. Distance from Y to Y $=13.57$ inches; then $13.57 : \text{rad} :: .02419 : \text{arc } 6'\ 7''.7=367''.7=24^s.51$. And $\frac{1}{7}$ of a turn was found on a mean of many trials to change the situation of the bubble, when near its usual limits, by 10.79 divisions. Therefore $\frac{24^s.51}{7}$ or $3^s.501=10.79$ div. and $1^{\text{div.}}$ or $k=0^s.325$. Consequently, for our observations, $b=+2.74\ k=+0^s.89$.

With the above value of r we will reduce the expressions for x to such as they would be at $21^h.0$; and also eliminate b from the equations. The corrections for r and b are tabulated below, and applying them to $\alpha-t$, we will reduce "Equations I" to "Equations II."

Correction for				No.	Equations II.
$r=+^s.34$	$b=+^s.89$				
$+.03$	$-.45$	$+42^s.60$		(1.)	$x=+42^s.60-.94\,a-1.07\,c$
$-.03$	$-.50$	$+43.11$		(2.)	$x=+43.11-.87\,a-1.04\,c$
$-.12$	$-.92$	$+48.07$		(3.)	$x=+48.07-.34\,a-1.09\,c$
$-.21$	$-.49$	$+42.85$		(4.)	$x=+42.85-.89\,a-1.05\,c$
$-.32$	$-.52$	$+43.29$		(5.)	$x=+43.29-.86\,a-1.03\,c$
$-.53$	$-.77$	$+47.10$		(6.)	$x=+47.10-.53\,a-1.01\,c$
$-.53$	$-.77$	$+48.89$		(7.)	$x=+48.89-.53\,a+1.01\,c$
$-.61$	$-.32$	$+43.35$		(8.)	$x=+43.35-1.10\,a+1.16\,c$
$-.66$	$-.82$	$+49.39$		(9.)	$x=+49.39-.47\,a+1.03\,c$
-1.04	$-.82$	$+49.58$		(10.)	$x=+49.58-.47\,a+1.03\,c$
-1.22	$-.47$	$+45.24$		(11.)	$x=+45.24-.92\,a+1.06\,c$
-1.29	$-.92$	$+48.16$		(12.)	$x=+48.16-.35\,a-1.08\,c$

By properly combining "Equations II" in the way recommended in Art. 75, we find the values of a and c as follows:

Adding (1.), (2.), (4.), (8.), (11.), to form the first equation, and (3.), (6.), (9.), (10.), (12.), to form the second, we have,

$$5\,x = +217^s.15 - 4.72\,a - 0.94\,c$$
$$5\,x = +242^s.30 - 2.16\,a - 1.12\,c$$

Subtracting $\quad 0 = -25^s.15 - 2.56\,a + 0.18\,c$

$$2.56\,a = -25^s.15 + .18\,c$$
$$a = -\ 9^s.82 + .07\,c$$

Again,

Adding (7.), (8.), (9.), (10.), (11.),

$$5\,x = +236^s.45 - 3.49\,a + 5.29\,c$$

Adding (2.), (4.), (5.), (6.), (12.),

$$5\,x = +224^s.51 - 3.50\,a - 5.21\,c$$

$$0 = +\ 11^s.94 + .01\,a + 10.50\,c$$
$$10.50\,c = -11^s.94 - .01\,a = -11^s.94 + 0^s.10 = -11.84$$
$$c = -\ 1^s.13$$

Correcting the value of a as above by this of c,

$$a = -9^s.82 - 0^s.08 = -9^s.90.$$

Finally, we substitute the values of a, b, and c, in "Equations I," and thus obtain "Equations III," or the corrections of the clock.

Equations III.	x
$x_1 = +43.02 + 9.31 - .45 + 1.21 =$	$+53^s.09$
$x_2 = +43.64 + 8.61 - .50 + 1.18 =$	$+52.93$
$x_3 = +49.11 + 3.37 - .92 + 1.23 =$	$+52.79$
$x = +43.55 + 8.81 - .49 + 1{\cdot}19 =$	$+53.06$

[redacted]

gradually increasing. The result on the first three wires of ζ Pegasi, not well agreeing with the rest of the series, should be cast out; then taking the mean of the first four, and of their corresponding times, and the mean of the next four, as also of the last three, we have

$$\begin{array}{r|l|l} \text{at } 21^h\ 15^m & & x = +52^s.97 \\ 22^h\ 33^m & 78^m \;\ldots\ldots & +53\ .47 \quad +.50 \\ 0^h\ 29^m & 116^m \;\ldots\ldots & +54\ .20 \quad +.73 \end{array}$$

Hence for any given time t' between $21^h\ 15^m$ and $22^h\ 33^m$

$$x = +52.97 + (t' - 21^h\ 15^m)\frac{0^s.50}{78},$$

and between $22^h\ 33^m$ and $0^h\ 29^m$

$$x = +53.47 + (t' - 22^h\ 33^m)\frac{0^s.73}{116}.$$

Then at $22^h\ 23^m$ $\quad x = +52.97 + 68\,\dfrac{0^s.50}{78} = +53^s.41$

at $23^h\ 15^m$ $\quad x = +53.47 + 42\,\dfrac{0^s.73}{116} = +53^s.73$

Im. of δ Capricorni $22^h\ 22^m\ 43^s.80 + 53.^s41 = 23^h\ 23^m\ 37^s.21$,

Em. " $23^h\ 14^m\ 42^s.5 + 53^s.73 = 23^h\ 15^m\ 36^s.2$,

true sidereal time at New Haven.

CHAPTER V.

OF PROBLEMS IN PRACTICAL ASTRONOMY.

77. In the calculations connected with practical astronomy, there are a number of operations so frequently required, that familiarity with at least a few is necessary to render farther progress easy or certain. The following are some of the most useful and universal in their application.

I. *On the use of Signs connected with the Logarithms of Numbers and Circular Functions.*

78. The sign + or − prefixed to a logarithm indicates the state of the natural number to which the logarithm belongs, and

not of the logarithm itself. Thus, in the operations of multiplication and division, the signs belonging to quantities are prefixed to their respective logarithms, and being disregarded in the addition or subtraction of the latter, are only of use to determine what sign shall be prefixed to the final product or quotient.

The sine, cosine, tangent, &c. of an arc, it will be recollected, have not the same signs in all the quadrants. The sine, for instance, is positive in the 1st and 2nd quadrants, but in the 3rd and 4th, being measured in an opposite direction, it is considered negative. For convenience of reference, Table I. contains a schedule of the changes of the different circular functions through the four quadrants, with their opposite signs in each. In common operations of trigonometry and astronomy, the change of signs caused by difference of quadrant may frequently be neglected; it is, however, the best, because the safest and most uniform method, to retain the distinction in all cases.

Ex. *What is the value of*

$$\frac{(-627''.1) \times \sin(-28^\circ\ 10') \times \cos(-28^\circ\ 10')}{\cos 157^\circ \tan 98^\circ \sin 30^\circ}$$

Remark.—As the sine of $+28^\circ\ 10'$ is the same as that of $360^\circ + 28^\circ\ 10'$, so the sine of $-28^\circ\ 10'$ is the same as of $360^\circ - 28^\circ\ 10'$. The sine and cosine of $-28^\circ\ 10'$, are therefore those of the 4th quadrant.

$l.\ (-627''.1)$	-2.7973
$l.\ \sin(-28^\circ\ 10')$	-9.6740
$l.\ \cos(-28^\circ\ 10')$	$+9.9453$

given for each day in the Nautical and American Almanacs, and usually in all ephemerides of any importance. Also in the Naut. Alm. is given the "Mean Time of Transit of the first point of Aries," or the *mean time* corresponding to *sidereal noon*. The sidereal time therefore corresponding to any hour, minute and second of Gr. mean time, is found by adding to the sidereal time of the preceding Gr. mean noon the sidereal equivalent of the mean time since then elapsed; and *vice versâ*. The reason of the process will perhaps be best understood by expressing the agreement between mean solar and sidereal time in the form of an equation.

Ex. 1. *To convert* $13^h\ 41^m\ 17^s.04$ *Gr. mean time, July* 1, 1840, *into Gr. sid. time.*

	Gr. mean time.		Gr. sid. time.	
July 1, 1840.	$0^h\ 0^m\ 0^s.00$	=	$6^h\ 38^m\ 24^s.89$	. . . p. II. N. A.
	13^h	=	13 2 8.13	Table II.
	41^m	=	41 6.74	Table II.
	$17^s.04$	=	17.09	Table II.
	$13^h\ 41^m\ 17^s.04$	=	$20^h\ 21^m\ 56^s.85$	

Ex. 2. *To convert* $3^h\ 10^m\ 19^s.76$ *sid. time into mean solar time, July* 1, 1840, *for the meridian of New Haven.*

	N. H. sid. time.		N. H. mean time.	
			$17^h\ 18^m\ 44^s.48$	Gr.m.t. of sid.noon.
			− 47.80	p. XXII. N. A.
Transit of 1st point of Aries,	$0^h\ 0^m\ 0^s.00$	=	17 17 56.68	N. H. " " "
	3	=	2 59 30.51	Table III.
	10	=	9 58.36	Table III.
	19.76	=	19.71	Table III.
	3 10 19.76	=	20 27 45.26	

Remark.—At any other meridian than Greenwich,—for instance, at New Haven, since we have no equation between mean and sidereal time at New Haven mean or sidereal noon, we convert New Haven into Gr. time by adding the difference of longitude, then convert Gr. mean into Gr. sid. time, or Gr. sid. into Gr. mean time, and again subtract the longitude to reconvert to New Haven time. These operations may be materially shortened, by adding $47^s.93$, or "the *acceleration* of sidereal time on solar" during $4^h\ 51^m\ 46^s$ (Table II.), to the Gr. sid. time of Mean Noon, to find the N. H. sid. time of Mean Noon; and by subtracting

$47^s.80$, or "the *retardation* of solar time on sidereal" during 4^h 51^m 40^s (Table III.), from "Mean Time of Transit of First Point of Aries," as given for Greenwich, to find the same for New Haven.

Some ephemerides give only "Sid. Time at Mean Noon;" in this case, Ex. 2 may be performed by reversing the operation in Ex. 1. For instance, to find the mean time answering to 20^h 21^m $56^s.85$ sid. time, take out from Table III. the equivalent of 20^h 21^m $56^s.85-6^h$ 35^m $24^s.89$ in mean time.

It should be always remembered, that all the quantities in the Nautical Almanac are in *Greenwich time*. The local time of any other place must therefore be invariably converted into the corresponding Greenwich time, before any reference can be made to this ephemeris.*

III. *Interpolation by Differences.*

80. It is frequently necessary in Practical Astronomy, from the values of any variable quantity at certain equal definite intervals, to determine its amount at any intermediate time. The most irregularly varying quantity, if of gradual change, and subject to the control of any, even an unknown, law of progress, may be concluded from a regular series of values, for any other point amidst the series by the *method of differences*. (See Day's Algebra, p. 273.)

If, for example, we have the moon's right ascension at mean noon for a number of days in succession, subtract the R.A. at each noon from that at the following noon, paying attention to

[redacted]

sents the first of a series of values, d', d'', d''', &c., the first of each successive order of differences, and A the value for the time t elapsed since the time of the first value, and reckoned in parts of the equal intervening intervals.

Ex. 1. *Given the sun's declination as follows:*

	At Mean Noon.	d'	d''	d'''	d''''
1838. Sept. 17,	+2° 20′ 14″.3*				
		−1396″.1			
18,	1 56 58 .2		−2″.6		
		−1398 .7		+0″.4	
19,	1 33 39 .5		−2 .2		+0″.1
		−1400 .9		+0 .5	
20,	1 10 18 .6		−1 .7		
		−1402 .6			
21,	+0 46 56 .0				

Required the same at Sept. 18ᵈ 9ʰ 30ᵐ.

Here, after taking the differences as above, $a = +2° 20' 14''.3$; $d' = -1396''.1$; $d'' = -2''.6$; $d''' = +0''.4$, &c., and $t = 1^d\ 9^h\ 30^m = 1\frac{19}{48} = 1.4$ nearly.

$$A = +2° 20' 14''.3 \quad -1396''.1 \times 1\tfrac{19}{48} \quad -2''.6 \times 1.4 \times \frac{0.4}{2} \quad +$$
$$0''.4 \times 1.4 \times \frac{0.4}{2} \times \frac{-0.6}{3} +, \text{ \&c.} \quad = +1° 47' 44''.9$$

When the series is so rapidly convergent, as to render calculation beyond the second differences unnecessary, the interpolation may be performed by means of Table IV., which contains the coefficients of d'' for every hundredth part of the unit of time elapsing between the successive values of the quantity required. The following rule will in this case be applicable:

"Take from the ephemeris the two values preceding, and the two following the required time. Subtract the first in order of time from the second, the second from the third, &c., for the first rank of differences. Subtract each of these from the following, for the second rank of differences, always paying attention to the algebraic signs of the quantities. Call the 2nd of the four values, a; the 2nd of the three first differences, d', and the mean of the two second differences, d''. Reduce the time elapsed since the date of the second value, to the decimal fraction of the unit of time chosen, calling the same t.

"Enter Table IV with this decimal at the side, and d'' at the top, in the column headed 'Seconds of Second Differences,' and take out the corresponding correction, which must have its sign always contrary to that of d''. Or if d'' be greater than 100,

* North declinations are positive; south declinations negative.

take out its natural or logarithmic coefficient from the proper columns, and multiply by d''. Add the correction so found to $a+td'$; the sum will be the value required."

It is not unfrequently the case, that we have but a limited number of values of the required quantity, and wish to interpolate for a time not included between them. In such a case we may extend the series artificially by addition of differences, until it will embrace the time for which we intend to calculate.

Ex 2. *In the calculation of the eclipse, Art.* 103, *we have only the value of* p—u *included between the lines as below, for* $8^h\ 30^m$, $9^h\ 30^m$, $10^h\ 30^m$; *required the values of the same for* $8^h\ 0^m$ *and* $10^h\ 45^m$.

	$p-u$		
$7^h\ 30^m$	$-2206''.3$		$+156.1$
		$+985.6$	
$8^h\ 30^m$	$-1220\ .7$		$+156.1$
		$+1141.7$	
$9^h\ 30^m$	$-\ \ 79\ .0$		$+156.1$
		$+1297.8$	
$10^h\ 30^m$	$+1218\ .8$		$+156.1$
		$+1453.9$	
			$+156.1$

Here $a=-2206''.3$; $d'=+985''.6$; $d''=+156''.1$; $t=.50$ or 30^m*

$$a+d't=a+\frac{d'}{2}=-1713''.5$$

$$\left.\begin{array}{lr} \text{Corr. for } +100'' & -12''.5 \\ \phantom{\text{Corr. for }} +\ 50 & -\ 6\ .3 \\ \phantom{\text{Corr. for }} +\ \ 6 & -\ \ \ .7 \end{array}\right\} \begin{array}{l}\text{Table IV.; Arg. at side.} \\ =30^m\end{array}$$

$$-1733\ .0$$

Again, $a=+1218.8$; $d'=+1453''.9$; $d''=+156''.1$; $t=15^m$*

$$a+d't\ =\ +1582''.3$$

$$\left.\begin{array}{lr} \text{Corr. for } +100'' & -\ 9.4 \\ 50'' & -\ 4.7 \\ 6'' & -\ \ .6 \end{array}\right\} \text{Arg. } 15^m. \quad \text{Table IV.}$$

$$+1567.6$$

A very useful application of the formula for interpolation is,—to find from a series of values of any quantity, the time when that quantity arrives at a maximum or minimum, as well as its amount at that time. By differentiation of the original formula, we have, for the moment that A is at a maximum, this equation:

$$d'+d''\frac{2t-1}{2}+\ \&c.\ =0$$

* Where the unit is an hour, the argument at the side may be found expressed in minutes.

whence $t = \frac{d'' - 2d'}{2d''}$

and with t thus found, we may easily obtain the greatest or least value, as for instance, of the sun's declination at the solstices, or the shortest distance between the centres of any two celestial bodies in case of an eclipse or near approach.

IV. *Corrections of the places of Fixed Stars.*

81. All well arranged catalogues of the fixed stars contain their mean right ascensions and declinations at a given epoch. In instrumental observation, however, we see them not in these mean places, but altered by the amount of their precession since the given epoch, and affected by nutation and aberration. The mean places must therefore be reduced to the apparent, before they can be compared with observations.

The algebraic expressions for these corrections have been so subdivided by a distinguished German astronomer,* that all the quantities relating to the places of the stars are expressed by four factors, a, b, c, d, accented and unaccented; and all those dependent on the time, by four others, A, B, C, D, such that the whole correction for the place of the star is as follows:

$$\triangle \alpha = a\text{A} + b\text{B} + c\text{C} + d\text{D}.$$
$$\triangle \delta = a'\text{A} + b'\text{B} + c'\text{C} + d'\text{D}.$$

where $\triangle \alpha$ and $\triangle \delta$ are the required corrections in R.A. and Decl. respectively.

The arrangement possesses this peculiar advantage,—that the factors, expressed by the small letters, being dependent on the places of the stars only, and therefore constant for a long period of years, may be calculated and inserted with the stars in a standard catalogue, while A, B, C, and D, which are the same for all the stars, but vary with the time, can be appended to the Nautical Almanac and similar series of tables, which have the element of time instead of space for their basis. The later catalogues, such as Schumacher's Catalogue of 500 stars in Pearson's Astronomy,—the Astron. Soc. Catal. of 2900 by Baily, and others, furnish the logarithmic constants, a, b, c, d, and a', b', c', d', for each star; and the "XXII. p. of the month, Naut. Alm.," contains the logarithms of A, B, C, D, for each day. The mean R. A. and

* Prof. Bessel.

Decl. for the epoch of the catalogue is reduced to that for the current year, by adding as many times the annual precession in R.A. and Decl., annexed to each star in the catalogue, as the number of even years elapsed since the given epoch.

Ex. 1. *What is the apparent right ascension of η Andromedæ on the 17th Oct. 1839?*

In the General Catalogue of the Royal Astron. Society, (Mem. Ast. Soc., vol. II.) the 96th star is η Andromedæ,—p. lx,—and we there find its "mean R.A. for Jan. 1, 1830," its "annual precession," and the "logarithms of a, b, c, and d," as below:

No. 96.—38 η *Andromedæ.*

LOGARITHMS OF

a	b	c	d	
+8.8486	+8.1774	+0.5029	+8.4315	Ast. Soc. Cat.
+1.2322	+0.9171	+9.9389	−0.9621	N. A. p. XXII.
0.0808	9.0945	0.4418	9.3936	
$+1^s.204$	$+^s.124$	$+2^s.766$	$-^s.248$	Nat. numbers.

		Ann. Prec.
R.A. Jan. 1, 1830,	$0^h\ 48^m\ 7^s.98$	$+3^s.183$
	+28 .647	9
	+ 1 .204	+28 .647
	+ .124	
	+ 2 .766	
	− .248	
	$0^h\ 48^m\ 40^s.47$ . . App. R.A. of η Andromedæ.	

The apparent declination may be calculated in the same way by employing the factors a', b', c', d'.

"The Apparent Places" of a select hundred fixed stars calculated for every tenth day of the year, are contained in the latter part of the Naut. Alm. If, of the objects observed by a transit or other instrument, any should be in this list, or among the four "Moon Culminating Stars" to be observed with the moon on that evening,—its apparent right ascension may be taken out without farther trouble; all others must be computed from the catalogues.

CHAPTER VI.

OF ECLIPSES OF THE MOON.

82. In calculating an eclipse of the moon, let us conceive ourselves encircled by a sphere, concentric with the earth, having its radius equal to the moon's distance, and of course, passing through the centre of the moon. The minute portion of this sphere occupied by the sections of the moon and the earth's shadow during a lunar eclipse, may without material error be regarded as a plane, and is called the *plane of projection;* and the great circles in which the centres of these sections move, will be straight lines in this plane. If the student has reviewed the general method of calculating a lunar eclipse, for which we would refer to Arts. 245–260 of Olmsted's Astronomy, he will recollect, that the cross sections of the earth's dark shadow and of the penumbra, are two concentric small circles of the sphere, whose semidiameters are respectively $P+\pi-D$ and $P+\pi+D$, where D is the sun's semidiameter, P his horizontal parallax, and π that of the moon.

83. *Methods of finding the places of the sun and moon in lunar and solar eclipses.*

The moon's *mean* place is the place she would occupy, if she revolved around the earth in a circle at her mean distance, and undisturbed by the attraction of the sun and planets. The *true* place of the moon differs from the mean place on account of the inequalities and perturbations of her motion. All the forces which tend to urge the moon from her mean orbit in various directions, may be resolved into such as shift her place in the direction of some great circle, and in a direction perpendicular to it. The effects of these forces in changing the moon's mean place, are expressed in equations, determined by analysis, and containing as factors the mean place of the moon, the place of her node, and the like. The numerical results of these equations are called the *corrections* of the moon's mean place, and may be computed directly for any particular case, or they may

be calculated for every degree of the great circle, and registered in separate tables. These Tables being prepared, all that is necessary is to take from them the several corrections required, and measure them off backwards and forwards according to their signs on the great circle employed, to arrive from the mean to the true place on that circle. The corrections for the moon's place on a perpendicular circle may be applied in the same way.

If this calculation be made for regular intervals of time, such as mean noon of every day, we shall have a new series of Tables, depending on the former, as the former do upon the original analytical equations. There will be a distinctive difference between them in this respect:—that since the former are computed for every degree in a great circle, their element is *space*, and they are complete as soon as the round of the circle is made; the latter, on the other hand, having *time* for their basis, are in their nature continuous, having no definite end. The first class of tables are the common Lunar Tables.* The latter are those

* Many of the older works treating of eclipses, and some of the more recent, include an abridged series of Lunar Tables, which have been omitted in the present treatise. The pages of the Nautical Almanac have been preferred as Tables of reference, and for the following reasons: The process of computing the moon's true place from the Lunar Tables is not necessarily connected with the calculation of an eclipse, any more than is the process of computing the Lunar Tables from the original equations. It is moreover of little profit to the student, since after a knowledge of the method of procedure as described above, he has only to enter a great number of tables in succession, and take from them their respective additive or subtractive corrections, which yet are so numerous, as to render the operation a very long and tedious one. This labor, before the wide diffusion of the Nautical Almanac, was indeed indispensable. But since that work is every where and easily obtainable, astronomers no longer regard this prefatory calculation as essentially connected with the work of an eclipse; and in the later European works on Practical Astronomy, instead of giving rules for obtaining the corrections from the "Lunar Tables" for the true place, they give rules for interpolating the true place for any given instant of time from the pages of the Nautical Almanac.

It is scarcely necessary to remark, that although the calculation by latitudes and longitudes had some advantages while the Lunar Tables were employed, the use of the Nautical Almanac renders that by right ascensions and declinations preferable. For in that work the moon's longitude and latitude are given only for every noon and midnight; while her right ascension and declination are giveq for every hour of the year, and we thus dispense with interpolation by second and third differences. Moreover, although in a lunar eclipse the same formulæ are equally applicable, whether the equator or the ecliptic be the circle of reference,—yet in a solar eclipse, the fixity of the pole of the equator, as regards the zenith, whence parallaxes are reckoned, renders the calculation by R.A. and Decl. more simple and more expeditious.

comprised in the Nautical Almanac,* and upon these we shall depend for our data in the subsequent calculations.

The sun's place, the moon's semidiameter and horizontal parallax, and other elements necessary to eclipses and astronomical computations in general, are also more easily, directly, and accurately obtained by proportion or interpolation between the quantities in the Naut. Alm., than from the ordinary tables.

84. *Demonstration of formulæ.*

Let A and $\varDelta$ be the true R.A. and Decl. of a point in the sphere diametrically opposite to the sun's place, that is, of the earth's shadow, and let α and δ represent the same quantities for the moon; aslo let T be the even hour nearest the middle of the eclipse. The quantities A and $\varDelta$ are the sun's R.A. and Decl., (pp. I–II, N. A.,) the first increased by 12^h, the latter with its sign changed; and α and δ are given for every hour of mean time, (pp. V–XII. N. A.) T is readily found for any eclipse by comparing the columns of the sun's and moon's R.A.; a simple inspection will show to the nearest hour of mean time *when* these are 12^h distant from each other. Then take out the quantities A, $\varDelta$, α, and δ, for the times $T-1^h$, T, $T+1^h$; the two first by interpolation, the two last directly.

In fig. 20, let ACD be the section of the earth's shadow, ASD the meridian passing through its centre, and CSF a great circle perpendicular to it. Let EMB be the *relative* orbit of the moon, the earth's shadow being supposed stationary; then $\alpha-A$, and $\delta-\varDelta$, will represent the *relative* differences of R.A. and Decl. of the moon from the centre of the shadow. The section of the moon and shadow being regarded as a plane, in which the

* The Nautical Almanac is a work, which in the present state of astronomy, no *observer* can easily dispense with, even for the simplest uses of the transit instrument and sextant. For the purposes of a *class in college*, who wish to calculate an eclipse of the moon or sun in advance, a single copy in the library of the institution will suffice; and the few data that are required may be transcribed in a few minutes. The work may generally be obtained for at least three years in advance in our large cities.

For most of the purposes indicated in the present treatise, and for all connected with eclipses, "Blunt's edition of the Naut. Alm." will suffice. Either this Abridgment (at $1½) or the larger London edition (at $2½) can always be obtained in advance by application to the agents, E. & G. W. Blunt, New York.

perpendiculars AD and CF are two axes of reference,—we must now ascertain the moon's perpendicular distance in seconds of space from AD and CF at the three times $T-1^h$, T, and $T+1^h$. Now, (Cor. 2, Note, Art. 63,) the distance of the moon's centre from the meridian $AD=(\alpha-A) \cos \delta$. Its distance from CF is very nearly equal to $\delta-\Delta$, and would be quite so, but that the great circle CF does not coincide with the small circle, parallel to the equator at that point, for any considerable distance, but departs from it by a minute quantity increasing as the square of $(\alpha-A.)$ This small difference $=\frac{1}{2r} \cos \delta \sin \Delta \ (\alpha-A)^2$,* where $r=$ the radius of a circle, expressed in seconds of the circumference $=206265''$.

Let then $p=(\alpha-A) \cos \delta$.

$$q=\delta-\Delta+\frac{1}{2r} \cos \delta \sin \Delta \ (\alpha-A)^2$$

These are the co-ordinates of the moon's centre with reference to CF and AD, and may be measured on those lines.

85. Take out by simple proportion from the Naut. Alm. the sun's semidiameter D and horizontal parallax P, and also those of the moon d, π, (pp. II–III. N. A.) Since the earth's shadow is nearly elliptical, being the shadow of an ellipsoid, π (which is in different latitudes proportioned to the earth's radius), should be reduced from the equatorial value to its value in lat. 45°, by applying the reduction contained in Table VI. For the adoption of the mean radius of the earth's shadow, will render the calcula-

[redacted]

$$s = \frac{61}{60}\,(P+\pi-D).$$

$$s' = \frac{61}{60}\,(P+\pi-D)+2D.$$

Now, when the moon just touches the dark shadow externally, as also when she is just quitting it, her distances, BS, ES, from the centre of the shadow $=s+d$; and at the beginning or end of total eclipse, her distance B′S or E′S $=s-d$. The lines p and q, or the co-ordinate distances of the moon from AD and CF, evidently form at these times with $s\pm d$ a right angled triangle, of which $s\pm d$ is the hypothenuse; and hence at these times we have the equation

$$(p^2+q^2)^{\frac{1}{2}} = s\pm d.$$

where s' may be put for s, when the contacts are those of the penumbra.

Now from the series of values which we have of p and q corresponding to $T-1^h$, T, and $T+1^h$, we might find, by a method of "trial and error," at what times the first member of the above equation should be equal to the last, and these times are, of course, those of beginning and end of partial and total eclipse. But in the present case, a more direct method presents itself.

86. If XIII, XIV, XV, in the figure represent the position of the moon at the times $T-1^h$, T, $T+1^h$,—then XIII G represents the change of p, and G XV that of q, in two hours; and XIII G : G XV :: rad : tan G XIII XV; which angle, being the inclination of the relative orbit BME to the equator, or to its parallel CSF, call ι. Let t be the time when p becomes nothing, found by simple proportion, and q' be the value of q at that time = SN. Draw RSU at right angles to BE, cutting it in M; then since MSN $=\iota$, MS = SN $\cos\iota$, and NM = SN $\sin\iota$. Farther, XIII G $\times$ $\sec\iota$ = XIII XV, or the motion of the moon in two hours, and dividing this by 2, we have h, the horary motion of the moon on its relative orbit BME. Then $60\,\frac{\text{NM}}{h} =$ the time in minutes of passing from N to M, and $t\pm 60\,\frac{\text{NM}}{h} = t'$, the time of central eclipse, and MS, the shortest distance, is already known. In the right angled triangle BMS, we have BS and MS

known, to find $BM=\sqrt{BS^2-MS^2}=\sqrt{(BS+MS)\,(BS-MS)}$.
Then $60\,\frac{BM}{h}$ = time of passing from B to M or from M to E,

and $t'\mp 60\,\frac{BM}{h}$ = the times of beginning and end of partial eclipse. So also for the beginning and end of total eclipse, and for the first and last contacts with the penumbra, we may employ $s-d$ and $s'+d$ instead of $s+d=BS$.

With this general illustration, the following more distinct formulæ will be easily understood.

Let ι = the inclination of the relative orbit to the circle parallel to the equator, being positive when δ is increasing.

$\Delta p=\frac{1}{2}$ the change of p from $T-1^h$ to $T+1^h$.

$\Delta q=$ " " q " "

q' = the value of q when p becomes nothing.

t = the time when p becomes nothing.

t' = " of central eclipse.

m = MS, the shortest distance.

h = moon's horary motion.

Then (1.) $\frac{\Delta q}{\Delta p}=\tan\iota$; ι to be taken between 0° and 90°, and + or − according to the sign of its tangent.

(2.) $\Delta p \sec\iota = h$.

(3.) $\pm q'\cos\iota = m$; m to be always positive.

(4.) $t-\frac{q'\sin\iota}{}=t'$; where the time is to be expressed in

[redacted]

ple the eclipse of Feb. 5, 1841, and proceed to calculate the times of its different phases for New Haven. And since the absolute times of the contacts are the same the earth over, we will first calculate them in Greenwich mean time, and reduce to New Haven mean time afterwards by subtracting the difference of longitude.

By comparing the columns of R.A. of the sun and moon, we see that on Feb. 5, the moon's R.A. at 13^h, 14^h, 15^h is roughly 9^h 16^m, 9^h 18^m, and 9^h 21^m respectively; while the sun's R.A. on the noons of the 5th and 6th is 21^h 16^m and 21^h 20^m respectively. Changing the 21^h to 9^h,—since 14^h of time is but little more than half-way from noon to noon, the opposition must take place nearest Feb. 5^d 14^h, which therefore = T.

Take from p. II the sun's R.A. for Feb. 5^d—13^h, 14^h and 15^h of mean time* by interpolation, and add to each 12^h; and from pp. V–XII that of the moon for the same times, directly. In the same way take out their declinations, changing the sign of that of the sun. Combine these quantities so as to obtain the co-ordinates $(\alpha-A)\cos\delta$ and $\delta-\Delta+\frac{1}{2r}\cos\delta\sin\Delta\,(\alpha-A)^2$. The latter term, being of very small amount, may be roughly computed with logarithms extending to only two or three places. The logarithm of $\frac{1}{2r}$ is constant, being in all cases +4.385.

Date.	A.	α	$\alpha-A$	Δ	δ	$\delta-\Delta$
Feb. 5^d 13^h	9^h 18^m $23^s.4$	9^h 16^m $4^s.3$	$-139^s.1$	$+15°$ 42′ 43″	$+16°$ 9′ 23″	+19′ 40″ = +1180″
14	" " 33.4	" 18 27.1	− 6.3	+ " 41 57	+15 47 55	+ 5 58 = + 358
15	" " 43.4	" 20 49.4	+126.0	+ " 41 11	+15 33 22	− 7 49 = − 469

	13^h	14^h	15^h
$l.\ \alpha-A$	− 2.1433	− 0.7993	+ 2.1004
$l.\ \cos\delta$	+ 9.9828	+ 9.9833	+ 9.9838
$l.$ 15″ (= 1^s)	+ 1.1761	+ 1.1761	+ 1.1761
$l.\ p$	− 3.3022	− 1.9587	+ 3.3603
p	− 2005″	− 91″	+ 1821″

	13^h	14^h	15^h
$2\,l.\ (\alpha-A)$	+ 4.29	+ 1.60	+ 4.20
$l.\ \cos\delta$	+ 9.98	+ 9.98	+ 9.98
$2\,l.$ 15″	+ 2.35	+ 2.35	+ 2.85
$l.\ \frac{1}{2r}$	+ 4.38	+ 4.38	+ 4.38
$l.\ \sin\Delta$	+ 9.43	+ 9.43	+ 9.43
	+ 0.43	+ 7.74	+ 0.34
$\frac{1}{2r}\cos\delta\sin\Delta\,(\alpha-A)^2$	+ 3″	+ 0″	+ 2″
$\delta-\Delta$	+1180″	+358″	−469″
q	+1183″	+358″	−467″

* If the American edition of the N. A. be used, which contains at present the sun's R.A., only for apparent noon, the interpolation must be performed for the *apparent* times corresponding to 13^h, 14^h, and 15^h of mean time, for which is given the equation of time on the same page.

$$\text{Then at } \begin{array}{r|l} & p \\ 15^h & +1821'' \\ 14 & -\ \ 91'' \end{array}$$

$$-1912'' : 1^h :: -91'' : 0^h.048 = \begin{array}{r} 14^h \\ 2^m\ 53^s \\ \hline 14^h\ 2^m\ 53^s = t \end{array}$$

$$\begin{array}{r|l} & p \\ 15^h & +1821'' \\ 13 & -2005'' \\ \hline & 2)+3826'' \\ \hline & \Delta p = +1913'' \end{array}$$

$$\text{Again at } \begin{array}{r|l} & q \\ 14^h & +358'' \\ 15 & -467'' \end{array}$$

$$1^h : -825'' :: 0^h.048 : \begin{array}{r} +358'' \\ -\ 40'' \\ \hline +318'' = q'. \end{array}$$

$$\begin{array}{r|l} & q \\ 15^h & -\ 467'' \\ 13 & +1183'' \\ \hline & 2)-1650'' \\ \hline & \Delta q = -\ 825'' \end{array}$$

$l.\ \Delta\ q - 2.9165$			
$l.\ \Delta\ p + 3.2817$ - - - -	$+\ 3.2817$	$l.\ (\pm q') \pm 2.5024$ - - - -	$+2.5024$
$l.\ \tan \iota\ -\ 9.6348$	$l.\ \sec \iota\ +10.0371$	$l.\ \cos \iota\ +9.9629$	$l.\ \sin \iota\ \ -9.5978$
$\iota = -23°\ 20'$	$l.\ \ h\ \ +\ 3.3188$	$l.\ m\ \ \ 2.4653$	$l.\ h\ (a.\ c.)\ +6.6812$
$l.\ \sec \iota\ +10.0371$*	$h\ =\ 2084''$	$m\ =\ 292''$	-8.7814
$l.\ \cos \iota\ +\ 9.9629$*			$\frac{q' \sin \iota}{h} = -\ 0^h.060$
$l.\ \sin \iota\ -\ 9.5978$*			

$$14^h\ 2^m\ 53^s + 3^m\ 36^s = 14^h\ 6^m\ 29^s = t'.$$

Now, by proportion between the quantities in the N. A., p. II–III, we have for Feb. $5^d\ 14^h$,

Moon's eq. hor. par. $= 60'\ 35''$	- -	$60'\ 35''$	
Sun's " " $= 9''$		$6''$	- - Reduction to 45° Lat. Tab. VI.
Moon's semidiam. $= 16'\ 30''$		$\pi = 60'\ 29''$	$s = 45'\ 8''$
Sun's " $= 16'\ 14''$		$P = 9''$	$2D = 32'\ 28''$

$$\begin{array}{rl} & 60'\ 38'' \\ D = & 16'\ 14'' \\ \hline P+\pi - D = & 44'\ 24'' \\ \frac{1}{60}(P+\pi-D) = & 44'' \\ \hline s = \frac{61}{60}(P+\pi-D) = & 45'\ 8'' \end{array}$$

$$\begin{array}{rl} s' = s+2D = & 77'\ 36'' \\ d = & 16'\ 30'' \\ \hline s'+d = & 94'\ 6'' \\ \hline s = & 45'\ 8'' \\ d = & 16'\ 30'' \\ \hline s+d = & 61'\ 38'' \\ s-d = & 28'\ 38'' \\ \hline m = & 4'\ 52'' \\ \hline s+d-m = 3406'' = & 56'\ 46'' \\ s+d+m = 3990'' = & 66'\ 30'' \\ \hline s-d-m = 1426'' = & 23'\ 46'' \\ s-d+m = 2010'' = & 33'\ 30'' \end{array}$$

For partial eclipse.		For total eclipse.
$l.\ 3406''$ - -	$+3.5322$	$l.\ 1426''$ - - $+3.1541$
$l.\ 3990''$ - -	$+3.6010$	$l.\ 2010''$ - - $+3.3032$
	$2)+7.1332$	$2)+6.4573$
	$+3.5666$	$+3.2287$
$l.\ h$	$+3.3188$ - - - - - -	$+3.3188$
	$+0.2478$	$+9.9099$
	$1^h.769 = 1^h\ 46^m\ 8^s$.	$0^h.813 = 48^m\ 47^s$.

Then $14^h\ 6^m\ 29^s = t$		$14^h\ 6^m\ 29^s$	
$1^h\ 46^m\ 8^s$		$48^m\ 47^s$	
$12^h\ 20^m\ 21^s$	Beg. of part. eclipse.	$13^h\ 17^m\ 42^s$	Beg. of total eclipse.
$15^h\ 52^m\ 37^s$	End of part. eclipse.	$14^h\ 55^m\ 16^s$	End of total eclipse.

* At the same time that ι is taken from the tables by means of its tangent, take

And, proceeding in the same way with $s'+d$, we have

$11^h\ 24^m\ 8^s$ First contact with Penumbra.
$16^h\ 48^m\ 50^s$ Last " " "

Collecting these results into a tabular form,

Gr. M. Time.	Long. of N. H.	N. H. Mean Time.	
11 24 8	4 51 46	6 32 22	First contact with Penumbra.
12 20 21	"	7 28 35	First contact with dark Shadow.
13 17 42	"	8 25 56	First total Immersion in dark Shadow.
14 6 29	"	9 14 43	Middle of Eclipse.
14 55 16	"	10 3 30	Last total Immersion in dark Shadow.
15 52 37	"	11 0 51	Last contact with dark Shadow.
16 48 50	"	11 57 4	Last contact with Penumbra.

Magnitude of the eclipse* $(=\frac{\text{RU}}{\text{RT}}$, see fig.) $=\frac{s+d-m}{2d}=\frac{3406''}{1980''}=1.720$ on the southern limb.

88. *To project a lunar eclipse.*

The same results may be obtained by means of an easy projection, with the advantage of having, as it were, an exact picture or representation of the particular features of the eclipse.

Draw the axes AD and CF (fig. 20) perpendicular to each other; and place the *plus* sign above, and the *minus* below, on the line AD,—and on the line CF, at the left and right respectively. Set off the values of p (Art. 87.) from S upon the line CF, in directions corresponding to their signs; thus $-2005''$ will be SP toward the right hand, and $+1821''$ will be SP′ toward the left. In the same way make P XIII, . . . P′ XV, equal to $+1183''$ . . . $-467''$ respectively. Through the points XIII, XIV, XV, draw

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out its logarithmic secant, cosine, and sine. This will save the trouble of a second, third, and fourth reference to the same place.

* The *magnitude of an eclipse*, in a partial eclipse, is the greatest distance to which the shadow advances on the moon, measured in parts of her diameter. In a total eclipse, the shadow being supposed to have advanced, not only upon the moon, but beyond it on the other side, the "magnitude" is still measured from the point of the moon's limb, nearest the centre of the shadow at the middle of the eclipse, across the moon's disc to the edge of the shadow beyond, and is therefore always equal to $\frac{s+d-m}{2d}$.

The magnitude of the eclipse is frequently given in digits, or 12ths of the moon's diameter; thus in the present case, 12×1.720, or 20.6 digits are eclipsed on the moon's southern limb.

The eclipse is said to be on that limb of the moon, which is nearest the centre of the shadow at the middle of the eclipse, and is therefore on the northern or southern limb, according as q' is negative or positive.

BE, the relative orbit of the moon, and through S, RSU at right angles to the same, cutting it in M.

With the semidiameter of the earth's shadow (s) and the centre S, describe the circle ACDF, and with the same centre, and the radii $s+d$ and $s-d$, cut the relative orbit in the points B, E, and B′, E′. With the radius d, or the moon's semidiameter, and centres B, B′, M, E′, E, describe circles, representing the moon, at the moments of the different phases. Then, by ascertaining the values of XIII XIV, or XIV XV on a scale of equal parts, the distances XII B, XIII B′, XIV M, XIV E′, and XV E, will show in parts of the value of an hour, how long after 12^h, 13^h, &c., respectively, the different phases take place.

By a scale of equal parts, $\frac{\mathrm{RU}}{\mathrm{RT}}$ will be the magnitude of the eclipse.

To determine what point of the moon's limb, reckoning from the north point v around to the *right hand*, the shadow will first touch, measure the arc $A\varphi$, and add 180°; this will equal the arc $vx\varphi$. So for the last contact, the arc $AD\varphi'+180° = v'\varphi'$. These two angles are for the present eclipse,

	Ang. from N. point.
For First Contact with dark Shadow,	241° 8′
For Last Contact with dark Shadow,	71° 12′

If carefully projected on a scale three or four times larger than that of the figure, the particulars of a lunar eclipse may be ascertained with sufficient exactness, since on account of the great indefiniteness of the edge of the moon's shadow, they are neces-

[redacted]

are but little inclined to the ecliptic, are occasionally covered in the same manner, but less frequently than the stars, in proportion as they are fewer. The only other body which the moon can thus occult is the sun, whose annual path around the sun is the ecliptic itself. The two former phenomena are called *occultations* of a *star* or *planet*, the latter an *occultation* or *eclipse of the sun.*

Let us regard, for a moment, the zodiac as a wide path around the heavens, in which stars of unequal magnitudes are scattered here and there, and in which the sun, planets, and moon are moving, but the moon nearer and swifter than any of the rest. Neither the moon or planets ever retrace the same exact line along this path, but in repeated revolutions mark out an endless variety of courses, all described within the zodiac. This is the reason why no two eclipses or occultations are ever exactly alike, for the moon never overtakes any one body on precisely the same course in successive revolutions, and the path of concourse is so wide that she much more frequently passes by it to the one side or the other.

It is evident from this illustration, that occultations, whether of the stars, the planets, or the sun, are all phenomena of the same class. The only difference between them is this, that the stars have neither sensible diameter, motion, nor parallax, while the sun and planets have all these. The calculation of an occultation, when a star is the body occulted, is of course much the simplest, and to this case, then, we will first give our attention; and since the same general rules are applicable to all bodies, it will not be difficult afterward to modify our formulæ, so that they shall include the sun and planets.

90. *The parallax of the moon.*

The *lunar parallax* is that which renders tne calculation of an occultation of the sun or a star more difficult than that of an eclipse of the moon. Any considerable change of place on the earth's surface always shifts the moon's apparent place in the heavens in an *opposite* direction, on account of the proximity of that body. If all the inhabitants in that hemisphere of the earth turned towards the moon were looking towards her at the same time, the different points to which they would refer her in the

heavens would be scattered over a circle about 2° in diameter. Suppose the sun or a star were at that moment situated within this circle; there would necessarily be an eclipse *somewhere* on the earth. But as the sun is only ½° in diameter, it would depend entirely on the place of the observer whether the moon were apparently so situated within this circle of 2° as to cover it, either partially or totally, or to be wholly separated from the sun's limb. Thus in the last eclipse of Sept. 1838, an observer at Washington would see the moon almost exactly on the centre of the sun; north of Washington, at Boston, it would only obscure his southern limb, while at Charleston, South Carolina, it would cover only his northern limb; still farther south, as at Rio Janeiro, South America, the moon would be apparently thrown wholly off of the sun to the north. So according as the observer was west or east of Washington, it would eclipse the sun earlier or later, because apparently thrown forward or backward in its path.

91. *Statement of the principles of calculation of a solar eclipse.* In a lunar eclipse, it will be recollected that the places of the earth's shadow and moon in right ascension and declination being given for several instants of time, and the differences between these being represented by p and q, the times of first and last contact were those at which

$$\{p^2+q^2\}^{\frac{1}{2}}=s+d,$$

s and d being the semidiameters of the moon and shadow.

Let us transfer our ideas to the small space in the heavens occupied by the bodies in a solar eclipse or occultation, making the sun correspond to the earth's shadow represented in fig. 20. The case *would be* precisely the same as in a lunar eclipse, if the moon had no parallax; and then at the moments of contact,

$$\{p^2+q^2\}^{\frac{1}{2}}=\mathrm{D}+d,$$

D and d being the semidiameters of the sun and moon.

And the case *will be* precisely the same as in a lunar eclipse, if we calculate the amount of the moon's parallaxes or displacements in right ascension and declination for each of the given instants, and applying them to her *true* places, obtain her *apparent* places. If, in our supposed circle of 2°, the limit of the

moon's possible displacements, we make $-u$ and $-v$ equal to the displacements in R.A. and Decl. as seen by us, the times of the contacts would be when

$$\left\{(p-u)^2+(q-v)^2\right\}^{\frac{1}{2}}=\mathrm{D}+d.$$

So that a solar eclipse only differs from a lunar in calculating the displacements or parallaxes of the moon in right ascension and declination, and so using the differences between her *apparent* places and those of the sun as the sides of a right angled triangle, of which the sum of their semidiameters is the hypothenuse. (See Art. 85.)

92. *Parallaxes of the moon in right ascension and declination.* We wish familiarly to illustrate to the student the effect which the vertical parallax or depression of the moon has in changing its right ascension and declination, at different hours of the day. Clear ideas on these points will aid him much in understanding the more intricate formulæ we shall presently have occasion to employ. The learner will see, by reference to fig. 21, without a minute explanation of its parts, that if HO represent the southern horizon, AB the meridian, and IX III XXI the moon's diurnal path through the southern sky from her rising to her setting, the moon will be depressed in the vertical circles by the spaces IX 9, VII 7, IV 4, III 3, &c., and less and less as she rises in proportion to the sine of her zenith distance, (Olmsted's Astron. Art. 83.) Her apparent path will be 9 7 4 3 M′ 21, every where *below* her true one IX VII IV III M XXI. Now, while she is rising from the left hand toward the right, on the east side of the meridian, the horary circles, or those of right ascension, being perpendicular to her apparent path, as PP′, QQ′, must slope downwards and to the right hand. The moon by her vertical depression will therefore be thrown forward in right ascension by the spaces C7, D4, but less and less as she approaches the meridian, where the parallax in right ascension is nothing at all. On the west side of the meridian, on the other hand, we see why, for a similar reason, the moon's right ascension is diminished by her parallax, and she is thrown backward in her course by the spaces E1, FM′, G 21.

The moon's parallaxes in declination are the same as the

breadths of the space between her true and apparent diurnal paths, and, unlike the parallaxes in R.A., vary but little throughout her course both above and beneath the horizon. The reason is, that while the parallaxes in declination III 3, IE, MF, obviously bear a smaller proportion to their respective vertical parallaxes as the moon goes down, the vertical parallaxes themselves grow larger, and thus compensate the effect. In fact, the parallaxes in declination increase slightly from the meridian to the horizon in this latitude.

We see, that the parallaxes in right ascension and declination form the two sides of a small right angled triangle of which the vertical parallax is the hypothenuse.

From this slight illustration of the parallaxes, we may already draw some obvious and useful conclusions. When the moon is at the *east* of the meridian, or *rising*, she is thrown forward in her course, and consequently an eclipse or occultation occurs *earlier* than the time of true conjunction in right ascension. And since the eclipse happens earlier in its ascent towards the meridian than it otherwise would have done, it is evident that it must happen also at a greater distance from the meridian. So, on the other hand, when the moon is *west* and *declining*, an occultation always occurs *later* than the conjunction in R.A.; yet, as before, at a greater distance from the meridian.

93. *Reduction to the sphere.*

The parallax of the moon is still farther modified by the spheroidal figure of the earth. The sine of the horizontal parallax of the moon is equal to the earth's radius divided by the moon's distance from the earth's centre. But in an oblate spheroid like the earth, the radius is not the same in different latitudes, but decreases from the equator to the pole; the sine of the horizontal parallax must, of course, decrease in the same ratio. Table VI. contains the corrections for every degree of latitude, to be subtracted from the horizontal *equatorial* parallax, as given in the Nautical Almanac for every 12 hours.

But the spheroidal figure of the earth affects parallax in still another way. The zenith from which we usually reckon zenith distances, is that point where a plumb-line, or a perpendicular to the surface of still water, if produced, would cut the sphere of

the heavens. Let PSQ (fig. 22.) represent the earth, P and Q being the compressed poles, S the place of the spectator, and ZSN a perpendicular to the surface of the spheroid at that point. The zenith to which parallaxes are referred, is the point where the radius OS if produced would cut the celestial sphere, because parallaxes are always measured from the moon's true place as seen at the centre of the earth. A reduction is therefore necessary of the true zenith Z to the *reduced zenith* Z′. The reduced zenith is always nearer to the equator than the true zenith, because in an ellipse, the angle SOP is always greater than SNP; the *reduced latitude* is consequently always less than the true latitude by the angle ZSZ′.

Table VI. contains the values of ZSZ′ for every degree of latitude; by simply subtracting the angle found in the table from the known latitude of a place, we obtain its reduced latitude. The parallaxes of the moon, calculated with this latitude, will be such in direction and amount as if reckoned from the reduced zenith.

Referring to fig. 22, it is easily seen, that by employing the reduced radius OS, and the reduced zenith Z′, or what is the same thing, the reduced latitude SOE, we render the parallaxes such as they would be in an imaginary sphere described around the centre O with the radius OS. Let P′SQ′ be such a sphere for the place S; Z′ would be the true zenith of S on such a sphere, and SOE its true latitude. Such an ideal sphere might be described for every different place on the earth's surface, and the reduction of parallaxes for any particular place is hence termed the *Reduction to the Sphere.*

94. *Augmentation of the moon's semidiameter.*

The immersion or emersion of a star takes place when its distance from the moon's centre is equal to her semidiameter. But this semidiameter is different for different places on the earth's surface. The earth's centre, as before, is the standard of reference. For this point the moon's *true* semidiameter MOD (fig. 22.) is calculated in the Nautical Almanac for every 12 hours. But the *apparent* semidiameter MSC for any point S is always larger than this when the moon is above the horizon, because MS is then less than MO. Let $\Delta = \frac{MS}{MO}$, or the proportion of

the moon's apparent distance to her true distance MO taken as unity, and let d and d' be the true and apparent semidiameters of the moon; then OM being unity or radius, $\sin d = \text{MD}$; also $\triangle \sin d' = \text{MC}$ $\therefore \sin d = \triangle \sin d'$, since MC and MD are radii of the same sphere. The *true* and *apparent* semidiameter of the sun are exactly the same in all calculations, the difference between them being imperceptible on account of his great distance.

The semidiameter of the moon is always so small, that no error of moment is introduced by supposing $\sin d = d$, and $\sin d' = d'$. And the same may be said of the parallaxes;—for instance $\sin \pi = \pi$ nearly.

95. *Investigation of the formulæ for calculating an occultation.* At this stage, some previous knowledge of Spherical Trigonometry will be found of much advantage, and in order to relieve the memory, and render reference to other text-books needless, one or two equations in common use are here inserted.

If the three angles of any spherical triangle be denoted by A, B, and C, and the three sides by a, b, and c, we have these equations:

In a triangle right angled at C,

Formula 1. $\sin a = \sin c \sin A.$

In an oblique angled triangle,

Formula 2. $\cos a = \cos b \cos c + \sin b \sin c \cos A.$

All the known methods employed by astronomers, in investigating rigorously the theory of occultations, presume in the reader a wider range of mathematical acquirement than is usually embraced in the course of most American colleges, and are too tedious and intricate to be introduced into the body of this work. It is therefore thought preferable to adopt a formula, which has been demonstrated analytically by Prof. Bessel of Germany,* and which, it is believed, possesses some advantages over every other; and assuming it as true, to give such a demonstration or explanation of it by geometrical illustration, as shall render its several parts clear and intelligible to the general student. Yet, for the benefit of those desirous of a more tho-

* Prof. Bessel's papers on this subject were originally published in Schumacher's Astronomische Nachrichten, and are translated into English in the 6th and 8th volumes of the Philosophical Magazine, New Series, 1829 and 1830.

rough investigation, we have thrown into the form of a note at the end of the work a demonstration, which, although rigorously exact, will be found to require no previous knowledge of Algebraical Geometry, nor any with Spherical Trigonometry, except of the few common formulæ there assumed as known.

96. In fig. 23, let EQX be the equator, and P its pole; and let E be the vernal equinox from which right ascensions are reckoned in the direction EQX. Let M be the true place of the moon, and S the star to be occulted; and PMR and PSQ hour circles passing through them. Let Z be the reduced zenith, and PZT the meridian of the place of observation.

Also, make QX = 90°, and join PX; then PQX is a triangle, having each side and each angle = 90°, and QX, PQ, PX are the arcs of great circles, whose poles are P, X, and Q.

Produce QP to Y, making PY = SQ, and join XY; then YSX is a triangle of the same kind with PQX. From X and Y draw arcs of great circles through Z and M.

Let EQ $= A =$ apparent R.A. } of the body occulted.
QS $(= \text{PY}) = \Delta =$ " Decl. }

ER $= \alpha =$ true R.A. }
RM $= \delta =$ " Decl. } of the moon.
$\varpi =$ horizontal parallax }
$d =$ horizontal semidiameter }

ET $= \mu =$ sidereal time } of the place of observation.
TZ $= \varphi =$ reduced latitude }

Then QR = angle BPM $= \alpha - A$
QT = angle CPZ $= \mu - A$

The quantity ϖ denotes the parallax of the moon reduced for the latitude of the place by Table VI.

We have then at the time of immersion or emersion of a star,

$$\text{(I.)}\ \sin^2 d = \{\cos\delta\sin(\alpha - A) - \sin\varpi\cos\varphi\sin(\mu - A)\}^2$$
$$+ \{\sin\delta\cos\Delta - \cos\delta\sin\Delta\cos(\alpha - A)$$
$$- \sin\varpi\{\sin\varphi\cos\Delta - \cos\varphi\sin\Delta\cos(\mu - A)\}\}^2.$$

This is, in effect, the equation demonstrated by Prof. Bessel in the Phil. Mag. for 1829, vol. VI. p. 338; slightly altered in notation, and cleared of fractional expressions. We shall give an approximate demonstration of this formula by a geometrical construction.

If we take the several parts of this equation, and refer them to the figure, we shall find

1. $\cos\delta\sin(\alpha-A)=\cos$ RM sin BPM = sin PM sin BPM = (Form. 1.) sin BM = cos XM.
2. $\cos\varphi\sin(\mu-A)=$ (for the same reason) cos XZ.
3. $\sin\delta\cos\varDelta-\cos\delta\sin\varDelta\cos(\alpha-A)=$ sin RM cos QS − cos RM sin QS cos BPM = cos PM cos PY + sin PM sin PY cos MPY* = (Form. 2.) cos YM.
4. $\sin\varphi\cos\varDelta-\cos\varphi\sin\varDelta\cos(\mu-A)=$ (for the same reason) cos YZ.

Therefore, if we let

$$\cos XM = p = \cos\delta\sin(\alpha-A).$$
$$\cos YM = q = \sin\delta\cos\varDelta-\cos\delta\sin\varDelta\cos(\alpha-A).$$
$$\sin\pi\cos XZ = u = \sin\pi\cos\varphi\sin(\mu-A).$$
$$\sin\pi\cos YZ = v = \sin\pi\{\sin\varphi\cos\varDelta-\cos\varphi\sin\varDelta\cos(\mu-A)\}.$$

the equation above stated becomes

$$\text{(II.)}\qquad (p-u)^2+(q-v)^2=\sin^2 d.$$

If, then, two great circles cut one another at right angles at the point of the star to be occulted, one of them being an hour circle, p and q will be the cosines of the arcs joining the poles of these two circles with the true place of the moon; and $\frac{u}{\sin\pi}$ and $\frac{v}{\sin\pi}$ will be the cosines of the arcs joining these two poles with the reduced zenith of the place.

97. The small portion BSAM of the heavens, in which the moon and star are situated, may be considered as a plane surface, and portions of great circles crossing it, as straight lines.† Let PS and XS (fig. 24.) be the two circles cutting one another at right angles at the place of the star S, of which PS is an hour circle; let M be the true place of the moon, and M′ its apparent place as depressed in a vertical circle. At the moment of immersion or emersion the distance between the star and the apparent place of the moon will be equal to her semidiameter,

* The sign is here changed, because cos MPY = −cos BPM. (See Table I.)

† In fig. 23, we look on the celestial sphere from an imaginary point outside of it; in figs. 24 and 25, from our real station within it. The small portion BSAM in fig. 23 is therefore reversed in a horizontal direction in figs. 24 and 25. The latter represent an occultation and eclipse occurring to the west of the meridian.

or M′S. Draw MB and M′F perpendicular to PS, and AM and GM′ parallel to the same. Then $M'S^2 = (BM-GM)^2+(BS-GM')^2$. This equation, it will be seen, is much like equation II. And since M′S = sin d or d, if we can show that BM, GM, BS, GM′ are equal, or very nearly equal to p, u, q, and v respectively, we shall demonstrate equation II, and consequently, its more expanded form in equation I, sufficiently for all the purposes of illustration.

(1.) p is equal to BM; for (see fig. 23.) p = cos XM = sin BM (fig. 24.) = BM very nearly, since in an eclipse BM is always very small.

(2.) q = BS; since q = cos YM = sin AM = AM = BS.

(3.) u = GM; for (see figs. 23 and 24,) MM′, or the parallax in altitude is equal (Olmsted's Astron. Art. 83,) to sin ϖ × sin. app. zen. dist.; but the app. zen. dist. of the moon differs from that of the star by a quantity less than her semidiameter; therefore MM′ = sin ϖ sin ZM′ = sin ϖ sin ZS very nearly. But GM = MM′ sin GM′M = sin ϖ sin ZS sin PSZ = (Form. 1.) sin ϖ sin CZ (see fig. 23,) = sin ϖ cos XZ. But u = sin ϖ cos XZ; therefore u = GM.

(4.) In the same way it may be shown that v is equal to GM′; since GM′ = MM′ cos GM′M = MM′ sin ZSX.

It is easily seen in fig. 24, that the portions of equations I. represented by p and q, express the differences between the star and the true place of the moon measured on the circles XS and PS; while the parts represented by the symbols u and v are equal to the parallaxes of the moon on the same circles. Thus we find our rough statement of the principles of calculation (see Art. 91.) confirmed.

We have in this demonstration supposed the sines of very small arcs equal to the arcs themselves, and have neglected the minute augmentation of the moon's semidiameter due to her altitude, (Art. 93.) But in the rigorous solution of the problem, (see Note at the end of the volume,) all these have been taken into account. It expresses the condition, that the moon, considered as a sphere, should have for a tangent the ray proceeding from a star to the observer, at a given point on the earth, considered as a spheroid; and as no limitation was introduced into the analysis, it may be regarded as mathematically accurate

And equation II, being identical with equation I, has the same foundation for perfect correctness.

98. *Formulæ for a solar eclipse.*

To adapt the foregoing formula to eclipses of the sun, some modifications are necessary. If we take the square root of eq. II, it becomes

$$\{(p-u)^2+(q-v)^2\}^{\frac{1}{2}}=\sin d,$$

or since, by Art. 93, $d=\sin d$ nearly,

(III.) $$\{(p-u)^2+(q-v)^2\}^{\frac{1}{2}}=d.$$

The same strict analysis by which the equation for occultations was obtained, when applied to eclipses of the sun, leads to an extremely intricate equation; from which, however, by striking out the higher powers of very small arcs, whose omission cannot introduce an error of more than one or two tenths of a second of space, another equation results of comparative simplicity, and of correctness amply sufficient for all purposes of calculation. It is as follows:

(IV.) $$\{(p-u)^2+(q-v)^2\}^{\frac{1}{2}}=\mathrm{D}+d-\mathrm{D}\sin\pi\cos z.$$

where D denotes the true semidiameter of the sun,

z " the sun's true zenith distance,

and π " the difference of the horizontal parallaxes of the moon and sun.

It is to be remarked that d and D in equations III and IV, are to be expressed in parts of radius as unity, and not in seconds of space.

99. It will be seen that this differs from the formula for occultations in two points; 1st, the term π, which enters as an element into the values of u and v, and is also contained in the last term of the equation, does not express, as before, the moon's horizontal parallax, but the difference between her own and the sun's; and 2nd, the distance between the two bodies at the moment of contact, or the *sum* of their semidiameters is diminished by the term $\mathrm{D}\sin\pi\cos z$.

To explain the reason of these differences, let us have recourse to fig. 25. Let M be the true place of the moon, and S that of the sun at the moment of first or last contact. The moon, as

before, is depressed from M to M′, in the direction of a vertical circle; the sun also, being about 400 times as far off as the moon, is depressed $\frac{1}{400}$ as much, or from S to S′. The distance S′M′ then equals the sum of their semidiameters, and $S'M'^2 = M'F^2 + S'F^2$, as before. But $M'F = BG = MH-(MG-SL)$; and $S'F = SH-(M'G-S'L)$. Now SH is the difference of declination between the true places of the sun and moon, or q (Art. 97), while $MH = p$, or their difference of place on the circle perpendicular to PS. Also $M'G-S'L =$ the difference of parallaxes on an hour circle, and MG−SL on the perpendicular circle. Therefore that the equation may be of the form $(D+d)^2 = (p-u)^2+(q-v)^2$, which corresponds to $S'M'^2 = \{MH-(MG-SL)\}^2+\{SH-(M'G-S'L)\}^2$,—since p and q are equivalent to MH and SH, u and v should correspond to MG−SL and M′G−S′L; that is, to the *differences* of parallax on the circles PS and XS. And, because the triangles MGM′ and SLS′ are similar, and the sines of the zenith distances of M′ and S′ are nearly the same, therefore these differences have the same ratio to the whole parallaxes of the moon on these circles, as the difference between the horizontal parallaxes has to the moon's whole horizontal parallax. Consequently, that u and v may correspond to the *differences* MG−SL and M′G−S′L, the quantity $\sin\pi$, which enters as a factor into their value, must refer to the *difference* between the horizontal parallaxes of the sun and moon.

Taking the square root of the equation above $\{(p-u)^2+(q-v)^2\}^{\frac{1}{2}} = D+d$. The addition of the term $-D\sin\pi\cos z$ in eq. IV, depends on the difference between the true and apparent semidiameters of the moon, (Art. 93.) A slight consideration of fig. 25 shows us, that the sun's true zenith distance (z) never differs much in an eclipse from the moon's apparent zenith distance; z is therefore nearly equal to Z′SM, (fig. 22,) or the moon's app. zen. dist. We have then by Art. 93,

$$d' : d :: MO : MS :: \sin OSM\ (\sin z) : \sin MOZ'.$$

Let $OMS = n$; then since $MOZ' = MSZ'-OMS$, $\therefore \sin MOZ' = \sin(z-n) =$ (Day's Trig. Anal. 208. II.) $\sin z\cos n - \cos z\sin n = \sin z - \sin\pi\sin z\cos z$, since $\cos n = 1$, very nearly, and $\sin n = \sin\pi\sin z$, (Olmsted's Astron. Art. 83.) Then by the proportion above, we have

$$\frac{d}{d'}=\frac{\sin z-\sin \varpi \sin z \cos z}{\sin z}=1-\sin \varpi \cos z.$$

Hence we see, that the semidiameter of the moon is always apparently augmented when above the horizon nearly in the proportion of $1-\sin \varpi \cos z : 1$.

Now it is manifest, that the distance between the moon's apparent place and the star, at the moment we see it occulted, is equal to her *apparent* semidiameter. But in eq. III, her true semid. d was used. In fact, the analysis by which that equation was obtained eliminates the quantity d' in the process, and introduces d; and, (since the other parts of the equation are proportionably diminished,) this change might be expressed in fig. 24 by supposing the scale on which it was projected to have been reduced in the proportion $d' : d$. When, therefore, we introduce the new quantity D (which represents either the sun's true or apparent semidiameter, Art. 94,) into eq. IV, or fig. 25, the same analysis requires that this also suffer first a similar reduction in the ratio $\frac{d}{d'}$, and the quantity M′S′ in fig. 25 becomes equal to

$d+\text{D}\frac{d}{d'}=d+\text{D}\ (1-\sin \varpi \cos z)=d+\text{D}-\text{D} \sin \varpi \cos z$, which is exactly the second member of eq. IV.

The fact may be thus expressed, that, to correspond with the substitution of the *true* for the *augmented* semidiameter of the moon, the sun's semidiameter is diminished in the same ratio.

100. The value of q in Art. 96 may easily be brought into a form better adapted for calculation. We there obtained,

$\sin \delta \cos \varDelta-\cos \delta \sin \varDelta \cos (\alpha-A)=q.$

$2 \sin^2 \frac{1}{2} (\alpha-A)=1-\cos (\alpha-A)\ldots$ (Day's Trig. Anal. Art. 210.)

Transposing the 1, and multiplying by $\cos \delta \sin \varDelta$, we have

$-\cos \delta \sin \varDelta+2 \cos \delta \sin \varDelta \sin^2 \frac{1}{2} (\alpha-A)=-\cos \delta \sin \varDelta \cos (\alpha-A).$

Substituting this value of $-\cos \delta \sin \varDelta \cos (\alpha-A)$ in the first equation,

$$\sin \delta \cos \varDelta-\cos \delta \sin \varDelta+2 \cos \delta \sin \varDelta \sin^2 \tfrac{1}{2} (\alpha-A)=q.$$

But (Day's Anal. Trig. Art. 208, II.)

$$\sin \delta \cos \varDelta-\cos \delta \sin \varDelta=\sin (\delta-\varDelta)$$

$$\therefore \sin (\delta-\varDelta)+2 \cos \delta \sin \varDelta \sin^2 \tfrac{1}{2} (\alpha-A)=q.$$

In formula IV, we have a quantity $\cos z$, which cannot be ta-

ken directly from the ephemeris, and it must therefore be brought into a different form. For this purpose we have

$\cos z = \cos$ ZS (fig. 23.) = (Form. 2) cos PZ cos PS + sin PZ sin PS cos SPZ = sin TZ sin QS + cos TZ cos QS cos SPZ $= \sin \varphi \sin \varDelta + \cos \varphi \cos \varDelta \cos (\mu - A)$. (G.)

All our formulæ are thus far expressed in parts of the radius of a sphere, whereas it is more convenient that they should be expressed in seconds of arc, of which 206264″.8 = radius. We will therefore change them so as to answer this purpose.

$$p = \sin (\alpha - A) \cos \delta.$$

Since $\sin (\alpha - A)$ is very small, we may substitute the arc, and

(C.) $p = (\alpha - A) \cos \delta$; . . . where $\alpha - A$, are expressed in seconds instead of parts of the radius.

Again, . . . $q = \sin (\delta - \varDelta) + 2 \cos \delta \sin \varDelta \sin^2 \frac{1}{2} (\alpha - A)$.

Here $\sin (\delta - \varDelta)$ and $\sin \frac{1}{2} (\alpha - A)$ are nearly equal to $\frac{\delta - \varDelta}{\text{rad.}}$ and $\frac{\frac{1}{2} (\alpha - A)}{\text{rad.}}$, and therefore

$$q = \frac{\delta - \varDelta}{\text{rad.}} + 2 \cos \delta \sin \varDelta \frac{(\alpha - A)^2}{4 \text{ rad.}^2}.$$

Multiplying, as before, by radius, to reduce from parts of radius to seconds,

(D.) $q = \delta - \varDelta + \frac{1}{2r} \cos \delta \sin \varDelta (\alpha - A)^2$; . . . where $\delta - \varDelta$, and $\alpha - A$, are expressed in seconds of arc, and $r = 206265''$.

In the expressions for u and v, $\sin \varpi = \frac{\varpi}{\text{rad.}}$ nearly, and multiplying by rad.,

(E.) $u = \varpi \cos \varphi \sin (\mu - A)$.

(F.) $v = \varpi \{ \sin \varphi \cos \varDelta - \cos \varphi \sin \varDelta \cos (\mu - A) \}$.

To correspond with these changes, let d and D, in equations III and IV, instead of being expressed in parts of radius, be converted into seconds of space, and we shall have

(A.) $\{ (p-u)^2 + (q-v)^2 \}^{\frac{1}{2}} = d$, . . . for occultations.

(B.) $\{ (p-u)^2 + (q-v)^2 \}^{\frac{1}{2}} = D + d - D \sin \varpi \cos z$, . . . for eclipses.

The advantage of this change in the formulæ consists in this, that the quantities d, D, ϖ, &c., as taken from the Naut. Alm., are expressed in minutes and seconds of arc, and not in parts of the radius.

The quantities $\alpha - A$, and $\mu - A$, must be converted from hours, minutes, and seconds, into degrees, minutes, and seconds, by multiplying by 15; they will thus be of the same denomination as the other quantities.

101. In the above equations (A) and (B), if we consider that p, q, u, v, and $\cos z$ are but abbreviated symbols of their values in equations (C), (D), (E), (F) and (G), we shall see, that these our two principal equations are made up entirely of known quantities. Most of these quantities, however, are to be taken from the Naut. Alm., and are therefore known only, when the *times* for which they are to be computed are given. Now the *times* of beginning and end of occultation are the very quantities of which we are in search, and are, of course, *unknown*. We must therefore find them by some method of approximation, or of trial and error.

Let T be the approximate time of greatest obscuration; and let $T-30^m$, T, $T+30^m$, in the case of a star,—and $T-1^h$, T, $T+1^h$, in the case of the sun, be three instants chosen for the computation of the equation (A), or (B). It is evident, that these three instants will include, or nearly include, the whole occultation or eclipse.

But to find T, or the time of the middle of the eclipse, is a matter of some little difficulty. It is always nearly the same as the time of apparent conjunction in R.A. of the sun and moon. In fig. 21, suppose the sun at M′; the moon, to be in app. conj. with it, must be apparently at M′, but truly at M; that is, she must have advanced in R.A. by the quantity HM, since the time of true conjunction. The time of app. conj. is therefore later west of the meridian, and earlier east of it, (see Art. 92, last paragraph,) than the time of true conj., by the time the moon occupies in describing HM.

The Gr. m. time of true conj. is easily found, (see our example, Art. 104.) Suppose New Haven to be the place of observation; convert this Gr. m. time into N. H. sid. time, (by Prob. II, Art. 79,) and call it μ'. Also let the unknown N. H. sid. time of app. conj. be μ''. Now HM = (see Art. 97. 3, and fig. 24.) u nearly = (Art. 99, eq. E,) $\pi \cos \varphi \sin (\mu'' - A)$, since HM is the parallax of the moon on the perpendicular circle at the time of

app. conj., or μ''. Take from the Naut. Alm. the difference between the moon's R.A. at two successive hours at the time of the eclipse, and that it may correspond with the quantity π, convert it from seconds of time to minutes of space by dividing by 4; and call it h. Since the moon's change of R.A. in 1 hour equals h, her *actual motion* at a declination δ, and in a direction parallel to the equator, will be (Art. 63.) $h \cos \delta$ per hour, and the time of describing $\mathrm{HM} = \frac{\mathrm{HM}}{h \cos \delta} = \frac{\pi \cos \varphi \sin (\mu'' - A) \sec \delta}{h}$.

Then since the time of app. conj. differs from that of true conj. by the time of describing HM, we have

$$\text{(Y.)} \qquad \mu'' = \mu' + \frac{\pi \cos \varphi \sec \delta}{h} \sin (\mu'' - A).^*$$

On the second side of this equation, we find μ'', which being the object of our search, is of course unknown, and requires that the equation, being of a transcendental nature, should be solved by successive approximations, as is done in our example, Art. 104.

Having found μ'', the N. H. sid. time of app. conj., convert it into the Gr. m. time of the same event. This will be nearly the middle time of occultation, or T; and subtracting and adding 30^m or 1^h, as the case may be, we have the instants of time required.

102. Now if we compute equation (A) or (B) for the three instants thus arbitrarily chosen, we shall find that at none of them are the two members of the equation (which we will represent by m and n respectively) equal to one another; for such equality takes place only at the two unknown instants of first and last contact. The second member, n, being the sum of the semidiameters of the two bodies, remains nearly constant during an eclipse; but the first member, m, which expresses the apparent distance between their centres at the instants for which it is computed, is of very rapid change. From a value greatly above that of n before the eclipse, it gradually diminishes, becomes equal to n at the moment of first contact, and still decreasing,

* Here $\sin (\mu'' - A)$ becomes negative at the east of the meridian, (because $\mu'' - A$ is in that case between 12^h and 24^h, or between -0^h and -12^h,) and therefore changes of itself the sign of the fraction, so as not to require the double sign after μ'.

reaches a minimum at the time of greatest obscuration; thence it increases, and again becomes equal to n at the moment of last contact. By considering these circumstances, and noticing whether at $T-30^m$ and $T+30^m$, (or at $T-1^h$ and $T+1^h$,) m is greater or less than n, and how much, we can form some idea where the times of first and last contacts in an occultation or an eclipse occur amidst our three assumed instants, and interpolate for them accordingly.

The quantity m is so variable in its rate of change, that it is unsafe to interpolate from three values for any intermediate ones; but $p-u$, and $q-v$, which form the sides of a right angled triangle, of which m is the hypothenuse, are comparatively uniform in their change. Therefore, from the three values of $p-u$, of $q-v$, and of n, which have been computed, interpolate intermediate values at intervals of 5, 10, or 15 minutes, to such an extent as will probably include the two required unknown times. For example, if at $T-1^h$, m is a little greater than n, interpolate for $T-50^m$ and $T-40^m$. Then if at $T-50^m$, the excess of m over $n=a$, and at $T-40^m$, m is less than n by b, say

$$\text{Time of first contact} = T-50^m+10^m \frac{a}{a+b}.$$

If the time of first contact, as thus obtained, is found to be about $T-43\frac{1}{2}^m$, to obtain a somewhat more accurate result, interpolate again for $T-44^m$, and $T-43^m$, and let the excess and defect of m at these instants be a' and b' respectively; then we have for a second approximation,

$$\text{Time of first contact} = T-44^m+1^m \frac{a'}{a'+b'}.$$

And this process, in all ordinary cases, will give the times of first and last contact with sufficient accuracy.

103. *Synopsis of formulæ for stellar and solar occultations.* We collect our formulæ into one general view, for the sake of more convenient reference during calculation.

First, in an occultation, at the moment of immersion or emersion,

$$\text{(A.)} \qquad \{(p-u)^2+(q-v)^2\}^{\frac{1}{2}} = d.$$

Secondly, in an eclipse of the sun or a planet, at the moment of first and last contact,

(B.) $\{(p-u)^2+(q-v)^2\}^{\frac{1}{2}} = D+d-D \sin \varpi \cos z.$

The auxiliary quantities are expressed by the following equations:

(C.) $p = (\alpha - A) \cos \delta.$

(D.) $q = \delta - \Delta + \frac{1}{2r} \cos \delta \sin \Delta \, (\alpha - A)^2.$

(E.) $u = \varpi \cos \varphi \sin (\mu - A).$

(F.) $v = \varpi \, \{\sin \varphi \cos \Delta - \cos \varphi \sin \Delta \cos (\mu - A)\}.$

(G.) $\cos z = \sin \varphi \sin \Delta + \cos \varphi \cos \Delta \cos (\mu - A).$

In these equations, ϖ always denotes the difference between the horizontal parallaxes of the moon and of the body occulted at the place of observation.

Finally, to find T, or the middle time for which the equations are to be computed, we have

(Y.) $\mu'' = \mu' + \frac{\varpi \cos \varphi \sec \delta}{h} \sin (\mu'' - A).$

Of the calculations belonging to Stellar and Solar Occultations.

104. Example. *Let it be required to find the times of first and last contact of the moon and sun in the annular eclipse of Sept. 18, 1838.*

From the tables of the Nautical Almanac for 1838, we require the following data:

Mean Time.

1838. Sept.	Sun's R.A. = A.	Sun's Decl. = Δ.	Sun's semid. = D.	Sid. Time of Gr. Mean Noon.	
$17^d\ 0^h$	$11^h\ 38^m\ 25^s.08$	$+2°\ 20'\ 14''.3$	$15'\ 56''.7$	$11^h\ 47^m\ 50^s.18$	From p. II. of the month.
18^d "	" 42 0 .61	+1 56 58 .2	" 57 .0		
19^d "	" 45 36 .16	+1 33 39 .5	" 57 .3		
20^d "	" 49 11 .75	+1 10 18 .6	" 57 .5		

Sept.	Moon's semid. = *d*.	Moons's eq. hor. par.	
$17^d\ 12^h$	$14'\ 42''.3$	$53'\ 57''.7$	From p. III. of the month.
18 0	" 41 .5	" 54 .8	
18 12	" 41 .1	" 53 .3	
19 0	" 40 .9	" 52 .8	

Sept. 18^d	Moon's R.A. = α.	Moon's Decl. = δ.	
$7^h\ 0^m$	$11^h\ 41^m\ 33^s.74$	$+2°\ 57'\ 14''.9$	From pages V-XII of the month.
8^h "	" 43 17 .79	+" 43 4 .6	
9^h "	" 45 1 .79	+" 28 53 .9	
10^h "	" 46 45 .75	+" 14 42 .6	
11^h "	" 48 29 .67	+" 0 31 .0	

	Sun's hor. par.	
Sept. 18	$8''.5$	From p. 266 immediately after the months.

The true conj. of the sun and moon in R.A. will plainly be about $18^d\ 8^h$, or a little before; and if we find the sun's R.A. for $18^d\ 7^h$ and $18^d\ 8^h$ respectively, we have

	A	a	$a - A$
$18^d\ 7^h$	$11^h\ 43^m\ 3^s.49$	$11^h\ 41^m\ 33^s.74$	$-89^s.75$
$18^d\ 8^h$	11 43 12 .47	11 43 17 .79	$+5$ .32
		Diff. between the two =	-95 .07

Then $-95^s.07 : -89^s.75 :: 60^m : 56^m\ 38^s$, and $18^d\ 7^h\ 56^m\ 38^s$ = Gr. mean time of true conj. in R.A.

Now to find the time of apparent conjunction by Art. 100, we have

On Sept. 18, $7^h\ 56^m\ 38^s$ Gr. m. t. $= 19^h\ 45^m\ 46^s.5$ Gr. sid. t.; by Prob. II.
4 51 46 Longitude of N. H.

$\mu' =$ 14 54 0 N. H. sid. t. of true conj.

For finding μ'' a very rough method will answer. Referring to eq. Y, Art. 102,

Take $\mu' - A = 14^h\ 54^m - 11^h\ 43^m = 3^h\ 11^m = 48°$.
$\phi = 41°.3 =$ Lat. of N. H.
$\pi = 53'.9$
$m = 11^h\ 45^m\ 1^s - 11^h\ 43^m\ 17^s = 104^s = 26'.0$
$\delta = 3°$
and $\mu'' - A = \mu' - A$, for a first approximation.

$l.\ \pi \ldots\ldots + 1.732$
$l.\ h\ (a.\ c.) + 8.585$
$l.\ \cos\phi\ . + 9.877$
$l.\ \sec\delta\ . + 10.001$

$l.\ \dfrac{\pi \cos\phi \sec\delta}{h}\quad + 0.195$
$l.\ \sin(\mu' - A)\quad + 9.87$

$+ 0.06$
Nat. num. $+1^h.15 = 1^h\ 9^m$.

Then $3^h\ 11^m + 1^h\ 9^m = 4^h\ 20^m = 65°$, or the 1st approximation to the value of $\mu'' - A$.

For a second approximation,

$l.\ \dfrac{\pi \cos\phi \sec\delta}{h} \ldots +0.195$
$l.\ \sin 65° \ldots\ldots\ldots +9.957$

$+0.152$
Nat. num. $+1^h.42 = 1^h\ 25^m$.
Then $3^h\ 11^m + 1^h\ 25^m = 4^h\ 36^m = 69°$, or the 2d approximation to the value of $\mu'' - A$.

A third trial would make $\mu'' - A = 4^h\ 39^m$; but the second is always sufficient.

Then $\mu'' - A$ being $4^h\ 39^m$,
$\mu'' = 16^h\ 22^m$ N. H. sid. t. of app. conj.
4 52 Long. of N. H.

21 14 Gr. sid. time $= 9^h\ 25^m$ = Gr. mean time of app. conj.
$T = 18^d\ 9^h\ 25^m$.

As it is more convenient to have the times some aliquot part of an hour, call the middle time either $18^d\ 9^h\ 20^m$, or $18^d\ 9^h\ 30^m$, neither of which differs essentially from the middle of the eclipse. Assuming $9^h\ 30^m$, for greater readiness in making proportions or interpolations, we have for the three times of calculation, Sept. 18^d, $8^h\ 30^m$, $9^h\ 30^m$, and $10^h\ 30^m$.*

* Although these assumed times may be chosen almost at pleasure, and loosely in even minutes, or aliquot parts of an hour, yet once determined upon, they serve as instants of exact reference, and all computations dependent upon them must be conducted accordingly.

$24^h : +215^s.54$	$:: 8^h\ 30^m : +76^s.34$	
	$:: 9\ 30 : +85.32$	
	$:: 10\ 30 : +94.30$	
For the sun's R.A., by simple proportion* we	$11^h\ 42^m\ 0.62$	
have at $8^h\ 30^m$	.. $A = 11^h\ 43^m\ 16^s.96$	
$9\ 30$	.. " = " " 25.94	
$10\ 30$	.. " = " " 34.92	

		Corr. for Interp.	
$24^h : -1398''.7$	$:: 8^h\ 30^m : -495''.4$	$+0''.3$	$= -\ 8'\ 15''.1$
	$:: 9\ 30 : -553.7$	$+0.3$	$= -\ 9\ 13.4$
	$:: 10\ 30 : -611.9$	$+0.3$	$= -10\ 11.6$
For the sun's Decl., by interpolation, we have at			$+1^\circ\ 56'\ 58''.2$
$8^h\ 30^m$	.... Δ		$= +1^\circ\ 48'\ 43''.1$
$9\ 30$	.... "		$= +$" $47\ 44.8$
$10\ 30$	.... "		$= +$" $46\ 46.6$

and so for α and δ, by simply halving the intervals between 8^h, 9^h, 10^h, and 11^h, we have

At $8^h\ 30^m$	$\alpha = 11^h\ 43^m\ 9^s.79$	$\delta = +2^\circ\ 35'\ 59''.3$	$\alpha - A = +\ 52^s.82 = +\ 792''.4$
$9\ 30$	" = " $45\ 53.77$	" = +" $31\ 48.3$	" $= +147.83 = +2217.4$
$10\ 30$	" = " $47\ 37.71$	" = +" $7\ 36.8$	" $= +242.77 = +3641.5$

Again,

	Gr. sid. time.	N. H. sid. time = μ.	$\mu - A$.
At $8^h\ 30^m$	$20^h\ 19^m\ 13^s.96$	$15^h\ 27^m\ 27^s.96$	$3^h\ 44^m\ 11^s.00 = 56^\circ\ 2'\ 45''$
$9\ 30$	$20\ 19\ 23.82$	$16\ 27\ 37.82$	$4\ 44\ 11.88 = 71\ 2\ 58$
$10\ 30$	$21\ 19\ 33.67$	$17\ 27\ 47.67$	$5\ 44\ 12.75 = 86\ 3\ 11$

For the moon's parallax,

		Moon's par. at $18^d\ 0^h$.		Corr. for Interp.	Corr. for Spheroid. Tab. VI.	Sun's hor. par.	π
At $8^h\ 30^m$	$12^h : 1''.5 :: 8^h\ 30^s : -1''.1$	$53^s\ 54''.8$	$53'\ 53''.7$	$-0''.1$	$-\ 4''.5$	$-\ 8''.5 =$	$53'\ 40''.6 = 3220''.6$
$9\ 30$	$:: 9\ 30 : -1''.2$	....	" 53.6	$-0''.1$	$-\ 4.5$	$-\ 8.5 =$	" $40.5 = 3220.5$
$10\ 30$	$:: 10\ 30 : -1''.3$	....	" 53.5	$-0''.1$	$-\ 4.5$	$-\ 8.5 =$	" $40.4 = 3220.4$

For all three times $d = 14'\ 41''.2$, and $D = 15'\ 57''.1$.
(Lat. of N. H.) $41^\circ\ 18'\ 28'' - 11'3''$ (corr. for spheroid, Tab. VI.) $41^\circ\ 7'\ 25'' = \phi$.

Logs. of	$8^h\ 30^m$	$9^h\ 30^m$	$10^h\ 30^m$	Logs. of	$8^h\ 30^m$	$9^h\ 30^m$	$10^h\ 30^m$
$\bar{\alpha} - A$†	$+2.89894$	$+3.34584$	$+3.56128$	$\sin \phi$	....	$+9.81802$	....
$\sin(\mu - A)$	$+9.91881$	$+9.97580$	$+9.99897$	$\cos \phi$	....	$+9.87097$	....
$\cos(\mu - A)$	$+9.74705$	$+9.51155$	$+8.83779$	π	$+3.50794$	$+3.50792$	$+3.50791$
$\cos \delta$	$+9.99955$	$+9.99963$	$+9.99970$	$\sin \pi$	$+8.19349$	$+8.19348$	$+8.19346$
$\sin \Delta$	$+8.49996$	$+8.49606$	$+8.49213$	D	....	$+2.98096$	....
$\cos \Delta$	$+9.99978$	$+9.99979$	$+9.99979$	$\frac{1}{2r}$	....	$+\bar{4}.385$	....

* When the second differences do not exceed $0^s.04$, or $0''.4$, interpolation will not add to accuracy, and simple proportion may be employed instead.

† The logarithms of all the factors which enter into equations (B), (C), (D), (E), (F), and (G), are here taken at once from the logarithmic tables, and arranged in order, so that during the progress of subsequent calculations, no logarithm need be looked out in the tables, but each may be copied at once from this short table in its order.

	$8^h\ 30^m$.	$9^h\ 30^m$.	$10^h\ 30^m$.
$l.\ (\alpha-A)$	+2.89894	+3.34584	+3.56128
$l.\ \cos\delta$	+9.99955	+9.99963	+9.99970
$l.\ (\alpha-A)\cos\delta$	+2.89849	+3.34547	+3.56098
$p=$	+791″.6	+2215″.5	+3639″.0
$l.\ \frac{1}{2r}$	+ $\bar{4}$.385	+ $\bar{4}$.385	+ $\bar{4}$.385
$l.\ \cos\delta$	+10.000	+10.000	+10.000
$l.\ \sin\delta$	+ 8.500	+ 8.496	+ 8.492
$2\ l.\ (\alpha-A)$	+ 5.797	+ 6.691	+ 7.122
	+ 8.682	+ 9.572	+ 9.999
$\frac{1}{2r}\cos\delta\sin\Delta\ (\alpha-A)^2$	+0″.0	+0″.4	+1″.0
δ	+2° 35′ 59″.3	+2° 21′ 48″.3	+2° 7′ 36″.8
Δ	+1° 48′ 43″.1	+1° 47′ 44″.8	+1° 46′ 46″.6
	+47′ 16″.2	+34′ 3″.5	+20′ 50″.2
$\delta-\Delta$	+2836″.2	+2043″.5	+1250″.2
	+0″.0	+0″.4	+1″.0
$q=$	+2836″.2	+2043″.9	+1251″.2
$l.\ \pi$	+3.50794	+3.50792	+3.50791
$l.\ \cos\phi$	+9.87697	+9.87697	+9.87697
$l.\ \sin(\mu-A)$	+9.91881	+9.97580	+9.99897
	+3.30372	+3.36069	+3.38385
$u=$	+2012″.4	+2294″.5	+2420″.2
$l.\ \pi$	+3.50794	+3.50792	+3.50791
$l.\ \sin\phi$	+9.81802	+9.81802	+9.81802
$l.\ \cos\Delta$	+9.99978	+9.99979	+9.99979
	+3.32574	+3.32573	+3.32573
$\pi\sin\phi\cos\Delta$	+2117″.1	+2117″.0	+2117″.0
$l.\ \pi$	+3.50794	+3.50792	+3.50791
$l.\ \cos\phi$	+9.87697	+8.87697	+9.87697
$l.\ \sin\Delta$	+8.49996	+8.49606	+8.49213
$l.\ \cos(\mu-A)$	+9.74705	+9.51155	+8.83779
	+1.63192	+1.39250	+0.71480
$\pi\cos\phi\sin\Delta\cos(\mu-A)$	+42″.8	+24″.7	+5″.2
$v=$	+2074″.3	+2092″.3	+2111″.8
$l.\ D$	+2.981	+2.981	+2.981
$l.\ \sin\pi$	+8.193	+8.193	+8.193
$l.\ \sin\phi$	+9.818	+9.818	+9.818
$l.\ \sin\Delta$	+8.500	+8.496	+8.492
	+9.492	+9.488	+9.484
	+0″.3	+0″.3	+0″.3
$l.\ D$	+ 2.981	+ 2.981	+ 2.981
$l.\ \sin\pi$	+ 8.193	+ 8.193	+ 8.193
$l.\ \cos\phi$	+ 9.877	+ 9.877	+ 9.877
$l.\ \cos\Delta$	+10.000	+10.000	+10.000
$l.\ \cos(\mu-A)$	+ 9.747	+ 9.512	+ 8.838
	+ 0.798	+ 0.563	+ 9.889
	+6″.3	+3″.7	+0″.8
$D\sin\pi\cos z$	+6″.6	+4″.0	+1″.1

	$8^h\ 30^m$.	$9^h\ 30^m$.	$10^h\ 30^m$.
D	$+\ 957''.1$	$+\ 957''.1$	$+\ 957''.1$
d	$+\ 881''.2$	$+\ 881''.2$	$+\ 881''.2$
$-D \sin \pi \cos z$	$-6''.6$	$-4''.0$	$-1''.1$
$D + d - D \sin \pi \cos z$	$+1831''.7$	$+1834''.3$	$+1837''.2$
p	$+\ 791''.6$	$+2215''.5$	$+3639''.0$
u	$+2012''.4$	$+2294''.5$	$+2420''.2$
$p-u$	$-1220''.8$	$-79''.0$	$+1218''.8$
q	$+2836''.2$	$+2043''.9$	$+1251''.2$
v	$+2074''.3$	$+2092''.3$	$+2111''.8$
$q-v$	$+\ 761''.9$	$-48''.4$	$-860''.6$

By rough trial:

At $8^h\ 30^m$.

$$(p-u)^2 = (1221'')^2 = (20')^2 = 400$$
$$(q-v)^2 = (762'')^2 = (13')^2 = 169$$
$$\overline{569}$$

$$1832'' = 31' = n. \qquad \sqrt{569} = 24' = m < n$$

At $10^h\ 30^m$.

$$(p-u)^2 = (1219'')^2 = (20')^2 = 400$$
$$(q-v)^2 = (861'')^2 = (14')^2 = 196$$
$$\overline{596}$$

$$1837'' = 31' = n. \qquad \sqrt{596} = 24' = m < n.$$

Interpolating for spaces of 15^m before $8^h\ 30^m$, and after $10^h\ 30^m$:

	$p-u$.	$q-v$.	$(p-u)^2$.	$(q-v)^2$.	m^2.	m.	n.	$m-n$.
$8^h\ 0^m$	$-1733''.2$	$+1166''.3$	300398–.*	136026–.	436424–.	$2089''.1$	$1830''.5$	$+258''.6$
8 15	$-1481\ .9$	$+\ 964\ .2$	219603–.	92968–.	312590–.	$1768\ .0$	$1831\ .1$	$-\ 63\ .1$
8 30	$-1220\ .8$	$+\ 761\ .9$						$+321\ .7$
$10^h\ 30^m$	$+1218''.8$	$-\ 860''.6$	148546–.	74063–.	222609–.	$1492''.0$	$1837''.2$	$-345''.2$
10 45	$+1567\ .6$	$-1063\ .9$	245742–.	113188–.	358930–.	$1894\ .5$	$1838\ .0$	$+\ 56\ .5$
11 0	$+1926\ .2$	$-1267\ .4$						$-401\ .7$

Then $+321''.7 : +258''.6 :: 15^m : 12^m\ 4^s$. | $8^h\ 12^m\ 4^s$. Gr. Time of first cont. at N. H.

and $-401''.7 : -345''.2 :: 15^m : 12^m\ 53^s$. | $10^h\ 42^m\ 53^s$. Gr. Time of last cont. at N. H.

104. We see at once that this eclipse is of extraordinary

* The unit's place is omitted in these squares, because the logarithms do not furnish it readily, and it adds nothing to accuracy.

length, being no less than $2^h\ 30^m$ in duration at New Haven. The reason is, that not only is the *true* motion of the moon about at its *minimum* of velocity, but the effect of parallax is to make its *apparent* motion still slower. Instead therefore of finding the two times of contact a little more accurately by the simple repetition indicated in Art. 101, we shall, on account of the peculiar circumstances of the eclipse, employ a longer method, of needless accuracy in ordinary cases; viz:—

By proportion from the values of $(\mu-\varDelta)$ and $\varDelta$ already known, find the same for $8^h\ 12^m$, $8^h\ 13^m$, and $10^h\ 42^m$, $10^h\ 43^m$; and for these times recompute by the formulæ, u, and that part of v which depends on $\cos\ (\mu-\varDelta)$. Interpolate p, q, the first term of v, and n, for the same instants; and finally recomposing $\{(p-q)^2+(q-v)^2\}^{\frac{1}{2}}$, ascertain at what two instants it is equal to n.*

	$8^h\ 12^m$.	$8^h\ 13^m$.	$10^h\ 42^m$.	$10^h\ 43^m$.
Δ $\mu-\mathrm{A}$	$+1^\circ\ 49'\ 0''.6$ $51^\circ\ 32'\ 41''$	$+1^\circ\ 48'\ 59''.6$ $51^\circ\ 47'\ 41''$	$+1^\circ\ 46'\ 29''.1$ $89^\circ\ 3'\ 14''$	$+1^\circ\ 46'\ 28''.1$ $89^\circ\ 18'\ 14''$

Logs. of	$8^h\ 12^m$.	$8^h\ 13^m$.	$10^h\ 42^m$.	$10^h\ 43^m$.
$\sin\ \Delta$	$+8.50112$	$+8.50105$	$+8.49095$	$+8.49088$
$\sin\ (\mu-\mathrm{A})$	$+9.89381$	$+9.89531$	$+9.99994$	$+9.99997$
$\cos\ (\mu-\mathrm{A})$	$+9.79372$	$+9.79133$	$+8.21780$	$+8.08454$
π	$+3.50795$	$+3.50795$	$+3.50791$	$+3.50791$

Calling the first member of v, v_1, and the 2nd, $-v_2$,—then since $l.\ \pi+l.\ \cos\ \varphi+l.\ \sin\ (\mu-\varDelta)=l.\ u$, and $l.\ \pi+l.\ \cos\ \varphi+l.\ \sin\ \varDelta+l.\ \cos\ (\mu-\varDelta)=l.\ v_2$, we have, by taking the logarithms from the above table, and the constant logarithms of $\sin\ \varphi$ and $\cos\ \varphi$ as before, and adding them:

	$8^h\ 12^m$.	$8^h\ 13^m$.	$10^h\ 42^m$.	$10^h\ 43^m$.
$l.\ u$	$+3.27873$	$+3.28023$	$+3.38482$	$+3.38485$
$u=$	$+1899''.9$	$+1906''.5$	$+2425''.6$	$+2425''.8$
$l.\ v_2$	$+1.67976$	$+1.67730$	$+0.09363$	$+9.96030$
v_2	$+47''.8$	$+47''.6$	$+1''.2$	$+0''.9$
v_1	$+2117''.1$	$+2117''.1$	$+2117''.0$	$+2117''.0$
$v=$	$+2069''.3$	$+2069''.5$	$+2115''.8$	$+2116''.1$

* The common rule for interpolation regards only second differences; and is not accurate, when, as in the present eclipse, the second differences are large, and values are interpolated for times much beyond the three instants chosen, on either side of

	$8^h\ 12^m$.	$8^h\ 13^m$.	$10^h\ 42^m$.	$10^h\ 43^m$.
$p=$	$+\ 364''.3$	$+\ 388''.0$	$+3923''.6$	$+3947''.3$
$p-u=$	$-1535''.6$	$-1518''.5$	$+1498''.0$	$+1521''.5$
$q=$	$+3073''.8$	$+3060''.6$	$+1092''.6$	$+1079''.4$
$q-v=$	$+1004''.5$	$+\ 991''.1$	$-1023''.2$	$-1036''.7$
$n=$	$1831''.0$	$1831''.0$	$1837''.8$	$1837''.9$

Then, as before,

	$p-u$.	$q-v$.	$(p-u)^2$.	$(q-v)^2$.	m^2.	m.	n.	$m-n$.
$8^h\ 12^m$	$-1535''.6$	$+1004''.5$	235807–.	100902–.	336709–.	$1835''.0$	$1831''.0$	$+\ 4''.0$
8 13	$-\ 518\ .5$	$+\ 991\ .1$	230584–.	98229–.	328813–.	1813 .3	1831 .0	$-17\ .7$
								$+21\ .7$
$10^h\ 42^m$	$+1498''.0$	$-1023''.2$	224399–.	104694–.	329093–.	$1814''.1$	$1837''.8$	$-18''.7$
10 43	$+1521\ .5$	$-1036\ .7$	231494–.	107475–.	338969–.	1841 .1	1837 .9	$+\ 8\ .2$
								$-26\ .9$

Then $+21''.7 : +\ 4''.0 :: 1^m : 11^s.06$.

and $-26''.9 : -23''.7 :: 1^m : 52^s.96$.

$8^h\ 12^m\ 11^s.1 =$ Gr. Time of first cont. at N. H.

$10^h\ 42^m\ 52^s.9 =$ Gr. Time of last cont. at N. H.

$8^h\ 12^m\ 11^s.1 - 4^h\ 51^m\ 46^s = 3^h\ 20^m\ 25^s.1$* . . . Beginning of eclipse, N. H. mean time.

$10^h\ 42^m\ 52^s.9 - 4^h\ 51^m\ 46^s = 5^h\ 51^m\ \ 6^s.9$ . . . End of eclipse, N. H. mean time.

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them. Thus the great length of this eclipse introduces an inaccuracy in these first results, not commonly to be apprehended.

To remedy this defect, without repeating the calculation of all the formulæ for times approximate to those of the contacts, let us examine whence the unequal variation of $p-u$ and $q-v$ arises. We at once see, that p and q, from their very nature, increase with tolerable regularity, while u and v, expressing the effects of parallax, do not. The reason of this is, that u and v depend on $(\mu-A)$, which being a large arc, and of very different amount at the different times chosen, its sines and cosines do not vary at a nearly uniform rate, as the arc itself, and all the other factors do. Therefore we need only recompute by the formulæ the two short terms, which contain sin $(\mu-A)$ and cos $(\mu-A)$; and all the other terms may be interpolated from known values without appreciable error.

* This result differs from that given for the same eclipse at New Haven in the American Almanac, 1838,—for two reasons. 1st, In that calculation the semidiameter of the sun was assumed to be 5″ greater than in the Naut. Alm.;—and 2nd, The longitude of New Haven was taken at $4^h\ 51^m\ 51^s$. Further, the latitude of N. H. in this work is assumed greater than that in the Amer. Alm. by 30″, causing a very trifling portion of the difference. The student, by increasing the numbers in

105. *For the time of Greatest Obscuration.*

This usually differs only by a few minutes from the assumed middle time of eclipse, in the present instance $9^h\ 30^m$. Therefore, interpolate for an interval of 5^m or 10^m both before and after $9^h\ 30^m$, the values of $p-u$, and $q-v$, and combine as before to form m. The formulæ for obtaining a minimum value (Problem III, Art. 80,) will then afford an easy method of finding the time of "shortest distance of centres," which is the same as that of greatest obscuration.

Time.	$p-u$.	$q-v$.	m.		
$9^h\ 25^m$	$-180''.0$	$+19''.2$	$+181''.0$		
				$-88''.3$	
$9^h\ 30^m$	$-79''.0$	$-48''.4$	$+92''.7$		$+113''.9$
				$+25''.6$	
$9^h\ 35^m$	$+23''.2$	$-116''.0$	$+118''.3$		

Then, Art. 80,

$$\text{Time of gr. obsc.} = 9^h\ 25^m + t \times 5^m:$$

$$t = \frac{+113''.9 + 176''.6}{227''.8} = 1.275$$

Time of gr. obsc. $= 9^h\ 31^m\ 22^s$; and by interpolation from the above three values for this time, $m = +88''.4$, or shortest distance.

This process is sufficiently accurate for ordinary purposes, and even in this case, if the number of digits is all that is required. But on account of the rapid change of second differences, let us interpolate again for $9^h\ 30^m$, $9^h\ 31^m$, and $9^h\ 32^m$, and our second result will be

Time of greatest obsc. at N. H. $= 9^h\ 31^m\ 36^s$, Gr. m. time.

$9^h\ 31^m\ 36^s - 4^h\ 51^m\ 46^s = 4^h\ 39^m\ 50^s$ . . . N. H. mean time of greatest obscuration.

$84''.0$ . . . Nearest approach of centres. This value is not in error by $0''.1$.

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column n by $5''$, and repeating the proportion, may obtain results which differ from those of the Amer. Alm. only by an amount due to the difference of 5^s in the assumed longitudes.

The longitude of New Haven by the eclipse of 1838, by subsequent occultations, and by late chronometric comparison with New York and Greenwich, is between $4^h\ 51^m\ 46^s$ and $4^h\ 51^m\ 47^s$.

106. *For the times of Formation and Rupture of the ring in an annular eclipse; and of Beginning and End of total darkness in a total eclipse.*

The equation for internal contact of the sun and moon differs from that expressing their external contact only in the sign which connects their semidiameters. $D\,(1 - \sin \pi \cos z)$ and d being, as before explained, the semidiameters of the sun and moon respectively, as proportioned to the other quantities of the formula, the equation of internal contact becomes

$$\{(p-u)^2 + (q-v)^2\}^{\frac{1}{2}} = D\,(1-\sin \pi \cos z) \backsim d.$$

When $d > D\,(1 - \sin \pi \cos z)$, the eclipse is *total;* when $d < D\,(1 - \sin \pi \cos z)$, an *annular* eclipse takes place.

It is farther evident, that when $\{(p-u)^2 + (q-v)^2\}^{\frac{1}{2}} (= m)$, at its minimum value, is greater than $D\,(1 - \sin \pi \cos z) \backsim d$, neither annular nor total eclipse can take place; when less, either one or the other must take place, according as $D\,(1 - \sin \pi \cos z)$ is greater or less than d.

In the present example, we have $(D - D \sin \pi \cos z) \backsim d$ as follows:

$8^h\ 30^m$	$69''.3$
$9^h\ 30^m$	$71''.9$
$10^h\ 30^m$	$74''.8$

$\therefore$ whence it equals $72''.0$ at $9^h\ 31^m\ 36^s$; but m is then at its minimum, and equal to $84''.0$. The eclipse is therefore neither annular nor total at New Haven; it is, however, very nearly annular, the moon, when nearest, overlying the sun's edge by only $12''.0$. Had the eclipse been annular, as the duration of the ring is always very short, the two times at which $m = (D - D \sin \pi \cos z) \backsim d$, could have been easily obtained from the values of these two quantities on this and the preceding page.

107. *For the points of first and last contact.*

Since the moon is altogether invisible before the first contact, it is desirable to know at what angle, reckoned from the vertex, or from the north point of the sun's limb, to the right hand around his circumference, the first indentation will be made upon his yet unbroken limb. For it is absolutely necessary to a good observation that the observer should know exactly at what point to look for the occurrence of an expected contact; and he may

three circles representing the moon, the three phases will be duly represented.*

For the further illustration of our method, suppose we have obtained the values of p, q, u, and v, at the beginning, middle, and end of an eclipse. Set off the three values of p, namely, OP, OP′, OP″; and also those of q, namely, Pm, P′m', P″m''. Then m, m', m'', will be the points in which the moon would be seen in her true relative orbit at these three moments respectively. Now from P, P′, P″, set off the values of $-u$, namely, PQ, P′Q′, P″Q″; and let M, M′, M″ differ from m, m', m'', as referred to the line AB, by the several values of $-v$, and join mM, m'M′, m''M″.

Now at the beginning, middle, and end of the eclipse, the moon as seen from the centre of the earth is in its true orbit at m, m', and m''. A vertical circle is at its [illegible] in the lower circle AB, at those three moments in the directions mM, m'M′, and m''M″, and the moon is apparently depressed by parallax in these vertical circles by the spaces mM, m'M′, m''M″ respectively. It will be seen then, that the general principle of our method consists simply in referring the moon's place, her parallax, &c., to the coordinate lines AB and CD passing through the centre of the sun.

Practical Use of Calculating Occultations and Eclipses.

The practical use of determining the times and phases of an apparent eclipse or occultation, is for the convenience and guidance of the observer. For this purpose it is obvious that a

	$8^h\ 12^m\ 10^s.8.$	$10^h\ 42^m\ 51^s.3.$
$l.\ (p-u)$	$-\ 3.18540$	$+\ 3.18130$
$l.\ (q-v)$	$+\ 3.00091$	$-\ 3.01481$
$l.\ \left(-\frac{p-u}{q-v}\right)$	$+10.18449$	$+10.16649$
$N = \psi =$	$56°\ 49'.2$	$235°\ 43'.4$
$V =$	$99°\ 23'.7$	$284°\ 37'.4$

108. *To project an occultation or solar eclipse.*

These results may be exhibited to the eye by a projection, analogous to that employed in the case of a lunar eclipse. In fig. 26, let AOB and COD, be the circle of declination passing through the sun, and a perpendicular to it; and let p and $p-u$ be reckoned on CD, to the left hand when negative, to the right when positive;* also let q and $q-v$ be reckoned on AB, upwards or downwards, as the quantities are positive or negative. With the radius D $(1 - \sin \varpi \cos z)$ from the centre O, describe the dotted circle NCC′Z to represent the sun; and with the radius d+D $(1-\sin \varpi \cos z)$, describe ACBD. The moon will of course just touch the sun, when her apparent place is any where on the circle ABCD.

Set off OU, OU′, OU″, equal to the three values of $p-u$ respectively, and perpendicular to these, UV, U′V′, U″V″, equal to the three values of $q-v$. Then V, V′, V″, will represent the apparent place of the moon with regard to the sun at the three times chosen. Through V, V′, V″, which are not in the same straight line, draw the curve MVV′V″M″ for the moon's apparent path, either by the rule for describing an arc of a circle through three points, or graphically with a rapid, but steady hand. The first contact will take place at the moment the moon arrives at M, and the last, when it reaches M″. Draw the perpendicular OM′ upon the moon's apparent path; the moon will then be at M′ at the moment of greatest obscuration, and the times of first contact, greatest obscuration, and last contact, may be approximately obtained, as in the case of the moon, by comparing the spaces MV, V′M′, V″M″, with the horary spaces VV′, V′V″,—noticing, however, that these latter are not quite equal to each other. And by describing with radius d, and centres M, M′, M″,

* The motion of the moon will then be from right hand to left on the paper, or as we see it in the heavens, when moving from west to east.

find for this occultation.... $13^h\ 19^m$ = Gr. m. t. of true conj.; $21^h\ 38^m\ 12^s = A$; and $-16°\ 51' = \varDelta$. From the Gr. m. t. of true conj., $13^h\ 19^m$, we find by eq. Y, (using logarithms to two places only,) $T = 13^h\ 35^m$. As a third approximation would of course tend to increase this value a little, make $T = 13^h\ 40^m$; then $T - 30^m = 13^h\ 10^m$, and $T + 30^m = 14^h\ 10^m$.

For the sake of brevity, our process in Art. 104 may be abbreviated in the following ways. Perform the computations for the two times $13^h\ 10^m$ and $14^h\ 10^m$ only, and employ but three places of logarithms; take out $\varDelta$, δ, ϖ, d, to the nearest tenth of a minute of space,—α, A, to the nearest second of time, and $\mu - A$ to the nearest minute of a degree; and in doing so, employ simple proportion only, interpolation being unnecessary. The computation of the term $\frac{1}{2r} \cos \delta \sin \varDelta\, (\alpha - A)^2$ may be dispensed with, and that of the term $\varpi \sin \varphi \cos \delta$ need be performed but once, being the same for both times. With these simplifications, we readily obtain

	p.	u.	$p-u$.	q.	v.	$q-v$.	d.
Oct. 17^d $13^h\ 10^m$	$-\ 4'.6$	$+\ 4'.4$	$-\ 9'.0$	$+28'.2$	$+49'.0$	$-20'.8$	$15'.8$
$14^h\ 10^m$	$+26'.4$	$+15'.5$	$+10'.9$	$+41'.8$	$+48'.2$	$-\ 6'.4$	"

An easy projection will now afford us the approximations of which we are in search. With the centre O, (fig. 27,) and radius 15.8, describe the circle ACBD, and draw the two co-ordinate axes AB, CD. Let the + and − values of $p-u$, $q-v$, &c., be set off in the same directions as in fig. 26.* Set off OU, OU′, equal to −9.0, +10.9; and perpendicular to these US, U′S′ equal to −20.8, −6.4, respectively. Through S and S′ draw SE for the *apparent* path of the star; of course, I will be the point of immersion, and E that of emersion, on the moon's limb.

* In this projection, for the sake of convenience, the moon is supposed to be stationary, and the star to move behind it. To correspond with this supposition, the directions in which + and − values are laid off should be in every way the reverse of those in fig. 26. But since the projection is adapted to the situation of the bodies as seen in an inverting telescope, the directions of the + and − values, remain the same as in fig. 26.

If the above projection be set off on a scale of 10′ to the inch, and still more, if of 5′ to the inch, the results may be obtained with little care, and with sufficient accuracy.

Then $13^h\ 10^m + \frac{SI}{SS'} = 13^h\ 10^m + 1^h \times \frac{8.8}{24.6} = 13^h\ 31^m.5 =$ Gr. mean time of immersion; and $14^h\ 10^m + \frac{S'E}{SS'} = 14^h\ 10^m + 1^h \times \frac{5.7}{24.6} = 14^h\ 23^m.9 =$ Gr. mean time of emersion.

Again, find u and v for the times of immersion and emersion, as follows:

	u.	v.
Oct. $17^d\ 13^h\ 31^m.5$	$+\ 8'.4$	$+48'.7$
$14^h\ 23^m.9$	$+18'.0$	$+48'.0$

Set off OP, OP′ equal to +8.4, +18.0; and PZ, P′Z′ equal to +48.7, +48.0 respectively. Then OZ, OZ′, will represent the directions of a vertical circle, at the times of immersion and emersion; and the vertex of the moon at those instants will be the points V and V′ respectively.* With a protractor measure from V and V′ around the moon's limb to the right hand the arcs VCI, and V′CE. These will be found equal to 183° and 279°, and are the "angles from moon's vertex" of the points of disappearance and reappearance. Collecting these results, we have

$13^h\ 31^m.5 - 4^h\ 51^m.8 = 8^h\ 39^m.7$ . . . N. H. m. time of Immersion.
$14^h\ 23^m.9 - 4^h\ 51^m.8 = 9^h\ 32^m.1$ . . . N. H. " " Emersion.

183° = Angle of point of disappearance from vertex.
279° = " " reappearance " "

Example 3. *Required the approximate New Haven times of immersion and emersion, and the corresponding angles from the moon's vertex, in the occultation of α Scorpii,* (1 *mag.*) *June* 20, 1842.

		Ang. from vertex.
Time of Immersion	$7^h\ 33^m.1$	355°
Time of Emersion	$8^h\ 23^m.4$	280°

* By holding the diagram so that OV and OV′ shall be successively perpendicular to the horizon, the position of the star as regards the moon at the time of immersion and emersion is represented. The observer at the telescope therefore, without having measured the angles instrumentally, may from an inspection of the diagram estimate the places where the star will immerge and emerge.

CHAPTER VIII.

ON THE METHODS OF DETERMINING THE LATITUDE AND LONGITUDE OF PLACES.

112. The great services which Astronomy has rendered to navigation, consist entirely in enabling the mariner to find any where on the ocean his latitude and longitude from celestial observations. And no less on the main land, the settlement of latitudes and longitudes by astronomical means, in sufficient numbers to determine the windings and conformations of coasts, the situation of important places, and the boundary lines of different countries and states, constitute the only *immediate* application of celestial science to the common wants and demands of civilized life. So one of the first objects of the private observer, the problem to which he naturally turns his earliest attention, as the basis of subsequent researches, is the determination of his own latitude and longitude.

Of the two, the latitude is much the most easily determined, since it is reckoned from a fixed circle, the equator,—or in effect, from the pole of the heavens, a point fixed in the sphere, and whose position relative to the observer's zenith is readily determined by simple instruments. The longitude, on the other hand, since no one terrestrial meridian is in any way distinguished from all others, must be reckoned from one arbitrarily chosen, whose position relative to the observer's meridian is indeterminable by any direct method.

113. *Methods of finding the Latitude.*

The latitude of a place is equal to the altitude of the pole, or to the complement of the altitude of the equator at that place, and may be found in many different ways. We shall endeavor to put the student or observer in possession of the principal and most useful of these methods, and of the means of reducing the observations. It is to be presumed that at this stage of the work, he is well versed in the practice of working out results from algebraic formulæ; and we shall therefore frequently omit ex-

amples, and thus gain room for a more complete exhibition of the subject.

First method. *By altitudes of the pole star at any hour of the night.*—With a sextant and artificial horizon, or other suitable instrument, take double altitudes of the pole star, and let them be included within a space of 10^m or 20^m in duration, or be divided into sets occupying as short intervals of time. Take the mean of these double altitudes, and dividing by 2, correct for refraction by the usual tables; also let a mean be taken of the times corresponding to the several altitudes. The Naut. Alm. for each year contains near the end "Tables for determining the Latitude by Observations of the Pole Star out of the Meridian," and since in the "Explanation of Articles" an example is worked out in full, it will be unnecessary to introduce one here. (See pp. 563–5, and pp. 602–3, Naut. Alm. for 1839.)

With an instrument competent to give a distinctly bright image of the pole star, this method is probably the best of those in common use, since it is practicable at any clear hour of the night, and to any degree of repetition of measures; it is also one of the easiest of reduction.

Second method. *By meridian altitudes of the pole star.*—The true time of the pole star's upper or lower culmination, as indicated by a good sidereal clock or chronometer, being known, an altitude taken at that time will give the latitude at once, after the usual corrections, by adding or subtracting the polar distance of the star, according as it is at its lower or upper culmination. A single altitude, however, is seldom satisfactory; and the pole star changes its altitude so slowly, that the mean of observations through a space of 15^m on either side of the meridian will not give a latitude in error as much as $5''$ from this cause,—a degree of accuracy usually sufficient for the sextant. If the time of observation be confined to 5^m on each side of the meridian, the latitude will not be affected to the amount of $1''$.

114. Third method. *By meridian altitudes of the sun or other heavenly body.*—The sun ascends and descends so rapidly, even near the meridian, that the mean of two or three altitudes would be materially too small. A single altitude may be taken, even without a knowledge of the time, if the observer, as noon approaches,

keeps, for instance, the lower limbs of the two images in exact contact as they gradually tend to recede from each other, until they recede no longer. The half of the reading, corrected for semidiameter, refraction, and parallax, is the true meridian altitude, whence the latitude is readily found.

We have, however, a ready means of reduction for altitudes observed through a considerable interval before and after noon. To the local mean times of observation, reduced from sid. times, or taken with a chronometer, apply the "equation of time," (N. A., p. II of the month,) corresponding to the Gr. m. times of observation, and the results will be the local *app.* sol. times, or hour angles of the sun from the meridian, at which the altitudes were taken. Now the versed sine of the sun's arc or hour angle, for a short period, varies very nearly as the number of seconds of his descent. Take the mean of the nat. versines* of the several hour angles from noon; this mean will bear the same ratio to the mean of the corrections of the sun's altitude, as any one of the versines to the corresponding correction. This ratio is expressed by the factor of reduction

$$\cos\delta \cos\varphi \operatorname{cosec} z \operatorname{cosec} 1'',\dagger$$

by which the mean of the versines being multiplied, we obtain the mean or final correction of the altitudes. The symbols δ, φ, and z, signify as usual, the decl., latitude, and zen. dist. of the body.

Ex. 1. *It is required to deduce the latitude of New Haven from the following altitudes of the sun, and corresponding mean*

necessary; if, however, it should be required, it may be found in the N. A., p. II of the month.

Double Altitudes of the sun.	Chron. Times.	N. H. Mean Times.	Equation of Time.	N. H. Apparent Times.	Sun's hour angle in arc.	Versines of arcs.
143° 37′ 13″	4h 47m 50s	− 3m 1s	−3m 19s	−6m 20s	−1° 35′ 0″	382
" 42 15	" 51 42	+ 0 51	" "	2 28	0 37 0	58
" 43 21	" 53 35	2 44	" "	−0 35	−0 8 45	3
" 42 47	" 56 6	5 15	" "	+1 56	+0 29 0	36
" 40 34	" 58 19	7 28	" "	4 9	1 2 15	164
" 32 16	5 2 52	+12 1	−" "	+8 42	+2 10 30	720
6) 238′ 26″						6) 1363
2) 143° 39′ 44″						227

71° 49′ 52″ ... App. altitude.*
−20″ ... Corr. for refr.
+ 3″ ... Corr. for par.
+1′ 44″ ... Corr. for mean of alts. by the above process.

71° 51′ 19″ ... True altitude.

$\delta = +28°\ 10'$	... cos.	9.96348
$\phi = 41°\ 18'$	... cos.	9.87579
$z = 18°\ 8'$	... cosec.†	10.50692
	cosec 1″	5.31443
		5.66062
227 (+4 to Index)‡		6.35603
104″ = corr. for mean of meridian altitudes.		2.01665

18° 8′ 41″ ... True zen. dist.
23° 9′ 53″ ... Sun's decl.

41° 18′ 34″ ... Latitude of New Haven.

Fourth method. *By double altitudes of the sun or of two stars.*—By taking two altitudes of the sun at times differing from each other an hour or more, both the latitude and the time may be determined, without previous knowledge of either. The space of time between noon and the observation nearest noon, should generally be less than that between the observations.

Let P (fig. 28,) be the pole, Z the zenith, S the place of the sun at the first observation, and S′ at the second. Find the app. sol. times of the observations as in the last example, and subtract the first from the last; then since the motion of the sun in R.A. is the measure of solar time, the difference of these two times

* These altitudes are too great for the sextant, but may be taken with a reflecting circle, used as a sextant.

† If the cosec. is not in the tables, take the arith. comp. of the sine.

‡ The numbers in the column of versines, taken from the tables to 6 places of decimals, are of course millionths of radius. Since radius is supposed to be divided into 10,000,000,000 parts, if the columnar numbers are considered as integers, 4 must be added to the index of the logarithm.

equals the hour angle SPS′. The sun's Decls. being taken out for each of the times, PS and PS′, their complements, are also known. ZS and ZS′ likewise, are the complements of the corrected altitudes. Therefore, in the triangle SPS′, having ∠SPS′ and the adjacent sides given, we may find first ∠PS′S, and then the side SS′. Again, in the triangle SZS′, with the three sides, we obtain ∠ZS′S, and subtracting from this ∠PS′S, we have ∠PS′Z. Lastly, in the triangle PZS′, with the ∠PS′Z, and the adjacent sides, we may find PZ, which is the complement of the latitude.

Let δ and δ' be the Decls. of the sun at S and S′, z and z' the zen. distances ZS and ZS′; also let the hour angle SPS′ $= h$, $\angle PS'S = \alpha$, $\angle ZS'S = \beta$, $SS' = \sigma$, and the latitude $= \varphi$. Then the formulæ necessary for the work will be as follows:

1. $\tan x^{*} = \cot\delta \cos h.$†
2. $\tan \alpha = \operatorname{cosec} \{90° \sim (x+\delta')\} \tan h \sin x.$
3. $\sin \sigma = \operatorname{cosec} \alpha \sin h \cos \delta.$
4. $y = \frac{1}{2}(z+z'+\sigma).$
5. $\cos \frac{1}{2}\beta = \sqrt{\sin y \sin (y \sim z) \operatorname{cosec} z' \operatorname{cosec} \sigma}.$
6. $\tan v = \cot \delta' \cos (\beta \sim \alpha).$
7. $\sin \varphi$‡ $= \sec v \cos (z' \sim v) \sin \delta'.$

Single altitudes of two bodies are equivalent to double altitudes of a single body, separated by a considerable interval of time; the sun and moon may be employed in the day time, and any two bright stars near the equator, and differing 1^h or more in

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* x is like or unlike $90°-\delta$, as h is $>$ or $<$ than $90°$; and α is like or unlike h, as x is $>$ or $<$ than $90°-\delta$. Again, v is like or unlike $90°-\delta$, and ϕ is like or unlike $z \sim v$, according as $\beta \sim \alpha$ is $>$ or $<$ than $90°$.

† *Remarks on the Formulæ.*—These are all from Hutton's Tables, London edition, 1834. For Nos. 1 and 2, see page xlii of Introduction, case 3, first formulæ; No. 3, page xliii, case 6, first formula; No. 5, page xlii, case 1, last formula; Nos. 6 and 7, page xlii, case 3, nearly the last formulæ. The only changes in our formulæ are easily recognized; for instance, in the 7th, (making $\Delta' =$ pol. dist. PS, or complement of δ',)—the proportion of Hutton, page xlii, case 3, ... $\cos v : \cos (z' \sim v) :: \cos \Delta' : \cos (90° - \phi)$ (or PZ) gives by multiplication, $\cos (90° - \phi) = \frac{\cos (z' \sim v) \cos \Delta'}{\cos v}$. But $\cos (90° - \phi)$ and $\cos \Delta'$ are equal to $\sin \phi$ and $\sin \delta'$, and $\cos v$ may be struck from the denominator, by inserting its reciprocal, $\sec v$, in the numerator. It thus becomes identical with our formula 7th.

‡ For a shorter process of finding ϕ,—substitute the rule I, p. lx, Hutton, for formulæ 6th and 7th.

R.A., are suitable objects for night observation. The sid. time elapsed between the two observations must be added to or subtracted from the diff. of the R.A.'s of the two stars, according as the star of greatest or least R.A. was the first in order of observation. The sum or difference will be the hour angle h, and the process of reduction will be the same as before by the above formulæ.

As we have in this chapter considered altitudes as taken by unfixed instruments, such as the sextant, and its different modifications of reflecting and repeating circles, we shall depart so far from the subject of the chapter, as to give the observer a formula, by which he may determine his time from a single altitude, the latitude being known. In the triangle PZS, (fig. 28,) the three sides are known, and to find the hour angle ZPS $= h$, we have the formulæ,

$$\sin \tfrac{1}{2} h = \sqrt{\sin \tfrac{1}{2} (z+(\varphi-\delta) \sin \tfrac{1}{2} (z-(\varphi-\delta) \sec \varphi \sec \delta.}\text{*}$$

R.A. of sun or star $+ h$† $=$ sid. time of observation.

115. *Methods of finding the Longitude.*

The principles of several ways of determining the longitude are illustrated in Olmsted's Astronomy, Arts. 272–278, so as to need little general explanation here. We will classify the numerous methods as follows:

1st. Such as show the difference of local times between any two places, by an event occurring at the same instant of absolute time to both.

Of this class are *lunar eclipses*, which, however, are uncertain to many seconds, by reason of great indefiniteness in the earth's shadow. Also the *eclipses of Jupiter's satellites*; but these also give a longitude uncertain to many seconds, because the satellites are bodies of considerable size, and disappear and reappear *gradually*. Yet from the frequency of these eclipses, observations may be multiplied to such an extent, as to give a tolerably accurate result. The Naut. Almanac, at p. XX of each

* Baily's Ast. Tables and Formulæ, Form. XV, p. 89. Hutton's Tables, p. xlii, case I, first formula.

† h is always reckoned by hours, in the direction of the apparent revolution of the heavens, and ranges from 0^h to XII^h on the west side of the meridian, and from XII^h to $XXIV^h$ or 0^h on the east.

month, furnishes a list of Greenwich immersions and emersions, to which ready reference may be made by the observer.

Belonging to this class are coincident observations, at different places, on *meteors*, *the explosion of rockets*, *the alternate appearance and disappearance of strong intermitted lights*, &c. These are almost instantaneous occurrences, and hence longitudes may be obtained from them to a higher degree of accuracy than by any other method. The application of these contrivances is, however, restricted, of course, to *small* differences of place.

The method by *chronometers* indirectly belongs to this class, since the arrival of the pointers of a Greenwich chronometer to $0^h\ 0^m\ 0^s$ at New Haven marks the occurrence of Greenwich mean noon at that instant, and the local time of New Haven can be noted at the same instant of absolute time. This method is susceptible of much greater accuracy than those of eclipses of the moon and of Jupiter's satellites, especially for short distances of transportation.* (See further, Olmsted's Astronomy, Art. 174.)

116. 2nd. Such as show the difference of local times between the places, by corresponding observations on a phenomenon not happening at exactly the same instant of absolute time to both, but at instants differing by an amount calculable from known data.

Occultations of stars and planets, and *eclipses of the sun*, are of this description. The immersion or emersion of a star may occur at instants of absolute time differing by two or more hours at two places, because the parallax of the moon displaces her differently for different observers; but, by the exact calculation and application of those displacements, an observation at any one place may be reduced to such as it would be at the centre of the earth, and it may then be compared with an observation at any other place, which has been reduced in a similar manner.

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* The recent introduction of Atlantic Steam-ships, and their rapid passages from land to land, has given rise to several novel experiments on determining differences of longitude between stations on the two continents, by the transportation of chronometers. The first three or four attempts have been very successful, and the longitude of New York, and incidentally that of New Haven, from Greenwich, has been well settled.

The problem of finding the longitude from an observed stellar or solar occultation, is nearly the reverse of that for finding the times of an occultation, at a place of known longitude. Suppose, for example, we take the observed time of immersion of δ Capricorni, as reduced to true sidereal time on page 69, Art. 76. The corresponding instant of mean time, when reduced to Greenwich mean time, by applying the assumed difference of longitude of New Haven, ($4^h\ 51^m\ 46^s$), in case the assumed longitude is *incorrect*, will *not* be the true Greenwich mean time of immersion, but will be in error to the same amount as the assumed longitude. Now if we calculate equation (A), Art. 103, for this erroneous Gr. time, a time not exactly agreeing with that of the observed immersion, it is evident from the conditions of the equation that the two members cannot be quite equal to one another; and from the calculated difference between them, we intend to deduce the unknown error of the assumed longitude. Let us examine what parts of (A.) will be affected by the error. In the first place, the true local sid. time of immersion (μ) is furnished at once by observation, and requires scarcely an approximate knowledge of the longitude. Now the terms u and v, as will be seen by their expanded values, depend on the quantity μ, and on others either unchanging, or changing imperceptibly in a very short interval of time; and therefore a small error in the assumed longitude does not affect them. But α and δ are of rapid change, and, if taken out for a slightly erroneous Gr. time, render the differences $\alpha-A$, $\delta-\Delta$, and consequently the terms p and q incorrect. If, therefore, we find for the assumed Gr. m. time of immersion, . . . $(p-u)^2+(q-v)^2-d^2=2$ E, the above remark renders it evident that the terms u and v are the correct values belonging to the instant of immersion, while p and q are values corresponding to a time as many seconds before or after the instant of immersion, as the assumed longitude is in error.

Let t be the true mean time of the observation at New Haven, l the assumed longitude, and $l+x$ the true unknown longitude; x to be expressed in parts of an hour. Also let p' and q' be the hourly variations of p and q. After calculating 2 E from the equation in the last paragraph, we may find x by the formula,

$$x=-\frac{\mathrm{E}}{(p-u)\,p'+(q-v)\,q'}.\text{*}$$

This equation *would* show the exact correction of the assumed longitude, if we could rely on the perfect accuracy of the Naut. Almanac. But as we have already remarked in Art. 109, the calculated place of the moon is liable to differ a little from the observed place. The errors of α and δ, affect those of p and q, and thus vitiate the result. If e is the minute fraction of an hour, by which the immersion, for instance, is delayed beyond the calculated time, on account of the errors of the tabular values of α and δ, then

$$x+e=-\frac{\mathrm{E}}{(p-u)\,p'+(q-v)\,q'}.$$

Now, if the occultation is observed any where else, a similar equation may be formed for the second station, and since e is constant at least for a few hours, we can eliminate it by subtraction, and obtain the true difference of longitude between the two places.

117. *Calculation of the longitude from an observed occultation.*

Ex. 2. *Assuming the longitude of Philadelphia to be* $5^{h}\ 0^{m}\ 42^{s}$, *and that of New Haven* $4^{h}\ 51^{m}\ 46^{s}$, *it is required to deduce the difference of longitude between these places from the following*

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* For $t+l+x$ is the true moment of immersion; and while u and v are values corresponding to this time, p and q are values corresponding to the time $t+l$. Let $p_{,}$ and $q_{,}$ be the unknown values answering to the time $t+l+x$; then $p_{,}-p$ and $q_{,}-q$ are the variations of p and q for the fraction of an hour x; and since these variations are proportioned to the time of change,

$$(1.)\qquad \frac{p_{,}-p}{p'}=\frac{q_{,}-q}{q'}=\frac{x}{1^{h}}=x.$$

Again, since $p_{,}$, $q_{,}$, u, v, are values belonging to $t+l+x$, the true moment of immersion,

$$(p_{,}-u)^2+(q_{,}-v)^2-d^2=0.$$
$$(p-u)^2+(q-v)^2-d^2=2\,\mathrm{E}.$$

Subtracting the lower from the upper,

$$(p_{,}-p)\,(p_{,}+p-2u)+(q_{,}-q)\,(q_{,}+q-2v)=-2\,\mathrm{E}.$$

Or since in this differential equation, $p_{,}+p$, when x is small, is very nearly equal to $2p$, and $q_{,}+q$ to $2q$,

$$(p-u)\,(p_{,}-p)+(q-v)\,(q_{,}-q)=-\mathrm{E}.$$

Dividing by eq. (1.),

$$(p-u)\,p'+(q-v)\,q'=-\frac{\mathrm{E}}{x}.$$

and $$(2.)\qquad x=-\frac{\mathrm{E}}{(p-u)\,p'+(q-v)\,q'}.$$

corresponding observations on the immersion of δ Capricorni, Oct. 17, 1839.

Imm. at Phil. .... $22^h\ 11^m\ 37^s.68$ ... *Phil. sid. time.*

" *N. H.* .. (*See Art.* 76, *pp.* 65–9,) .. 22 23 37 .21 .. *N. H. sid. time.*

It is evident at once that $\mu = 22^h\ 11^m\ 37^s.68$ for Philadelphia, and $22^h\ 23^m\ 37^s.21$ for New Haven. Adding to each of these the respective longitude of the two places, as assumed above, and converting from Gr. sid. to Gr. m. time, we have

Assumed Gr. m. time of immersion, $\begin{cases} 13^h\ 28^m\ 53^s.50 \ldots \text{for Phil.} \\ 13\ \ 31\ \ 56\ .53 \ldots \text{for N.H.} \end{cases}$

Calculating for these times equations (C.), (D.), (E.) and (F.) in Art. 103, we have,

	Gr. m. time of calc.	*p.*	*u.*	*p—u.*	*q.*	*v.*	*q—v.*
For Phil.	$13^h\ 28^m\ 53^s.50$	$+311''.1$	$+387''.3$	$-\ 76''.2$	$+1945''.2$	$+2887''.4$	$-942''.2$
" N.H.	13 31 56 .53	$+405\ .6$	$+514\ .2$	$-108\ .6$	$+1986\ .6$	$+2924\ .8$	$-938\ .2$

Here in the progress of $3^m\ 3^s.03$, p undergoes a variation of $+94''.5$, and q of $41''.4$; therefore, their hourly variations (which, for the determination of a few seconds of change, may be safely deduced in this rude way,) are as follows: $p' = +\,1858''.7$; $q' = +\,814''.3$. Then,

For Phil. $(p-u)^2+(q-v)^2-d^2 = 2E = -1748''$ $\therefore$ $E = -\ 874''$.

For N. H. " $= 2E = -3250''$ $\therefore$ $E = -1625''$.

Again we have

For Phil. $\quad -\dfrac{E}{(p-u)\,p'+(q-v)\,q'} = x+e = -3^s.46.$

For N. H. $\quad$ " $\quad = x+e = -6^s.06.$

Giving the N. H. quantities an accent, to distinguish them from those obtained for Philadelphia,

$$l+x+e = 5^h\ 0^m\ 42^s - 3^s.46 = 5^h\ 0^m\ 38^s.54.$$
$$l'+x'+e = 4\ \ 51\ \ 46\ -6\ .06 = 4\ \ 51\ \ 39\ .94.$$

Since e is the same in both, we obtain by subtraction,

$(l'+x')-(l+x) = -8^m\ 58^s.60$ . . . True diff. of longitude of N. H. from Phil.

If the longitude of Philadelphia is well determined in comparison with that of New Haven, and may be considered for our

purpose as needing no correction, then, making $x=0$, we have $e=-3.46$, and by substitution

$l'+x'=4^h\ 51^m\ 43^s.40$ . . . True longitude of New Haven.

118. 3rd. We bring under a third class such methods of finding the difference of longitude, as depend on the measurement of the moon's angular motion in the interval between two instants of time; the two observed instants being in the local times of the two places respectively.

The solution of these methods depends on this simple principle,—that as we can calculate the hourly motions of the moon for any given time in given directions, we can readily deduce the *absolute* time of her describing any known and measured arc. But the absolute time elapsing between any two instants, expressed in the local times of the two places, is all that is necessary to determine their difference of longitude.

The method of *lunar distances*, and that of moon-culminating stars belong to this class. The mode of taking the distance of the moon from a star or the sun by the sextant, has been explained in the chapter on that instrument, Art. 43. Such an observation gives the distance from the apparent edge of the moon to the app. place of the star,—whereas the distance between their true places is wanted. Preliminary to the formulæ for obtaining the true distance, the app. alts. or zen. distances of the bodies must be reduced from the true ones, and the observed angle must be corrected for the moon's augmented semidiameter. For the true altitude—of the moon, for instance, take out for that body α and δ from the Naut. Alm.; then μ (the sid. time of the observation) $-\alpha=h$, the hour angle of the moon from the meridian. In the triangle PZS, (fig. 28,) we have $ZPS=h$, and the adjacent sides are the complement of φ and δ; the formulæ, of course, are like Nos. 6 and 7, in the "Fourth method of finding the latitude."

$$\tan x=\cot\text{an}\ \varphi \cos h.$$

$$\cos z=\sec x \cos\{90^\circ \sim (x+\delta)\} \sin \varphi.$$

The true is now to be converted into the app. zen. dist. by applying the parallax and refraction. Call the moon's par. in alt. Π, and finding the reduced horizontal parallax ϖ, as in the chapter "on Occultations," &c., make

1st approx. of parallax $= \varpi \sin z$.

Then $\Pi = \varpi \sin (z + \text{1st approx. of parallax})$.

And so for the sun or a star, except that only one approximation of parallax is needed for the former, and none for the latter. The next step is to apply the moon's augmented semidiameter to the observed angle. Calling the true semid. d, the augmented semid. $= \dfrac{d \sin (z+\Pi)}{\sin z} = d \sin (z+\Pi) \operatorname{cosec} z$. If the sun is one of the bodies observed, the semidiameter of that body also is to be added to the observed angle to obtain the app. dist. of centres. Call the app. and true altitudes of the moon A and A′; the same quantities for the sun or star a and a'; and the app. and true distances between the bodies Δ and Δ'. We are now prepared to find Δ' by the following formulæ:

$$\left.\begin{aligned} x &= \tfrac{1}{2}\,(\Delta + A + a). \\ \sin y &= \frac{(\cos x \cos (x-\Delta) \cos A' \cos a' \sec A \sec a)^{\frac{1}{2}}}{\cos \frac{1}{2}\,(A'+a')}. \\ \sin \tfrac{1}{2}\,\Delta' &= \cos \tfrac{1}{2}\,(A'+a') \cos y. \end{aligned}\right\}*$$

We have now the angular distance as it would appear at the centre of the earth at the local time of observation. The Naut. Alm. gives this distance for intervals of three hours of Gr. time, and we may find at what moment of Gr. time the true central distance was the same as the calculated, by interpolating between these values. And having both the Gr. and local times at which the moon was at a certain true central distance from a star, the longitude is at once determinable.

The method of *moon-culminating stars* consists in measuring by the transit instrument the arc of R.A. described by the moon in passing from the Greenwich to another meridian. Let l, as before, be the assumed longitude of New Haven, or any other place, and $l+x$ the true longitude to be determined. Let α and α' be the true R.A.'s of the moon at the moments of its passing the Gr. and N. H. meridians respectively. These will be the sidereal times of her transit at those places, because on the meridian the depression of parallax does not affect her right ascen-

* The demonstration of these formulæ being omitted for want of space, the reader may be referred for authority to "Baily's Tables and Formulæ," Form. XLVIII. p. 121.

sion. In the Naut. Alm., under the head of "Moon-Culminating Stars," are given for every day in the year, the moon's R.A., and "var. for 1 hour of longitude," both at her upper or visible, and lower or invisible passage, across the meridian of Greenwich. These quantities being thus calculated for every 12 hours of *longitude*, we can interpolate their values for any intermediate longitude, in the same way as we interpolate between equal intervals of *time*. Find therefore by interpolation the moon's R.A., and var. of R.A. in 1 hour of long., for the assumed longitude l, and call them α'' and α'''. Now α', the sid. time of the moon's transit at N. H., is of course her R.A. for the true longitude $l+x$; and consequently $\alpha'-\alpha''$ is the *increase* of the moon's R.A. for the small space of longitude x, immediately after passing the meridian l. Then α''' (the increase for 1 hour of longitude at the meridian l) : $\alpha'-\alpha''$ :: 1^h of longitude : x; and we have x in parts of an hour of longitude by the following equation:

$$x=\frac{\alpha'-\alpha''}{\alpha'''}.$$

118. *Calculation of the longitude from an observed moon-culmination.*

Ex. 3. *The assumed longitude of New Haven being* $4^h\ 51^m\ 46^s$, *it is required to deduce the true longitude from the observed meridian passage of the moon at New Haven on the 17th of Oct. 1839, as given in the transit observations of that evening*, (Art. 76.)

We find among the equations for stars in the column eqs. I,

$21^h\ 24^m$. The moon crosses the N. H. meridian about 12^m after this, and therefore by the exam. on p. 69, a correction of $0^s.08$ must be added to $+53^s.12$, making it $+53^s.20$. The difference between this correction by the moon-culminating stars, and the erroneous correction caused by the advance of the moon in R.A., is of course the moon's true increase of R.A. in her passage between the two meridians $=\alpha'-\alpha=+10^m\ 50^s.71$. We are now to interpolate from the Naut. Alm. for the quantity α'', and to do this with sufficient accuracy, we must calculate the formula on p. 72, as far as the 3rd or 4th term:

Oct. 1839.	a.	d'.	d''.	d'''.
$17^h\ 0^m$	$21^h\ 25^m\ 1^s.66$	$+26^m\ 40^s.70$		
" 12	" 51 42 .36	" 26 .22	$-14^s.48$	$+6^s.61$
18 0	22 18 8 .58	+ " 18 .35	$-7\ .87$	
" 12	" 44 26 .93			

$$t=\frac{4^h\ 51^m\ 46^s}{24^h}=0^d.4052.$$

$$d'=1600''.7;\ d''=-14^s.48;\ d'''=+6^s.61.$$

$$
\begin{aligned}
21^h\ 25^m\ 1^s.66 &= a.\\
+10\ \ 48\ .65 &= +d't.\\
+1\ .75 &= -d''t\,\frac{t-1}{2}.\\
+.43 &= +d'''t\,\frac{t-1}{2}\cdot\frac{t-2}{3}.\\
\hline
21^h\ 35^m\ 52^s.49 &\\
\hline
+10\ \ 50\ .83 &= a''-a.\\
+10\ \ 50\ .71 &= a'-a.\\
\hline
-0^s.12 &= a'-a''.
\end{aligned}
$$

For α''', from the values $134^s.15$, $132^s.71$, $131^s.76$, and their differonces,—we have $134^s.15-0^s.58-0^s.08=133^s.50=\alpha'''$. Then

$$\frac{\alpha'-\alpha''}{\alpha'''}=\frac{-0.12}{133.50}=-0^d.000899=-3^s.24=x.$$

$l+x=4^h\ 51^m\ 46^s-3^s.24=4^h\ 51^m\ 42^s.76\ldots$ Longitude of N. H.

If by a similar observation at Greenwich, or other foreign observatory, the R.A. of the moon in the Naut. Alm. is found to be too great by the quantity $\alpha_{,,}$ the equation becomes

$$x+c=\frac{\alpha'-\alpha''+\alpha_{,}}{\alpha'''}.$$

One peculiarity of the methods belonging to this class is,—that an error in the measurement will produce one between 20 and 30 times as great in the longitude. Thus, an error in observing the transit of the moon of only 1^s at either station, would make a difference, on the average, of more than 26^s; and an observer who takes a lunar distance $10''$ too small, makes a difference in his longitude of about 18^s of time. The reason of this is, that the moon's monthly motion is measured by means of sidereal time, whose rate of revolution is nearly 27 times more swift. But where a distinct phenomenon is observed, as in all the methods under the first two classes, no such multiplication of error can take place.

119. We have now arrived at the conclusion of our work,—and with this partial review of the terrestrial contrivances and means by which astronomers have acquired their knowledge of the celestial bodies, we shall turn with increased pleasure to the consideration of the results of their labors, which constitute the departments of descriptive and physical astronomy. The student would regard, for example, with none the less interest the return of the long expected comet, and the exact verification of the calculations of mathematicians, because he was acquainted with the means by which the observer tracked the body in its passage through the heavens, and with his diminutive, but refined apparatus, recorded unerringly the data from which the physical astronomer should predict, without hazard of failure, the exact positions which it should in future assume. Nor will an eclipse be viewed with less pleasure and satisfaction, after he has become able to foretell its time and aspect. And all the data of astronomy, in their immensity of extent,—its processes, and magnificent conclusions, will now seem to him far more stable and secure, for he has derived confidence from penetrating to the very basis of the science, from reviewing the delicate resources of the observer in his instruments, and the processes of observation, until he has arrived at valuable measurements and primitive data.

The observer, too, who has the means of deriving results of his own from the heavens, needs no stimulus to prosecute a study, which few, who thoroughly engage in it, will easily re-

linquish. Every failure of agreement in his conclusions will but urge him patiently to solve the difficulty; every instance of success will inspire him with fresh ardor and enterprise: and he will find no pursuit more constantly beckoning him forward to what lies beyond him, more absorbing in its prosecution, more elevating to the mind, or impressing him with a deeper sense of the power and wisdom of the Creator

NOTE TO ART. 95.

Being a simple Geometrical and Algebraic demonstration of the Formula for Occultations on page 93.

PRELIMINARY to this investigation, we shall introduce three formulæ necessary to the following demonstration, and shall refer for authority to Young's Spherical Trigonometry, a common elementary work and text-book.

In a triangle ABC right angled at C,

Formula 1. $\cos c = \cos a \cos b$.

(See Young's Spher. Trig., Art. 52, eq. 3 of right angled triangles.)

In an oblique angled triangle,

Formula 2. $\sin a \sin B = \sin b \sin A$.

(For . . . (Y. S. T., Art. 52, eq. 1,) $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b}$; and clearing of fractions, we have formula 2.)

Formula 3. $\sin c \cos A = \cos a \sin b - \sin a \cos b \cos C$.

(For . . . (Y. S. T., Art. 48, eq. 1,) $\cos a \sin b = \sin a \cos b \cos C + \sin c \cos A$; and transposing, we have formula 3.)

Let O (fig. 29,) be the centre of the earth, S the place of a spectator on its surface, and M the position of the moon in space. Suppose with the centre O and radius OM, a spherical surface to be described, in which let us take X, Y, Z, any points 90° distant from each other, and join them by the quadrantal arcs XY, YZ, ZX; join also OX, OY, OZ, OM, and produce OS to meet the sphere in A; A will then represent on the sphere the geocentric zenith of the spectator, and M the *true* place of the moon. Through O draw OM′ parallel to SM, and complete the parallelogram OSMN; then since SM is the apparent direction of the moon from the spectator, the observer will refer the moon on an infinite sphere to a point corresponding to M′, in the same direction from the centre of the earth that M is from S. M′ therefore represents the *apparent* place of the moon on the sphere. We have then three planes, XOY, YOZ, ZOX, at right angles to each other, and to which we can refer the points M, S, and N.

From M, S, and N, draw MD, SB, and NC perpendicular to OX; then will OC = BD. For if we suppose three planes, P, P′, P″, passing through the points M, S, and N, parallel to YOZ, and cutting OX at right angles,—MD, SB, and NC will lie in those planes, because they are at right angles to OX; the planes will therefore cut OX in the points D, B, and C respectively, and DB will be the perpendicular distance between the planes P and P′, and CO between the planes P″ and YOZ. But SM unites the planes P and P′, and ON the planes P″ and YOZ, and these lines, being parallel, have the same inclination to their respective planes; if this angle of inclination be called ι, the perpendicular distance between P and P′ (DB) = SM $\sin \iota$, and that between P″ and YOZ (OC) = ON $\sin \iota$. But SM and ON are equal; therefore OC = DB.

Since OCN, OBS, and ODM are right angles, OC = ON cos CON = ON cos XM′; OB = OS cos XA; and OD = OM cos XM. But OD = OB + BD = OB + OC, and therefore

I. $\quad OM \cos XM = OS \cos XA + SM \cos XM'$.

By referring S, M, and N, to the plane ZOX by means of the line OY, precisely as we

have already referred them to YOZ by means of the line OX, we shall obtain the similar equation

II. $\quad$ OM cos YM = OS cos YA + SM cos YM′.

And referring them to the plane XOY by the line OZ,

III. $\quad$ OM cos ZM = OS cos ZA + SM cos ZM′.

Now in the right angled triangle YHM, we have (formula 1.), cos YM = cos HM cos YH. For the same reason, cos YA = cos IA cos YI; and cos YM′ = cos GM′ cos YG.

Again, by the same formula, cos ZM = cos HM cos ZH = cos HM sin YH; so cos ZA = cos IA sin YI, and cos ZM′ = cos GM′ sin YG.

Substituting these values in the two last equations,—equations (I.), (II.) and (III.), after transposition, become as follows:

(1.) $\quad$ SM sin GM′ = OM sin HM − OS sin IA

(2.) $\quad$ SM cos GM′ cos YG = OM cos HM cos YH − OS cos IA cos YI.

(3.) $\quad$ SM cos GM′ sin YG = OM cos HM sin YH − OS cos IA sin YI.

With M′, the apparent place of the moon for the centre, and with radius M′T = the moon's *apparent* semidiameter, describe a circle representing the moon as it appears in situation and magnitude on the infinite sphere to the observer. Let T be a star distant from the moon's centre by her apparent semidiameter; that is, apparently in contact with the moon's limb, and therefore undergoing either immersion or emersion. Then in the spherical triangle XTM′, we have

(4.) $\quad$ sin M′T cos XTM′ = cos XM′ sin XT − sin XM′ cos XT cos M′XT. $\quad$ (Form. 3.)

(5.) $\quad$ sin M′T sin XTM′ = sin XM′ sin M′XT $\quad$ (Form. 2.)

Let us now make

SM = Δ = distance of the moon from the place of observation in parts of OM as radius or unity.

OS = $\sin\pi$ = distance of the observer from the centre of the earth in parts of OM as radius; π being the eq. hor. par. of the moon, reduced in the ratio of $\dfrac{\text{OS}}{\text{earth's eq. rad.}}$.

YH = a
HM = δ
YG = a'
GM′ = δ
YI = μ
IA = ϕ

Let also
YL = A
LT = Δ
M′T = d'
$\angle$XTM′ = τ, ... when $a' <$ A,
$\quad$ = 360° − τ, ... when $a' >$ A.
$\angle$M′XT = −(a' − A), . . . when $a' <$ A.
$\quad$ = (a' − A), . . . when $a' >$ A.

By the substitution of these symbols, equations (1.), (2.) and (3.) become

(1′.) $\quad \Delta \sin\delta' = \sin\delta - \sin\pi \sin\phi.$

(2′.) $\quad \Delta \cos\delta' \cos a' = \cos\delta \cos a - \sin\pi \cos\phi \cos\mu.$

(3′.) $\quad \Delta \cos\delta' \sin a' = \cos\delta \sin a - \sin\pi \cos\phi \sin\mu.$

And changing the notation of eqs. (4.) and (5.) in the same manner,

(4′.) $\quad \sin d' \cos\tau = \sin\delta' \cos\Delta - \cos\delta' \sin\Delta \cos(a' - \text{A}).$

(5′.) $\quad \sin d' \sin\tau = -\cos\delta' \sin(a' - \text{A}).$

The angle τ is the angle included between the meridian XT and the great circle TM passing through the star T and moon M′,—reckoned from 0° to 360° in the direction XOY, or so that τ shall be between 0° and 180°, when $a' <$ A, and between 180° and 360°, when $a' >$ A.

The next step will be to transform in eqs. (4′.) and (5′.), the apparent semidiameter, R.A., and Decl. of the moon, into the true ones,—by combination with eqs. (1′.), (2′.) and (3′.) For eq. (4′.) the process will be as follows:

Multiplying by $\triangle$, and expanding $\cos(a'-A)$ . . . Day's Trig. Anal., Art. 208, IV.;

$$\triangle \sin d' \cos \tau = \triangle \sin \delta' \cos \Delta - \triangle \cos \delta' \sin \Delta \cos a' \cos A$$
$$- \triangle \cos \delta' \sin \Delta \sin a' \sin A.$$

Here $\triangle \sin \delta'$ occurs in the first term of the second member, $\triangle \cos \delta' \cos a'$ in the second, and $\triangle \cos \delta' \sin a'$ in the third; substituting the values of these quantities as given in eqs. (1'.), (2'.) and (3'.):

$$\triangle \sin d' \cos \tau = \sin \delta \cos \Delta - \sin \pi \sin \phi \cos \Delta$$
$$- \cos \delta \sin \Delta \cos a \cos A + \sin \pi \cos \phi \sin \Delta \cos \mu \cos A$$
$$- \cos \delta \sin \Delta \sin a \sin A + \sin \pi \cos \phi \sin \Delta \sin \mu \sin A.$$

Recombining by Art. 208, IV., Day's Anal. Trig.:

$$\triangle \sin d' \cos \tau = \sin \delta \cos \Delta - \cos \delta \sin \Delta \cos (a-A)$$
$$- \sin \pi \sin \phi \cos \Delta + \sin \pi \cos \phi \sin \Delta \cos (\mu - A).$$

And since in Art. 94 we have proved that $\triangle \sin d' = \sin d$;

$$(7.) \quad \sin d \cos \tau = \sin \delta \cos \Delta - \cos \delta \sin \Delta \cos (a-A)$$
$$- \sin \pi \left\{ \sin \phi \cos \Delta - \cos \phi \sin \Delta \cos (\mu - A) \right\}.$$

For eq. (5'.), a similar, but shorter process brings out this equation:

$$(8.) \quad \sin d \sin \tau = - \cos \delta \sin (a-A) + \cos \phi \sin \pi \sin (\mu - A).$$

Squaring each of these equations, and adding their squares, (since $\sin^2 d \sin^2 \tau + \sin^2 d \cos^2 \tau = \sin^2 d$,);

$$(\mathrm{I.}) \quad \sin^2 d = \left\{ \cos \delta \sin (a-A) - \sin \pi \cos \phi \sin (\mu - A) \right\}^2$$
$$+ \left\{ \sin \delta \cos \Delta - \cos \delta \sin \Delta \cos (a-A) \right.$$
$$\left. - \sin \pi \left\{ \sin \phi \cos \Delta - \cos \phi \sin \Delta \cos (\mu - A) \right\} \right\}^2.$$

which equation is the same with the expanded formula for occultations in Art. 96, p. 93.

The formula just demonstrated is, however, far more general than the other. The points X, Y, Z, were chosen *any* three points 90° distant from each other, and our equation is therefore universal, and may be practically applied to *any* great circle of the heavens, and its pole. The equator is the one to which the quantities in eq. (I.) p. 96, refer; but the same symbols, in the equation now demonstrated, are referable also to the ecliptic and its pole, the moon's orbit and its pole, the horizon and zenith, or to any imaginable great circle in the heavens at the pleasure of the calculator.

Having demonstrated eq. (I.) to be thus general in its application, eq. (II.) Art. 96, becomes by the manner of deduction from eq. I, equally universal; and the conclusion at which we arrive immediately after eq. (II.) in Art. 97, may be stated without limitation as follows:

"If, then, *any* two great circles cut one another at right angles at the point of the star to be occulted, p and q will be the cosines of the arcs joining the poles of these two circles with the true place of the moon," &c.

TABLE I.

Changes of the circular functions in sign and algebraic expression through the four quadrants.

	0°	*For an angle A between 0° and 90°.*	90°	*For an angle A between 90° and 180°.*	180°	*For an angle A between 180° and 270°.*	270°	*For an angle A between 270° and 360°.*	360°
Sine A	0	+ sin A	+1	+sin (180° − A) +cos (A − 90°)	0	−sin (A − 180°) − cos (270° − A)	−1	−sin (360° − A) −cos (A − 270°)	0
Cos A	+1	+cos A	0	− cos (180° − A) −sin (A − 90°)	−1	−cos (A − 180°) −sin (270° − A)	0	+cos (360° − A) +sin (A − 270°)	+1
Tan A	0	+tan A	∞	−tan (180° − A) −cot (A − 90°)	0	+tan (A − 180°) +cot (270° − A)	∞	−tan (360° − A) −cot (A − 270°)	0
Cot A	∞	+cot A	0	−cot (180° − A) −tan (A − 90°)	∞	+cot (A − 180°) +tan (270° − A)	0	−cot (360° − A) −tan (A − 270°)	∞
Sec A	+1	+sec A	∞	−sec (180° − A) −cosec (A − 90°)	−1	−sec (A − 180°) −cosec (270° − A)	∞	+sec (360° − A) +cosec (A − 270°)	+1
Cosec A	∞	+cosec A	+1	+cosec (180° − A) +sec (A − 90°)	∞	−cosec (A − 180°) −sec (270° − A)	−1	−cosec (360° − A) −sec (A − 270°)	∞
Ver-Sin A	0	+versin A	+1	2 − versin (180° − A) 2 − coversin (A − 90°)	+2	2 − versin (A − 180°) 2 − coversin (270° − A)	+1	+versin (360° − A) +coversin (A − 270°)	0
Co-Versin A	+1	+coversin A	0	+coversin (180° − A) +versin (A − 90°)	+1	2 − versin (270° − A) 2 − coversin (A − 180°)	+2	2 − coversin (A − 270°) 2 − versin (360° − A)	+1

TABLE II.

For converting Intervals of Mean Solar Time into Equivalent Intervals of Sidereal Time.

HOURS.				MINUTES.									SECONDS.					
h	h	m	s	m	m	s	m	m	s	m	m	s	s	s	s	s	s	s
1	1	0	9.86	1	1	0.16	21	21	3.45	41	41	6.74	1	1.00	21	21.06	41	41.11
2	2	0	19.71	2	2	0.33	22	22	3.61	42	42	6.90	2	2.01	22	22.06	42	42.11
3	3	0	29.57	3	3	0.49	23	23	3.78	43	43	7.06	3	3.01	23	23.06	43	43.12
4	4	0	39.43	4	4	0.66	24	24	3.94	44	44	7.23	4	4.01	24	24.07	44	44.12
5	5	0	49.28	5	5	0.82	25	25	4.11	45	45	7.39	5	5.01	25	25.07	45	45.12
6	6	0	59.14	6	6	0.99	26	26	4.27	46	46	7.56	6	6.02	26	26.07	46	46.13
7	7	1	9.00	7	7	1.15	27	27	4.44	47	47	7.72	7	7.02	27	27.07	47	47.13
8	8	1	18.85	8	8	1.31	28	28	4.60	48	48	7.89	8	8.02	28	28.08	48	48.13
9	9	1	28.71	9	9	1.48	29	29	4.76	49	49	8.05	9	9.02	29	29.08	49	49.13
10	10	1	38.56	10	10	1.64	30	30	4.93	50	50	8.21	10	10.03	30	30.08	50	50.14
11	11	1	48.42	11	11	1.81	31	31	5.09	51	51	8.38	11	11.03	31	31.08	51	51.14
12	12	1	58.28	12	12	1.97	32	32	5.26	52	52	8.54	12	12.03	32	32.09	52	52.14
13	13	2	8.13	13	13	2.14	33	33	5.42	53	53	8.71	13	13.04	33	33.09	53	53.15
14	14	2	17.99	14	14	2.30	34	34	5.59	54	54	8.87	14	14.04	34	34.09	54	54.15
15	15	2	27.85	15	15	2.46	35	35	5.75	55	55	9.04	15	15.04	35	35.10	55	55.15
16	16	2	37.70	16	16	2.63	36	36	5.91	56	56	9.20	16	16.04	36	36.10	56	56.15
17	17	2	47.56	17	17	2.79	37	37	6.08	57	57	9.36	17	17.05	37	37.10	57	57.16
18	18	2	57.42	18	18	2.96	38	38	6.24	58	58	9.53	18	18.05	38	38.10	58	58.16
19	19	3	7.27	19	19	3.12	39	39	6.41	59	59	9.69	19	19.05	39	39.11	59	59.16
20	20	3	17.13	20	20	3.29	40	40	6.57	60	60	9.86	20	20.05	40	40.11	60	60.16
21	21	3	26.99															
22	22	3	36.84															
23	23	3	46.70															
24	24	3	56.56															

TABLE III.

For converting Intervals of Sidereal Time into Equivalent Intervals of Mean Solar Time.

HOURS.				MINUTES.									SECONDS.					
h	h	m	s	m	m	s	m	m	s	m	m	s	s	s	s	s	s	s
1	0	59	50.17	1	0	59.84	21	20	56.56	41	40	53.28	1	1.00	21	20.94	41	40.89
2	1	59	40.34	2	1	59.67	22	21	56.40	42	41	53.12	2	1.99	22	21.94	42	41.89
3	2	59	30.51	3	2	59.51	23	22	56.23	43	42	52.96	3	2.99	23	22.94	43	42.83
4	3	59	20.68	4	3	59.34	24	23	56.07	44	43	52.79	4	3.99	24	23.93	44	43.88
5	4	59	10.85	5	4	59.18	25	24	55.90	45	44	52.63	5	4.99	25	24.93	45	44.88
6	5	59	1.02	6	5	59.02	26	25	55.74	46	45	52.46	6	5.98	26	25.93	46	45.87
7	6	58	51.19	7	6	58.85	27	26	55.58	47	46	52.30	7	6.98	27	26.93	47	46.87
8	7	58	41.36	8	7	58.69	28	27	55.41	48	47	52.14	8	7.98	28	27.92	48	47.87
9	8	58	31.53	9	8	58.53	29	28	55.25	49	48	51.97	9	8.98	29	28.92	49	48.87
10	9	58	21.70	10	9	58.36	30	29	55.09	50	49	51.81	10	9.97	30	29.92	50	49.86
11	10	58	11.87	11	10	58.20	31	30	54.92	51	50	51.64	11	10.97	31	30.92	51	50.86
12	11	58	2.05	12	11	58.03	32	31	54.76	52	51	51.48	12	11.97	32	31.91	52	51.86
13	12	57	52.22	13	12	57.87	33	32	54.59	53	52	51.32	13	12.96	33	32.91	53	52.86
14	13	57	42.39	14	13	57.71	34	33	54.43	54	53	51.15	14	13.96	34	33.91	54	53.85
15	14	57	32.56	15	14	57.54	35	34	54.27	55	54	50.99	15	14.96	35	34.90	55	54.85
16	15	57	22.73	16	15	57.38	36	35	54.10	56	55	50.83	16	15.96	36	35.90	56	55.85
17	16	57	12.90	17	16	57.21	37	36	53.94	57	56	50.66	17	16.95	37	36.90	57	56.84
18	17	57	3.07	18	17	57.05	38	37	53.77	58	57	50.50	18	17.95	38	37.90	58	57.84
19	18	56	53.24	19	18	56.89	39	38	53.61	59	58	50.33	19	18.95	39	38.89	59	58.84
20	19	56	43.41	20	19	56.72	40	39	53.45	60	59	50.17	20	19.95	40	39.89	60	59.84
21	20	56	33.58															
22	21	56	23.75															
23	22	56	13.92															
24	23	56	4.09															

TABLE IV.

Interpolation by Second Differences.

Parts of the unit of time.	*Minutes of an hour.*	*Nat. coefficient of d″.*		*Log. coefficient of d″.*		SECONDS OF SECOND DIFFERENCES.										*Minutes of an hour.*	*Parts of the unit of time.*
						10″	20″	30″	40″	50″	60″	70″	80″	90″	100″		
		—	*Diff.*	—	*Diff.*	1″ .1	2″ .2	3″ .3	4″ .4	5″ .5	6″ .6	7″ .7	8″ .8	9″ .9	10″ 1″		
	m															m	
.01	0.6	.0049	49	7.6946	2966	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5	59.4	.99
.02	1.2	.0098	48	7.9912	1717	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	58.8	.98
.03	1.8	.0145	47	8.1629	1204	0.1	0.3	0.4	0.6	0.7	0.9	1.0	1.2	1.3	1.5	58.2	.97
.04	2.4	.0192	46	8.2833	924	0.2	0.4	0.6	0.8	1.0	1.2	1.3	1.5	1.7	1.9	57.6	.96
.05	3.0	.0237	45	8.3757	745	0.2	0.5	0.7	0.9	1.2	1.4	1.7	1.9	2.1	2.4	57.0	.95
.06	3.6	.0282	44	8.4502	624	0.3	0.6	0.8	1.1	1.4	1.7	2.0	2.3	2.5	2.8	56.4	.94
.07	4.2	.0325	43	8.5126	532	0.3	0.7	1.0	1.3	1.6	2.0	2.3	2.6	2.9	3.3	55.8	.93
.08	4.8	.0368	42	8.5658	465	0.4	0.7	1.1	1.5	1.8	2.2	2.6	2.9	3.3	3.7	55.2	.92
.09	5.4	.0409	41	8.6123	409	0.4	0.8	1.2	1.6	2.0	2.5	2.9	3.3	3.7	4.1	54.6	.91
.10	6.0	.0450	40	8.6532	366	0.5	0.9	1.4	1.8	2.2	2.7	3.1	3.6	4.1	4.5	54.0	.90
.11	6.6	.0489	39	8.6898	328	0.5	1.0	1.5	2.0	2.4	2.9	3.4	3.9	4.4	4.9	53.4	.89
.12	7.2	.0528	38	8.7226	298	0.5	1.1	1.6	2.1	2.6	3.2	3.7	4.2	4.8	5.3	52.8	.88
.13	7.8	.0565	37	8.7524	272	0.6	1.1	1.7	2.3	2.8	3.4	4.0	4.5	5.1	5.7	52.2	.87
.14	8.4	.0602	36	8.7796	249	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6.0	51.6	.86
.15	9.0	.0637	35	8.8045	229	0.6	1.3	1.9	2.5	3.2	3.8	4.4	5.1	5.7	6.4	51.0	.85
.16	9.6	.0672	34	8.8274	211	0.7	1.3	2.0	2.7	3.4	4.0	4.6	5.4	6.0	6.7	50.4	.84
.17	10.2	.0705	33	8.8485	196	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	7.1	49.8	.83
.18	10.8	.0738	32	8.8681	181	0.7	1.5	2.2	3.0	3.7	4.4	5.2	5.9	6.6	7.4	49.2	.82
.19	11.4	.0769	31	8.8862	169	0.8	1.5	2.3	3.1	3.8	4.6	5.4	6.2	6.9	7.7	48.6	.81
.20	12.0	.0800	30	8.9031	157	0.8	1.6	2.4	3.2	4.0	4.8	5.6	6.4	7.2	8.0	48.0	.80
.21	12.6	.0829	29	8.9188	147	0.8	1.6	2.5	3.3	4.1	5.0	5.8	6.6	7.5	8.3	47.4	.79
.22	13.2	.0858	28	8.9335	137	0.9	1.7	2.6	3.4	4.3	5.1	6.0	6.9	7.7	8.6	46.8	.78
.23	13.8	.0885	27	8.9472	128	0.9	1.8	2.7	3.5	4.4	5.3	6.2	7.1	8.0	8.9	46.2	.77
.24	14.4	.0912	26	8.9600	120	0.9	1.8	2.7	3.6	4.6	5.5	6.4	7.3	8.2	9.1	45.6	.76
.25	15.0	.0937	25	8.9720	112	0.9	1.9	2.8	3.7	4.7	5.6	6.6	7.5	8.4	9.4	45.0	.75
.26	15.6	.0962	24	8.9832	105	1.0	1.9	2.9	3.8	4.8	5.8	6.7	7.7	8.6	9.6	44.4	.74
.27	16.2	.0985	23	8.9937	98	1.0	2.0	3.0	3.9	4.9	5.9	6.9	7.9	8.9	9.9	43.8	.73
.28	16.8	.1008	22	9.0035	91	1.0	2.0	3.0	4.0	5.0	6.0	7.1	8.1	9.1	10.1	43.2	.72
.29	17.4	.1029	21	9.0126	86	1.0	2.1	3.1	4.1	5.1	6.2	7.2	8.3	9.3	10.3	42.6	.71
.30	18.0	.1050	38	9.0212	154	1.1	2.1	3.2	4.2	5.2	6.3	7.4	8.4	9.5	10.5	42.0	.70
.32	19.2	.1088	34	9.0366	134	1.1	2.2	3.3	4.4	5.4	6.5	7.6	8.7	9.8	10.9	40.8	.68
.34	20.4	.1122	30	9.0500	115	1.1	2.2	3.4	4.5	5.6	6.7	7.8	9.0	10.1	11.2	39.6	.66
.36	21.6	.1152	26	9.0615	96	1.2	2.3	3.5	4.6	5.8	6.9	8.0	9.2	10.4	11.5	38.4	.64
.38	22.8	.1178	24	9.0711	81	1.2	2.4	3.5	4.7	5.9	7.1	8.2	9.4	10.6	11.8	37.2	.62
.40	24.0	.1200	18	9.0792	65	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6	10.8	12.0	36.0	.60
.42	25.2	.1218	14	9.0857	49	1.2	2.4	3.7	4.9	6.1	7.3	8.5	9.8	11.0	12.2	34.8	.58
.44	26.4	.1232	10	9.0906	35	1.2	2.5	3.7	4.9	6.2	7.4	8.6	9.9	11.1	12.3	33.6	.56
.46	27.6	.1242	6	9.0941	21	1.2	2.5	3.7	5.0	6.2	7.5	8.7	9.9	11.2	12.4	32.4	.54
.48	28.8	.1248	2	9.0962	7	1.2	2.5	3.7	5.0	6.2	7.5	8.7	10.0	11.2	12.5	31.2	.52
.50	30.0	.1250		9.0969		1.3	2.5	3.8	5.0	6.3	7.5	8.8	10.0	11.3	12.5	30.0	.50

TABLE V.

Coefficients of the errors a, b, *and* c, *for the Transit Instrument, corresponding to every degree of Declination from the zenith of New Haven to the southern horizon.*

Decl. of star.	a.		b.		c.		*Decl. of star.*	a.		b.		c.	
	$\frac{\sin(\phi-\delta)}{\sec\delta}$	*Diff.*	$\frac{\cos(\phi-\delta)}{\sec\delta}$	*Diff.*	$\sec\delta$	*Diff.*		$\frac{\sin(\phi-\delta)}{\sec\delta}$	*Diff.*	$\frac{\cos(\phi-\delta)}{\sec\delta}$	*Diff.*	$\sec\delta$	*Diff.*
+41°	0.007	23	1.325	20	1.325	21	− 4°	0.712	13	0.705	11	1.002	1
40	0.030	23	1.305	20	1.305	20	5	0.726	13	0.694	12	1.004	1
39	0.052	22	1.286	19	1.287	19	6	0.739	13	0.682	12	1.006	2
33	0.073	21	1.267	19	1.269	18	7	0.752	13	0.670	12	1.008	2
37	0.094	21	1.249	18	1.252	17	8	0.765	13	0.658	12	1.010	2
36	0.114	20	1.231	18	1.236	16	9	0.779	13	0.647	12	1.012	3
35	0.134	20	1.213	18	1.221	15	10	0.792	14	0.635	12	1.015	3
34	0.153	19	1.196	17	1.206	15	11	0.806	14	0.623	12	1.019	3
33	0.172	19	1.180	17	1.192	14	12	0.820	14	0.611	12	1.022	4
32	0.191	19	1.164	16	1.179	13	13	0.833	14	0.599	12	1.026	4
31	0.209	18	1.148	16	1.167	12	14	0.847	14	0.587	12	1.031	4
30	0.226	18	1.132	16	1.155	12	15	0.861	14	0.574	12	1.035	5
29	0.244	17	1.117	15	1.143	11	16	0.876	14	0.562	13	1.040	5
28	0.261	17	1.102	15	1.133	11	17	0.890	14	0.549	13	1.046	5
27	0.277	17	1.087	15	1.122	10	18	0.904	14	0.537	13	1.052	6
26	0.294	16	1.073	14	1.113	10	19	0.918	15	0.524	13	1.058	6
25	0.310	16	1.059	14	1.103	9	20	0.933	15	0.511	13	1.064	6
24	0.326	16	1.045	14	1.095	9	21	0.948	15	0.498	13	1.071	7
23	0.341	15	1.032	14	1.086	8	22	0.963	15	0.485	13	1.078	7
22	0.356	15	1.018	13	1.078	8	23	0.979	15	0.471	14	1.086	8
21	0.371	15	1.005	13	1.071	7	24	0.994	16	0.457	14	1.095	8
20	0.386	15	0.991	13	1.064	7	25	1.010	16	0.443	14	1.103	9
19	0.401	15	0.978	13	1.058	6	26	1.026	16	0.429	14	1.113	9
18	0.416	15	0.966	13	1.052	6	27	1.043	16	0.415	15	1.122	10
17	0.430	14	0.953	13	1.046	6	28	1.059	17	0.400	15	1.133	10
16	0.445	14	0.941	12	1.040	5	29	1.076	17	0.385	15	1.143	11
15	0.459	14	0.928	12	1.035	5	30	1.094	17	0.370	16	1.155	11
14	0.473	14	0.916	12	1.031	5	31	1.111	17	0.355	16	1.167	12
13	0.487	14	0.904	12	1.026	4	32	1.129	18	0.339	16	1.179	12
12	0.500	14	0.892	12	1.022	4	33	1.148	18	0.323	17	1.192	13
11	0.514	14	0.880	12	1.019	4	34	1.167	19	0.306	17	1.206	14
10	0.528	14	0.868	12	1.015	3	35	1.186	19	0.289	17	1.221	15
9	0.541	14	0.856	12	1.012	3	36	1.206	20	0.272	18	1.236	15
8	0.554	14	0.844	12	1.010	3	37	1.226	20	0.254	18	1.252	16
7	0.568	13	0.832	12	1.008	2	38	1.247	21	0.236	19	1.269	17
6	0.581	13	0.821	12	1.006	2	39	1.268	21	0.217	19	1.287	18
5	0.594	13	0.809	12	1.004	2	40	1.290	22	0.197	20	1.305	19
4	0.608	13	0.797	12	1.002	1	41	1.313	23	0.177	20	1.325	20
3	0.621	13	0.786	12	1.001	1	42	1.336	23	0.157	21	1.346	21
2	0.634	13	0.774	11	1.001	1	43	1.360	24	0.136	22	1.367	22
+ 1	0.647	13	0.763	11	1.000	0	44	1.385	25	0.114	23	1.390	23
0	0.660	13	0.751	11	1.000	0	45	1.411	26	0.091	23	1.414	24
− 1	0.673	13	0.740	11	1.000	0	46	1.438	27	0.068	24	1.440	25
2	0.686	13	0.728	11	1.001	0	47	1.466	28	0.043	25	1.466	26
− 3	0.699	13	0.717	11	1.001	1	−48	1.494	29	0.018	26	1.494	28
		13		11		1			30				29

TABLE VI.

Angles of the vertical with the earth's radius, and reduction of the equatorial horizontal parallax for every degree of latitude.

Compression of the earth = $\frac{1}{309}$.

Latitude.	Angle of Vertical with earth's radius.	Corr. of Parallax of 58'.	Diff. for 1'.	Latitude.	Angle of Vertical with earth's radius.	Corr. of Parallax of 58'.	Diff. for 1'.
1°	0' 23".3	0".00	0".00	46°	11' 8".8	5".82	0".10
2	0 46 .5	0 .01	0 .00	47	11 7 .7	6 .01	0 .10
3	1 9 .7	0 .03	0 .00	48	11 5 .7	6 .21	0 .11
4	1 32 .8	0 .06	0 .00	49	11 2 .9	6 .41	0 .11
5	1 55 .8	0 .09	0 .00	50	10 59 .3	6 .59	0 .11
6	2 18 .7	0 .12	0 .00	51	10 54 .9	6 .79	0 .12
7	2 41 .4	0 .17	0 .00	52	10 49 .7	6 .98	0 .12
8	3 3 .8	0 .22	0 .00	53	10 43 .8	7 .18	0 .12
9	3 26 .1	0 .28	0 .00	54	10 37 .0	7 .36	0 .13
10	3 48 .1	0 .34	0 .01	55	10 29 .5	7 .55	0 .13
11	4 9 .9	0 .41	0 .01	56	10 21 .2	7 .74	0 .13
12	4 31 .4	0 .48	0 .01	57	10 12 .0	7 .92	0 .14
13	4 52 .5	0 .57	0 .01	58	10 2 .3	8 .10	0 .14
14	5 13 .2	0 .65	0 .01	59	9 51 .7	8 .28	0 .14
15	5 33 .7	0 .75	0 .01	60	9 40 .4	8 .45	0 .15
16	5 53 .6	0 .85	0 .01	61	9 28 .4	8 .63	0 .15
17	6 13 .2	0 .96	0 .02	62	9 15 .7	8 .79	0 .15
18	6 32 .3	1 .06	0 .02	63	9 2 .2	8 .95	0 .15
19	6 50 .9	1 .18	0 .02	64	8 48 .3	9 11	0 .16
20	7 9 .0	1 .31	0 .02	65	8 33 .7	9 .26	0 .16
21	7 26 .7	1 .44	0 .02	66	8 18 .3	9 .41	0 .16
22	7 43 .7	1 .57	0 .03	67	8 2 .4	9 .55	0 .16
23	8 0 .2	1 .71	0 .03	68	7 45 .9	9 .70	0 .17
24	8 16 .2	1 .85	0 .03	69	7 28 .8	9 .83	0 .17
25	8 31 .5	2 .00	0 .03	70	7 11 .1	9 .97	0 .17
26	8 46 .2	2 .15	0 .04	71	6 53 .0	10 .08	0 .17
27	9 0 .3	2 .31	0 .04	72	6 34 .3	10 .21	0 .18
28	9 13 .7	2 .46	0 .04	73	6 15 .2	10 .32	0 .18
29	9 26 .5	2 .63	0 .05	74	5 55 .6	10 .43	0 .18
30	9 38 .5	2 .80	0 .05	75	5 35 .6	10 .54	0 .18
31	9 40 .9	2 .98	0 .05	76	5 15 .0	10 .64	0 .18
32	10 0 .6	3 .15	0 .05	77	4 54 .2	10 .72	0 .19
33	10 10 .5	3 .32	0 .06	78	4 33 .0	10 .80	0 .19
34	10 19 .7	3 .51	0 .06	79	4 11 .4	10 .88	0 .19
35	10 28 .0	3 .69	0 .06	80	3 49 .6	10 .94	0 .19
36	10 35 .7	3 .87	0 .07	81	3 27 .4	11 .01	0 .19
37	10 42 .6	4 .06	0 .07	82	3 5 .0	11 .07	0 .19
38	10 48 .7	4 .25	0 .07	83	2 42 .4	11 .12	0 .19
39	10 54 .1	4 .45	0 .08	84	2 19 .6	11 .16	0 .19
40	10 58 .6	4 .64	0 .08	85	1 56 .6	11 .20	0 .19
41	11 2 .3	4 .83	0 .08	86	1 33 .4	11 .23	0 .19
42	11 5 .2	5 .03	0 .09	87	1 10 .2	11 .26	0 19
43	11 7 .4	5 .22	0 .09	88	0 46 .8	11 .27	0 .19
44	11 8 .6	5 .42	0 .09	89	0 23 .4	11 .29	0 .19
45	11 9 .1	5 .62	0 .10	90	0 0 .0	11 .29	0 .19

PLATE IV.

Fig. 20.

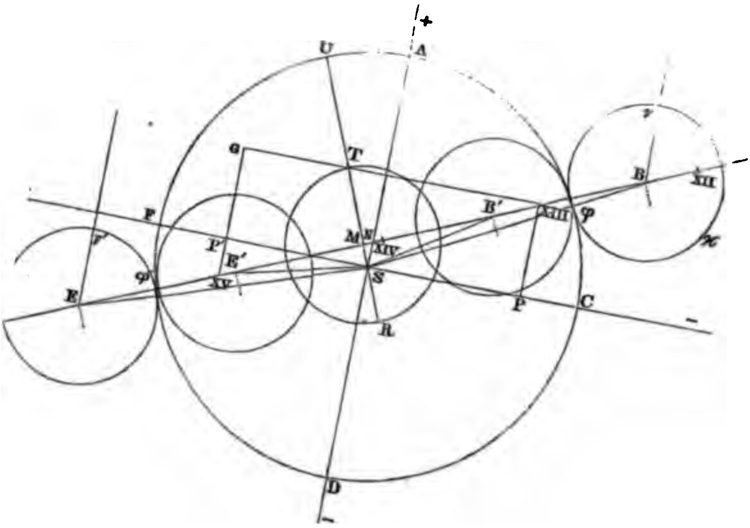


Fig. 21.

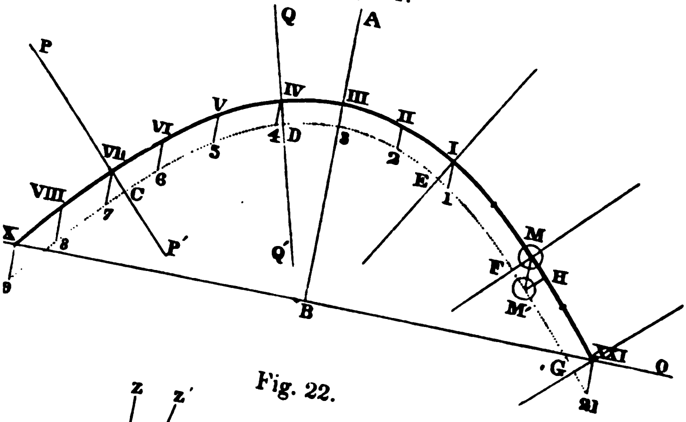


Fig. 22.

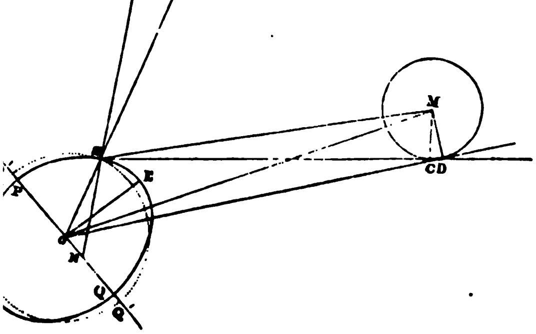


PLATE II.

Fig. 12.

Fig. 13.

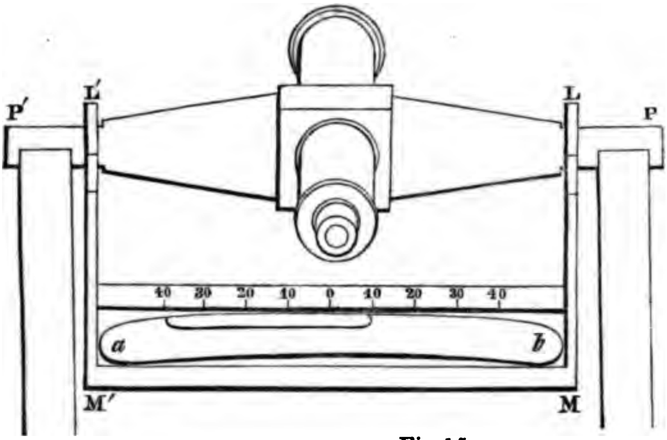
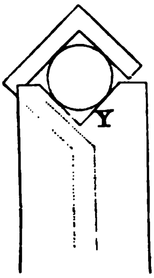
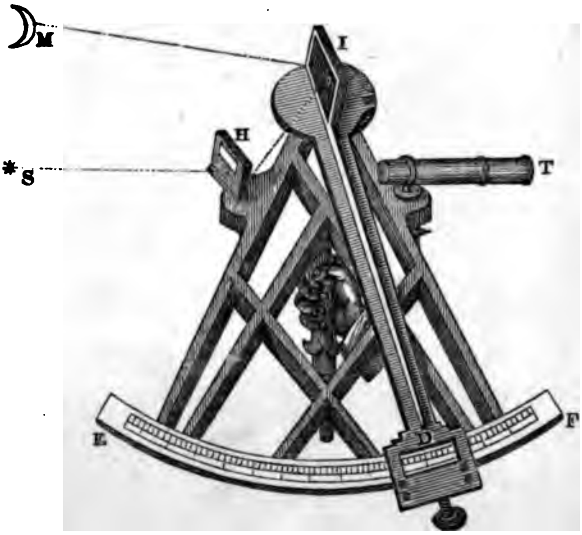


Fig 15.

Fig. 14.





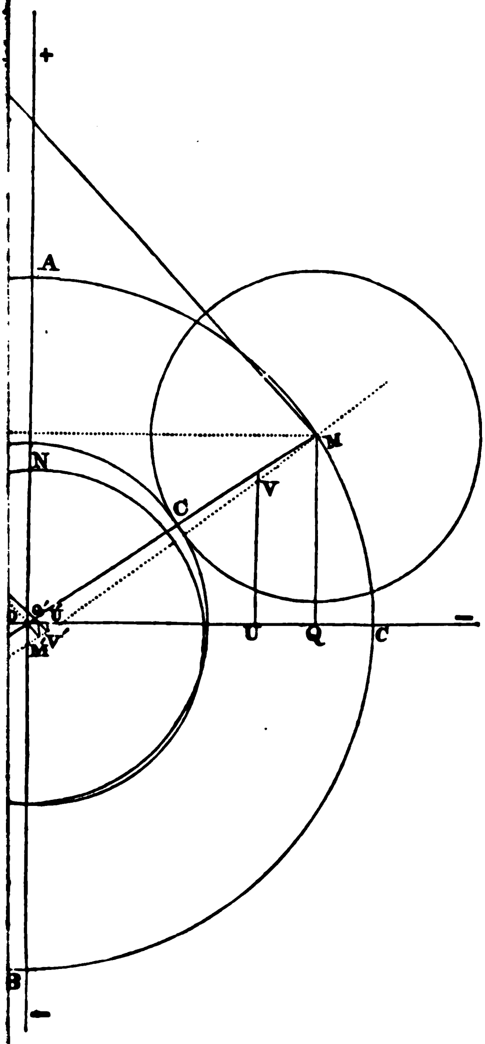

+
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PLATE IV.

Fig. 20.

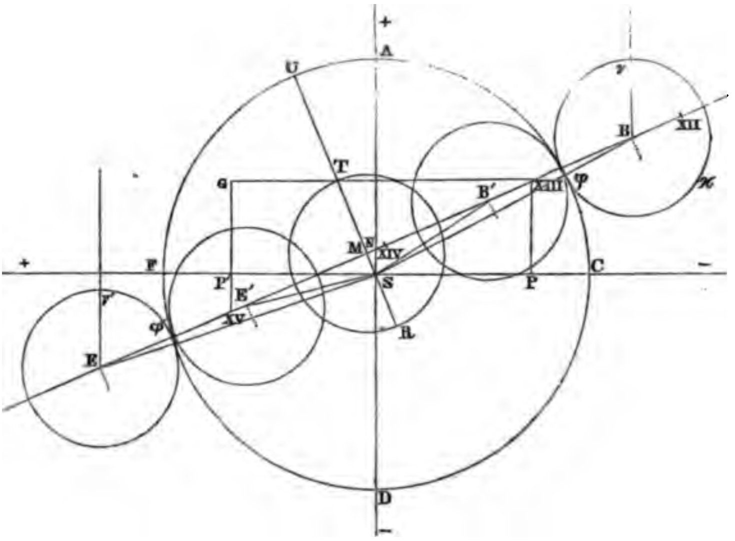


Fig. 21.

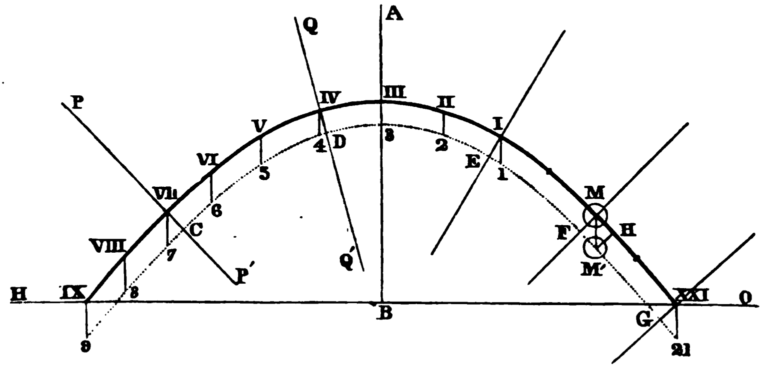


Fig. 22.

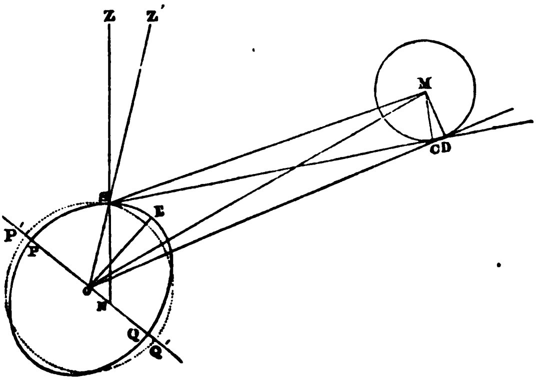


PLATE III.

Fig. 16.

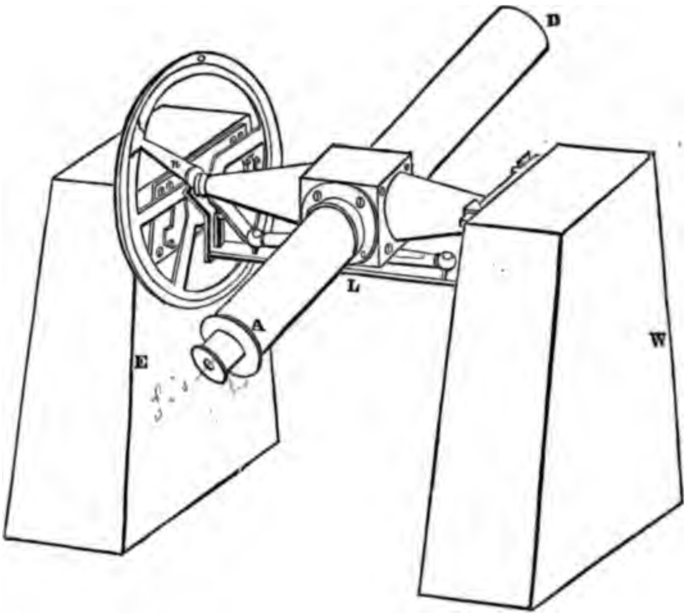


Fig. 17.

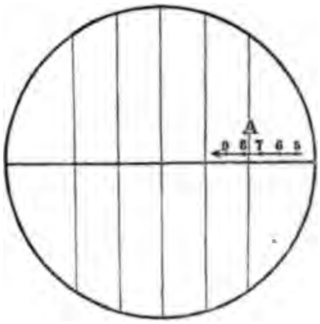


Fig. 18.

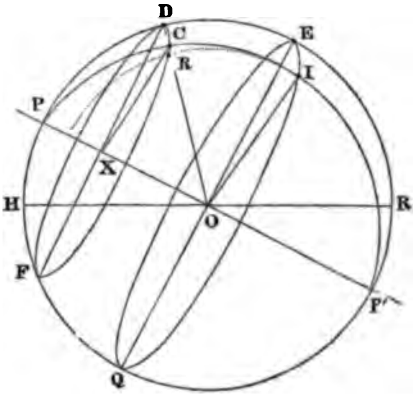


Fig. 19.

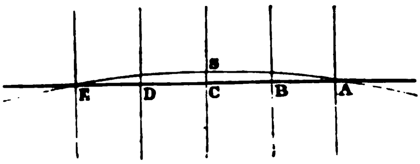


PLATE IV.

Fig. 20.

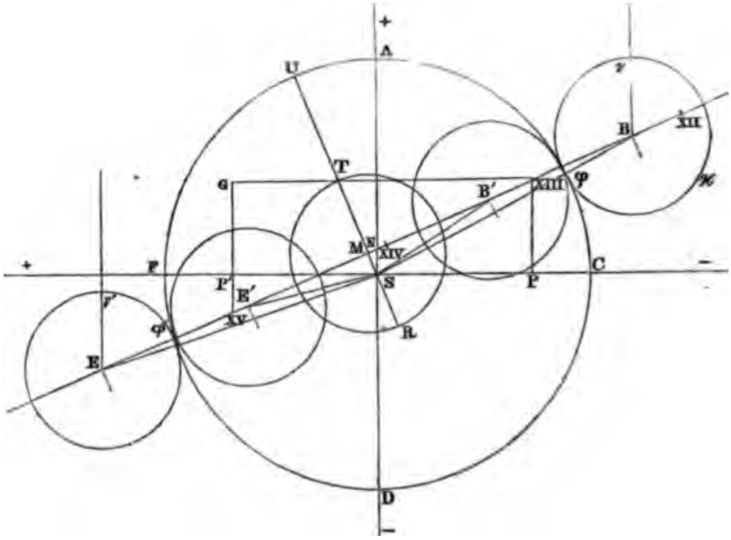


Fig. 21.

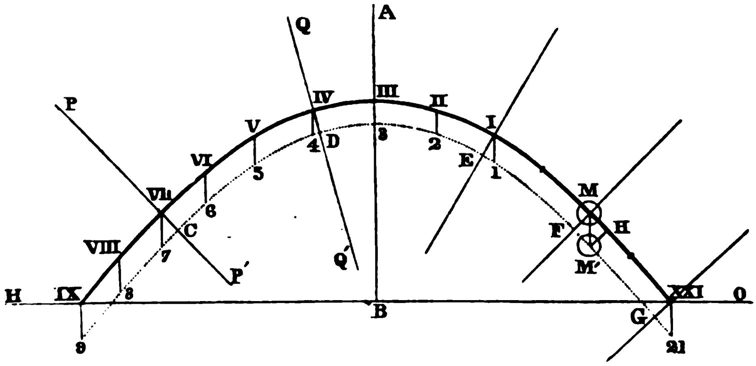


Fig. 22.

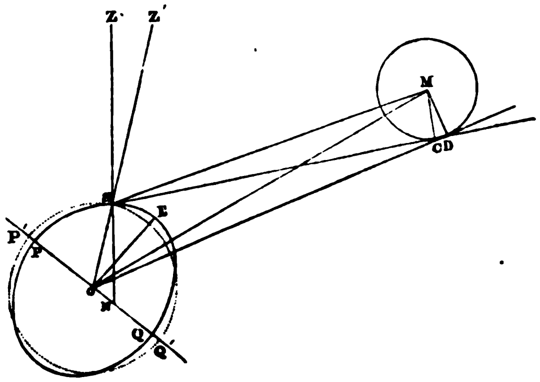


PLATE IV.

Fig. 20.

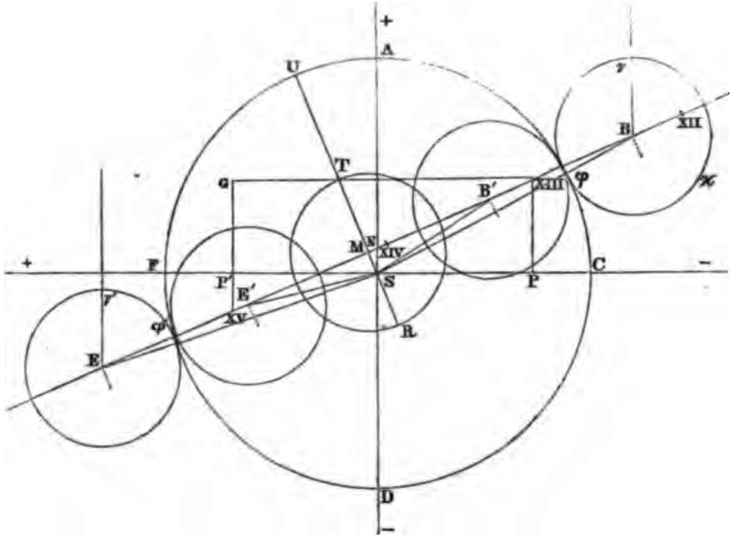


Fig. 21.

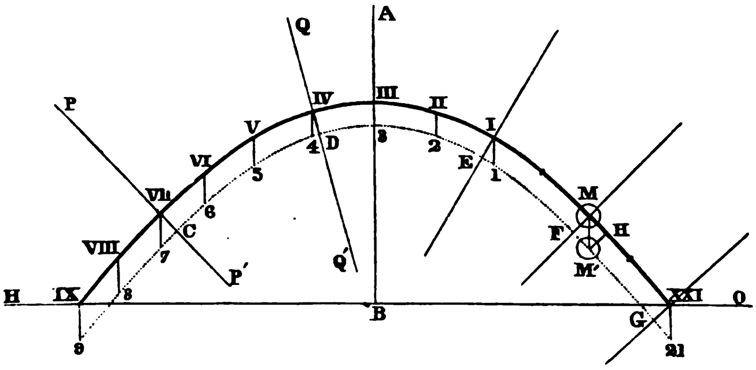
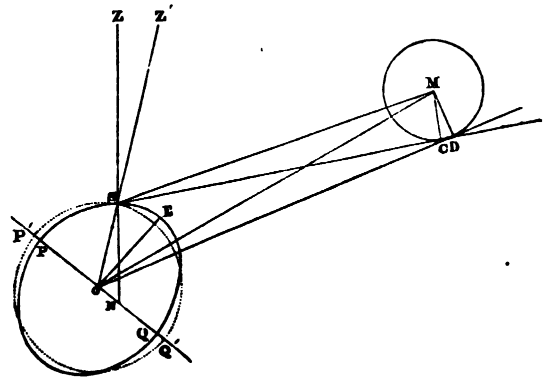
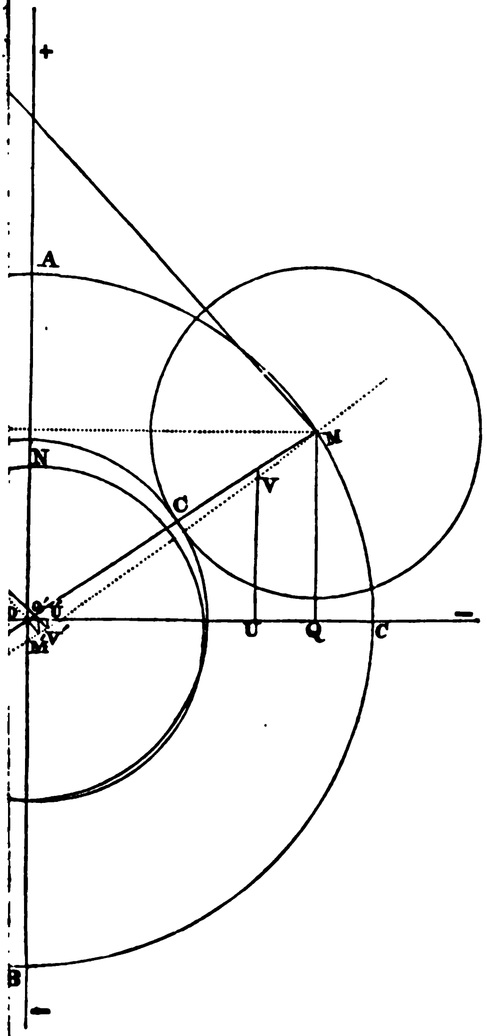


Fig. 22.

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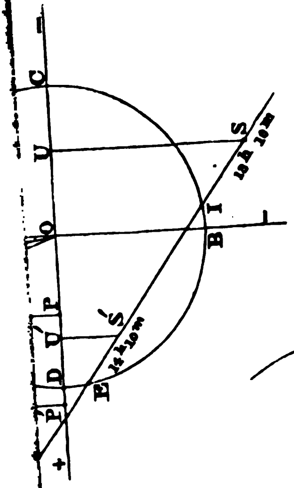
TE VII.

ig. 27.



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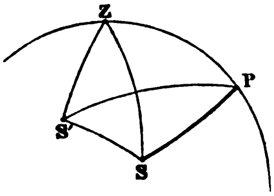
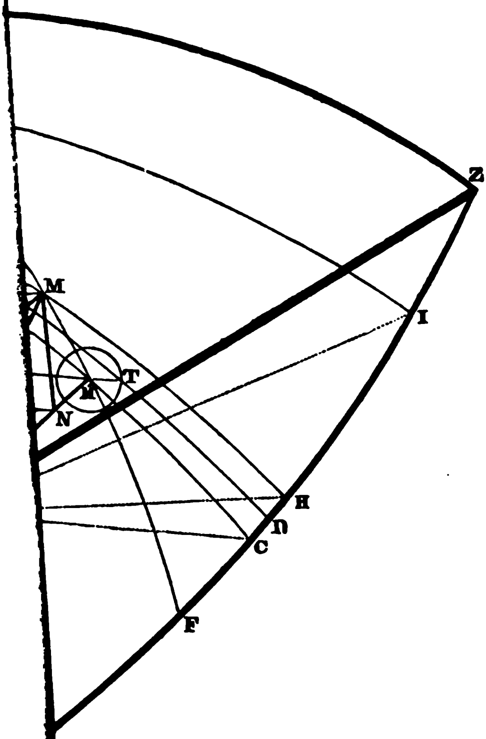


g. 29.



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